THE PRINCIPLES OF MECHANICS.

EXPLAINING and DEMONSTRATING

The general LAWS of MOTION,

The LAWS of GRAVITY,
MOTION of DESCENDING BODIES,
PROJECTILES,
MECHANIC POWERS,
PENDULUMS,
CENTERS of GRAVITY, &c.
STRENGTH and STRESS of TIMBER,
HYDROSTATICS, and
CONSTRUCTION of MACHINES.

A WORK very neceſſary to be known, by all Gentlemen, and Others, that deſire to have an Inſight into the Works of NATURE and ART. And extremely uſeful to all Sorts of ARTIFICERS; particularly to ARCHITECTS, ENGINEERS, SHIPWRIGHTS, MILL-WRIGHTS, WATCHMAKERS, &c. or any that Work in a Mechanical Way.

The THIRD EDITION, CORRECTED.

Illuſtrated with FORTY-THREE COPPER-PLATES.

Multáque per Trochleas, et Tympana pondere magno
Commovet, atque levi ſuſtollit Machina niſu.

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THE

PREFACE.

THE art of MECHANICS *being the first that men had occasion to make use of, it is reasonable to suppose that it took its beginning with man; and was studied in the earliest ages of the world. For no sooner did mankind begin to people the earth, than they wanted houses to dwell in, cloaths to wear, and utensils to till the ground, to get them bread, with other necessaries of life; and being thus destitute of proper habitations, and other conveniences of living, their wants must immediately put them upon the study of mechanics. At their first setting out, they would be content with very little theory; endeavouring to get that more by experience than reasoning, and being unacquainted with numbers, or any sort of calculation; and having neither rule nor compass to work by, nor instruments to work with, but such as they must invent first of all, nor any methods of working: with all these disadvantages, we may judge what sort of work they were likely to make. All their contrivances must be mere guessing, and they could but ill execute what they had so badly contrived; and must be continually mending their work by repeated trials, till they got it to such a form as to make a shift to serve for the use designed. And this is the first and lowest state of mechanics, which was enough to give a beginning to it; and in this state it doubtless remained for a long time, without much improvement. But at length as men found more leisure and opportunity, and gained more experience, manual arts began to take their rise, and by degrees to make some progress in the world.*

But we meet with no considerable inventions in the mechanical way, for a long series of ages; or if there had been any, the accounts of them are now lost, through the length of time; for we have nothing upon record for two or three thousand years forward. But afterwards we find an account of several machines that were in use. For we read in Genesis *that ships were as old, even on the Mediterranean, as the days of* Jacob. *We likewise read that the* Philistines *brought 30 thousand chariots into the field against* Saul; *so that chariots were in use 1070 years before* Christ. *And about the same time architecture was brought into* Europe. *And*

1030 *years before* Christ, Ammon *built long and tall ships with sails, on the Red Sea and the Mediterranean. And about 90 years after, the ship* Argo *was built; which was the first* Greek *vessel that ventured to pass through the sea, by help of sails, without sight of land, being guided only by the stars.* Dædalus *also, who lived* 980 *years before* Christ, *made sails for ships, and invented several sorts of tools, for carpenters and joiners to work with. He also made several moving statues, which could walk or run of themselves. And about* 800 *years before* Christ, *we find in* 2 Chron. xv. *that* Uzziah *made in* Jerusalem, *engines invented by cunning men, to be on the towers and upon the bulwarks, to shoot arrows and great stones withal. Corn mills were early invented; for we read in* Deuteronomy, *that it was not lawful for any man to take the nether or the upper mill stone to pledge; yet water was not applied to mills before the year of* Christ 600, *nor wind mills used before the year* 1200. *Likewise* 580 *years before* Christ, *we read in* Jeremiah xviii. *of the potter's wheel.* Architas *was the first that applied mathematics to mechanics, but left no mechanical writings behind him: he made a wooden pigeon that could fly about.* Archimedes, *who lived about* 200 *years before* Christ, *was a most subtle geometer and mechanic. He made engines that drew up the ships of* Marcellus *at the siege of* Syracuse; *and others that would cast a stone of a prodigious weight to a great distance, or else several lesser stones, as also darts and arrows; but there have been many fabulous reports concerning these engines. He also made a sphere which showed the motions of the sun, moon, and planets. And* Posidonius *afterwards made another which shewed the same thing. In these days the liberal arts flourished, and learning met with proper encouragement; but afterwards they became neglected for a long time.* Aristotle, *who lived about* 290 *years before* Christ, *was one of the first that writ any methodical discourse of mechanics. But at this time the art was contained in a very little compass, there being scarce any thing more known about it, than the* 6 *mechanical powers. In this state it continued till the* 16*th century, and then clock work was invented, and about* 1650 *were the first clocks made. At this time several of the most eminent mathematicians began to consider mechanics. And by their study and industry have prodigiously enlarged its bounds, and made it a most comprehensive science. It extends through heaven and earth, the whole universe, and every part of it is its subject. Not one particle of matter but what comes under its laws. For what else is there in the visible world, but matter and motion; and the properties and affections of both these, are the subject of mechanics.*

To the art of mechanics is owing all sorts of instruments to work with, all engines of war, ships, bridges, mills, curious roofs and arches, stately theatres, columns, pendent galleries, and all other grand works in building. Also clocks, watches, jacks, chariots, carts and carriages, and even the wheel barrow. Architecture, navigation, husbandry, and military affairs, owe their invention and use to this art. And whatever hath artificial motion by air, water, wind, or cords; as all manner of musical instruments, water-works, &c. This is a science of such importance, that without it we could hardly eat our bread, or lie dry in our beds.

By mechanics we come to understand the motions of the parts of an animal body; the use of the nerves, muscles, bones, joints, and vessels. All which have been made so plain, as proves an animal body to be nothing but a mechanical engine. But this part of mechanics, called anatomy, is a subject of itself. Upon mechanics are also founded the motions of all the celestial bodies, their periods, times, and revolutions. Without mechanics, a general cannot go to war, nor besiege a town, or fortify a place. And the meanest artificer must work mechanically, or not work at all. So that all persons whatever are indebted to this art, from the king down to the cobler.

Upon mechanics is also founded the Newtonian, *or only true philosophy in the world. For all the difficulty of philosophy consists in this; from some of the principal phænomena of motions to investigate the forces of nature. And then from these forces to demonstrate the other phænomena; all which is to be done upon mechanical principles. Thus from the distances and revolutions of the heavenly bodies, the forces of gravity are derived; and from these forces thus known, are deduced the motions of the planets, comets, the moon, and the sea; as well as the motions of bodies upon the surface of the earth. These relate to the visible bodies of the universe. But there are also certain forces belonging to the small particles of matter, which we are still ignorant of; by which they are either impelled towards one another, and cohere in regular figures; or are repelled, and so recede from each other. For the particles of different sorts of bodies have different laws; since the small particles of some bodies attract one another, whilst those of other sorts repel each other; and that by forces almost infinitely various. Upon these forces the cohesion, solidity, and fluidity of bodies depend. The nature of elasticity, electricity, and magnetism. Upon these also depend the principles of fermentation, putrefaction, generation, vegetation, and dissolution of bodies; digestion and secretion in animal bodies; the motion of the blood and fluids in animals,*

mals, and the moving of the members by the command of the will; the exciting sensations in the mind; the emission, reflection, refraction, and inflection of light; freezing by cold, burning by fire; all operations in chemistry, &c. If these forces could be found out, it would open to us a new field in the science of mechanics. But for want of proper experiments, these forces among the invisible and imperceptible particles of matter, are utterly unknown, and exceeding difficult to be discovered, and therefore make no part of the ensuing Treatise. Nor shall I meddle with astronomy, as being a subject of itself: nor with experimental philosophy, any further than concerns mechanics.

And although architecture has a great dependance upon mechanics; yet there are a great many precarious rules in this art, invented purely for ornament, and the sake of beauty; which have nothing to do with mechanics. And therefore mechanical beauty (that is, strength in due proportion) is all that I have any business to meddle with here.

It has been ignorantly objected by some, that the Newtonian philosophy, like all others before it, will grow old and out of date, and be succeeded by some new system, which will then be as much cried up as this is now. But this objection is very falsly made. For never a philosopher before Newton ever took the method that he did. For whilst their systems are nothing but hypotheses, conceits, fictions, conjectures, and romances, invented at pleasure, and without any foundation in the nature of things. He on the contrary, and by himself alone, set out upon a quite different footing. For he admits nothing but what he gains from experiments, and accurate observations. And from this foundation, whatever is further advanced, is deduced by strick mathematical reasoning. And where this thread does not carry him, he stops, and proceeds no further; not pretending to be wise above what is written in nature. Being rather content with a little true knowledge, than, by assuming to know every thing, run the hazard of error. Contrary to all this, these scheming philosophers, being men of strong imaginations and weak judgments, will run on ad infinitum, and build one fiction upon another, till their Babel thus erected, proves to be nothing but a heap of endless confusion and contradiction. And then it is no wonder, if the whole airy fabric tumbles down, and sinks into ruin. And yet it seems, such romantic systems of philosophy, will please some people as well as the strictest truth, or most regular system. As if philosophy, like religion, was to depend on the fashion of the country, or on the fancies and caprice of weak people. But surely this is nothing but rambling in the dark, and saying that the nature of things depends upon no steady principles at all. But

in truth, the business of true philosophy is to derive the nature of things from causes truly existent; and to enquire after those laws on which the Creator choosed to found the world; not those by which he might have done the same, had he so pleased. It is reasonable to suppose that from several causes, something differing from each other, the same effect may arise. But the true cause will always be that from which it truly and actually does arise; the others have no place in true philosophy. And this can be known no way, but by observations and experiments. Hence it evidently follows, that the Newtonian *philosophy, being thus built upon this solid foundation, must stand firm and unshaken; and being once proved to be true, it must eternally remain true, until the utter subversion of all the laws of nature. It is therefore a mere joke to talk of a new philosophy. The foundation is now firmly laid: the* Newtonian *philosophy may indeed be improved, and further advanced; but it can never be overthrown: notwithstanding the efforts of all the* Bernouilli's, *the* Leibnitz's, *the* Green's, *the* Berkley's, *the* Hutchinson's, *&c. And even the* French *themselves have at last adopted it, and given up the* Cartesian *scheme*

Practical mechanics might be very much improved, if the secrets of all trades were to lie open; and the several machines used in each trade, duly explained. And experimental philosophy would be thereby much improved, as well as the trades themselves. And one trade might borrow many great helps in working, from another trade. It is not my design to treat at all on the lowest part of mechanics, which concerns manual arts or working by hand. For there is no theory required here, but only a habit of working, to be acquired by frequent practice.

I have, in the following book, confined myself entirely to the mathematical part, and what depends on it, and is deduced from it. And therefore I have first of all laid down and demonstrated the general laws of motion, as a foundation for all the rest. Then follows the laws of gravity, the descent of bodies, and motion of projectiles in free space; the mechanical powers; the descent of bodies upon inclined planes, the vibration of pendulums; center of gravity, and others; the pressure, strength, and stress, of beams of timber; then you have the principles of hydrostatics, hydraulics, and pneumatics; the resistance of fluids; the powers of engines, and the description of machines. In each of these, I have delivered all the fundamental principles both in theory and practice. And to make it more universally useful, I have demonstrated every thing geometrically; or at most, by the help only of the lowest and easiest rules of algebra, for the sake of brevity; avoiding all operations by fluxions: so that the reader need not be scared with the thoughts of any difficulty of that kind.

Con-

Concerning the machines, I have given an account of their structure, as far as is necessary to explain their motions and effects; omitting the description of their minuter parts, not so necessary for this end, and which are easily understood of themselves. I hope the reader will pardon my inserting among the rest, some machines that may seem trifling, put in here and there to fill up vacant places in the schemes. Yet even in these, there is something curious in their structure or motion, that may be worth observing. I might have given the cuts of many more machines; but perhaps what I have already done, may be thought too much, in such a nation as this, where natural knowledge wants due encouragement, and where no Mæcenas *appears to patronize and protect it; and where arts and sciences hang, as it were, in suspence, whether they shall stand or fall; and where public spirit and* English *generosity are just expiring. This decline of arts and sciences, is wholly, or in a great measure owing to the ambition, and most extreme avarice of the present age. Where men, not being able to lift their eyes above this earth, think nothing worth their care, but raking together the dross it affords; striving, like the toad, who shall die with the most earth in his paws. The duller part of mankind are entirely engaged in the pursuit of filthy lucre; and the brighter sort are wholly devoted to low, trifling, and often barbarous diversions. In such momentous concerns as these, it is no wonder if arts and sciences flag; and natural knowledge meet with nothing but contempt; and* Minerva *give place to* Plutus. *And indeed, if the general temper and disposition of men had been the same in all ages, as it is in this; I am in doubt whether we had ever had any such thing as a mill to grind us corn for bread, or a pump to draw us water. It is a trifling excuse for men of exalted station, to urge, that they are unacquainted with such arts or sciences. For learning has always been esteemed to be under the peculiar care and superintendency of the* great; *who ought to protect and encourage both that, and the professors of it; or else arts and sciences can never flourish. And as the encouragement of these evidently tends to the benefit of mankind, and the promoting of the public good; nothing can excuse so gross a neglect, or such a manifest disregard, as they shew, for the happiness of their fellow creatures. The industrious students only have the fatigue; whilst all the world reap the advantage of their labours:*

Scire volunt omnes, mercædem solvere nemo. JUVEN.

It is reported of Alexander *that he allowed* 800 *talents a year to* Aristotle, *to defray the expences of procuring all sorts of living creatures; so that by his own particular experience, he might be enabled*

enabled to write of the nature and properties of them. And the reason why the world hath now so few Aristotles, *is because there are no* Alexanders.

But as to the fate of this book, it is indifferent to me what reception it shall meet with in the world. Not that I am in the least diffident of the principles here delivered; for I know they will stand the strictest examination. Nor would I be thought careless concerning the advantage, which my few readers may receive from it: for on the contrary, I have done all I could to instruct them, and lead them regularly through this noble and useful science. But in a mercenary age, where there is so little encouragement for works of this nature, I am under no concern what judgment may be passed upon it by the ignorant multitude. Yet I sincerely wish, that my more ingenious readers may find what they expect here; and am in hopes, that they will meet with no difficulties, but what they will easily surmount. To effect which, I have made every thing in this book as full and clear as my own abilities, and the nature of the thing would permit me.

W. EMERSON.

P. S. *The kind reception the former editions of this book has met with from the publick, has induced me to revise and correct it; and to make such further additions thereto, as I thought necessary for completing so useful a science. Accordingly I have made such alterations and improvements in this third edition, as to me seemed absolutely necessary for the benefit of my readers.*

I cannot find that I have omitted any thing material in this Treatise pertinent to my subject. And as my professed design was to write a book of Principles, *no body of common sense would expect that I should go contrary to that design, and stuff my book with calculations of difficult problems, fit only for books of Algebra and Fluxions; and not at all proper for an* elementary Treatise. *If I had done this, I had not only deserted my subject, and all good method; but also imposed upon my readers, by swelling the book, and making them pay dear for* superfluities; *and that to please a few trifling critics, who know no better. These sort of animals live by detraction, and are always* snarling *at what they don't understand, and cannot mend.*

If any such set of critics, *assuming the privilege of being* dictators *to the public, and* censors *of other men's writings, should start up, and cry down my book, because not written after their crude notions, I would not have my readers be at all surprized at this:*

this: for it would be more surprizing if any thing of value should escape, without being degraded and condemned by them. And for my own part, the only concern it will give me, will be to despise their dull criticisms, and laugh at their ignorance.

Men' moveat cimex Pantilius? aut crucier, quòd
Vellicit absentem Demetrius? aut quòd ineptus
Fannus Hermogenis lœdat Conviva Tigelli?
Plotius, & Varius, Mæcenas, Virgiliusque,
Valgius, —— probet hæc ————

HOR.

W. E.

THE

THE

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SECT.

EXPLANATION of the CHARACTERS.

$\propto$ as, or in a given proportion to.
Perp. perpendicular.
Rad. radius.
S. the ſine.
Coſ. the coſine.
Tan. the tangent.
Cotan. the cotangent.
Sec. the ſecant.

MECHANICS.

MECHANICS.

DEFINITIONS.

1. MECHANICS is a ſcience, which teaches the proportion of the forces, motions, velocities, and in general the actions of bodies upon one another.

2. *Body* is the maſs or quantity of matter. If a body yields to a ſtroke and recovers its former figure again, it is called an *elaſtic body:* If not, it is *inelaſtic.*

3. *Denſity* of a body is the proportion, of the quantity of matter contained in it, to the quantity of matter in another body of the ſame bigneſs. Thus the denſity is ſaid to be double or triple, when the quantity of matter contained in the ſame ſpace is double or triple.

4. *Force* is a power exerted on a body to move it. If it act but for a moment it is called the force of *percuſſion* or *impulſe.* If it act conſtantly, it is called an *accelerative force:* If conſtantly and equally, it is called a *uniform accelerative force.*

5. *Velocity* is an affection of motion, by which a body paſſes over a certain ſpace in a given time. The velocity is ſaid to be greater or leſs, according as the body paſſes over a greater or leſs ſpace in the ſame time.

6. *Motion* is a continual and ſucceſſive change of place. If a body moves through equal ſpaces in equal times, it is called *equable motion.* If its velocity continually increaſes, it is called *accelerated motion*; if it decreaſes, it is *retarded motion.* If it increaſes or decreaſes uniformly, it is *equally accelerated* or *retarded.* Likewiſe if its motion be conſidered in regard to ſome other body at reſt, it is called *abſolute motion.* But if its motion be conſidered with reſpect to other bodies alſo in motion, then it is *relative motion.*

7. *Direction of motion* is the way the body tends, or the right line it moves in.

8. *Quantity of motion*, is the motion a body has, both in regard to its velocity, and quantity of matter. This is called the *momentum* of a body, by ſome mechanical writers.

9. *Vis inertiæ*, is that innate force of matter by which it reſiſts any change, and endeavours to preſerve its preſent ſtate of motion or reſt.

10. *Gravity* is that force wherewith a body endeavours to deſcend towards the center of the earth. This is called *abſolute gravity*, when the body tends downwards in free ſpace: and *relative gravity* is the force it endeavours to deſcend with in a fluid.

11. *Specific gravity* of bodies, is the greater or leſs weight of bodies of the ſame magnitude; or the proportion between theſe weights. The ſpecific gravity is ſaid to be double or triple, when the weight of the ſame bulk of matter is double or triple.

12. *Center of gravity* of a body is a certain point in it, upon which the body being freely ſuſpended, it would reſt in any poſition.

13. *Center of motion* of a body, is a fixed point about which the body is moved. And the *axis of motion*, is the fixed axis it moves about.

14. *Weight and power* when oppoſed to one another, ſignify the body to be removed, and the body that moves it. That body which communicates the motion is called the *power*; and that which receives it, the *weight*.

15. *Equilibrium* is when two or more forces acting againſt one another, none of them overcome the others, but deſtroy one another's effects, and remain at reſt.

16. *A fluid* is a body whoſe parts yield to any impreſſed force; and by yielding are eaſily moved among themſelves.

17. *Hydroſtatics* is a ſcience that treats of the properties of fluids.

18. *Hydraulics* is the art of raiſing or conveying water by the help of engines.

19. *Pneumatics* is a ſcience that treats of the properties of the air.

20. *Machine* is any mechanical inſtrument contrived to move bodies, or to perform ſome particular motions. The mechanical powers are ſimple machines.

21. *Engine* is a mechanic inſtrument compoſed of levers, wheels, pullies, ſcrews, &c. in order to move, lift, or ſuſtain ſome great weight, or perform ſome great effect. This is the largeſt and moſt compounded ſort of machines.

22. Me-

22. *Mechanic powers*, are the ballance, the lever, the wheel, the pulley, the ſcrew, and the wedge. To which ſome add the inclined plane.

23. *Streſs* is the effect of a force acting againſt a beam, or any thing to break it, or the violence it ſuffers by that force. The contrary to this is *ſtrength*, which is the reſiſtance any beam is able to make againſt a force endeavouring to break it.

24. *Friction* is the reſiſtance ariſing from the parts of machines, or of any bodies rubbing againſt one another.

POSTULATA.

1. THAT a ſmall part of the ſurface of the earth, or the horizon, may be looked upon as a plane. Though this is not ſtrictly true, yet it differs inſenſibly in ſo ſmall a ſpace as we have any occaſion to conſider it.

2. Heavy bodies deſcend in lines parallel to one another, and perpendicular to the horizon: And they always tend perpendicular to the horizon by their weight. For this is true as to ſenſe, becauſe the lines of their direction meet only at the center of the earth, taken as a perfect ſphere.

3. The weight of any body is the ſame in all places at or near the ſurface of the earth. For the difference is inſenſible at any heights to which we can aſcend. Though in ſtrictneſs the force of gravity decreaſes in aſcending, from the earth's ſurface, in the reciprocal ratio of the ſquares of the heights from the earth's center.

4. We are to ſuppoſe all planes perfectly even and regular, all bodies perfectly ſmooth and homogeneous; and moving without friction or reſiſtance; lines perfectly ſtreight, and inflexible, without weight or thickneſs; cords extremely pliable; *&c.* For tho' bodies are defective in all theſe; and the parts or matter, whereof engines are made, ſubject to many imperfections; yet me muſt ſet aſide all theſe irregularities, till the theory is eſtabliſhed; and afterwards make ſuch allowance as is proper.

AXIOMS.

1. EVERY body preſerves in its preſent ſtate, whether of reſt, or moving uniformly in a right line; till it is compelled to change that ſtate, by ſome external force.

2. The alteration of motion, or the motion generated, or deſtroyed in any body, is proportional to the force applied: And is made in the direction of that right line in which the force acts.

3. The action and re-action between two bodies are equal, and in contrary directions.

4. The motion of the whole body is made up of the sum of the motions of all the parts.

5. The weights of all bodies in the same place, are proportional to the quantities of matter they contain; without any regard to their bulk, figure, or kind. For twice the matter will be twice as heavy, and thrice the matter thrice as heavy; and so on.

6. The vis inertiæ of all bodies, is proportional to the quantity of matter.

7. Every body will descend to the lowest place it can get to.

8. Whatever sustains a heavy body, bears all the weight of it.

9. Two equal forces acting against one another in contrary directions destroy one another's effects.

10. If a body is acted on with two forces in contrary directions; it is the same thing as if it were only acted on with the difference of these forces, in direction of the greater.

11. If a body is kept in equilibrio; the contrary forces, in any one line of direction, are equal, and destroy one another.

12. Whatever quantity of motion any force generates in a given time; the same quantity of motion will an equal force destroy in the same time; acting in a contrary direction.

13. Any active force will sooner or more easily overcome a lesser resistance than a greater.

14. If a weight be drawn or pushed by any power; it pushes or draws all points of the line of direction equally. And it is the same thing, whatever point of that line the force is applied to.

15. If two bodies be moving the same way in any right line; their relative motion will be the same, as if one body stood still, and the other approached, or receded from it with the difference of their motions: or with the sum of their motions, if they move contrary ways.

16. If a body is drawn or urged by a rope, the direction of that force is the same as the direction of that part of the rope next adjoining to the body.

17. If any force is applied to move or sustain a body, by means of a rope; all the intermediate parts of the rope are equally distended, and that in contrary directions.

18. If a running rope go freely over several pullies; all the parts of it are equally stretched.

19. If any forces be applied againſt one end of a free lever or beam; the other end will thruſt or act with a force, in direction of its length.

20. The parts of a fluid will yield, and recede towards that part where it is leaſt preſſed.

21. The upper part of a fluid is ſuſtained by the lower part.

SECT. I.

The general laws of MOTION.

PROP. I.

The quantities of matter in all bodies are in the complicate ratio of their magnitudes and densities.

FOR by Def. 3. if the magnitudes be equal, the matter will be as the densities. And if the densities be equal, the matter will be as the magnitudes. Therefore the matter is universally in the compound ratio of both.

Cor. 1. *The quantities of matter in similar bodies, are as the densities and cubes of their like dimensions, in a sphere.* The magnitude is as the cube of the diameter.

Cor. 2. *The quantities of matter are as the magnitudes and specific gravities.* For the specific gravities are as the densities, by Ax. 5.

PROP. II.

The quantities of motion, in all moving bodies whatever, are in the complicate ratio of the quantities of matter and the velocities.

For if the velocities are equal, it it manifest (by Ax. 4.) that the quantities of motion will be as the quantities of matter. And if the quantities of matter are equal, the motions will be as the velocities. Therefore universally, the quantities of motion are in the compound ratio of the velocities and quantities of matter.

Cor. *In any sort of motion, the quantity of motion is as the sum of all the products of every particle of matter multiplied by its respective velocity.*

For the quantity of motion of any particle is as that particle multiplied by its velocity: And as each particle of the fame body moves with equal velocity, the motion of the whole will be the fum of the motions of all the parts.

PROP. III.

In all uniform motions, the fpace defcribed is in the complicate ratio of the time and velocity.

For it is evident, if the velocity be given, the fpace defcribed by any body, will be as the time of its moving. And if the time be given, the fpace defcribed will be greater or lefs, according as the velocity is greater or lefs; that is, the fpace will be as the velocity. Therefore if neither be given, the fpace will be in the compound ratio of both the time and velocity.

Cor. The time is as the fpace directly and velocity reciprocally.

PROP. IV.

The motion generated by any momentary force, is as the force that generates it.

For if a certain quantity of force generates any motion, a double quantity of force, will generate double the motion; and a triple force, triple the motion, and fo on.

Cor. The fpace defcribed is as the force and time directly, and quantity of matter reciprocally.

For by this Prop. the force is as the motion, that is (by Prop. II.) as the matter and velocity; therefore the velocity is as the force directly and matter reciprocally. Alfo (by Prop. III.) the fpace is as the time and velocity, and therefore as the time and force directly, and matter reciprocally.

SCHOLIUM.

Let b = *body or quantity of matter to be moved.*
f = *force of impulfe acting on the body b.*
m = *momentum or quantity of motion generated in b.*
v = *velocity generated in b.*
s = *fpace defcribed by the body b.*
t = *time of defcribing the fpace s with the velocity v.*

Then

Then by the three laſt Props. we ſhall have $m \propto bv$, $s \propto tv$, and $f \propto m$. By the help of theſe three general proportions, the relation of the ſpaces, times, velocities, &c. may be found, upon all ſuppoſitions. And thence all the laws and proportions belonging to uniform motion, may be readily and univerſally reſolved: Expunging ſuch as are not concerned in the queſtion; and rejecting all thoſe that are given or conſtant; and alſo thoſe that are in both terms of the proportion. Thus we ſhall get in general,

$$f \propto m \propto bv \propto \frac{bs}{t}.$$
$$m \propto f \propto bv \propto \frac{bs}{t}.$$
$$s \propto tv \propto \frac{tm}{b} \propto \frac{tf}{b}.$$
$$v \propto \frac{m}{b} \propto \frac{s}{t} \propto \frac{f}{b}.$$
$$t \propto \frac{s}{v} \propto \frac{sb}{m} \propto \frac{sb}{f}.$$
$$b \propto \frac{m}{v} \propto \frac{f}{v} \propto \frac{mt}{s} \propto \frac{ft}{s}.$$

PROP. V.

In any motion, generated by a uniformly accelerating force: the motion generated in any time, is in the complicate ratio of the force and time.

For in any given time, the motion generated will be proportional to the force that generates it, this being its natural and genuine effect. And ſince in all the ſeveral parts of time, the force is the ſame, and has the ſame efficacy; therefore the motion generated will alſo be as the time: whence univerſally, the motion generated is in the compound ratio of both the force and the time of acting.

Cor. 1. *This Prop. is equally true in reſpect to the motion loſt or deſtroyed in a moving body, by a force acting in a contrary direction.* By Ax. 12.

Cor.

Cor. 2. *If the space thro' which a body is moved by any force, be divided into an indefinite number of small equal parts; and if, in each part, the accelerative force acts differently upon the body, according to any certain law. And if there be taken the product of the accelerating force in each part multiplied by the time of passing through it. Then I say,*

As any uniform accelerative force × by the time of acting (FT):
To the motion generated in that time (M) ::
So the sum of all the products of each particular force and time:
To the motion generated in this whole time, by the variable force.

For (by this Prop.) the time of describing any part × by the force: motion generated in that time :: $F \times T$: M. therefore by composition, the sum of all the products, to the whole motion generated; is also as FT, to M.

Cor. 3. *The velocity generated (or destroyed) in any time, is as the force and time directly, and the quantity of matter reciprocally.*

For (by Prop. II.) the quantity of motion is as the matter and velocity; therefore the velocity is as the motion directly and the matter reciprocally, that is, (by this Prop.) as the force and time directly, and the matter reciprocally.

Cor. 4. *The increase or decrease of any velocity, generated or destroyed in any time; is as the force and time directly, and the matter reciprocally.*

This follows from Cor. 3. because all effects are proportional to their causes.

PROP. VI.

In any motion generated by a uniform accelerative force; the space described from the beginning of the motion, is in the complicate ratio of the last velocity, and the time wherein it is generated.

For suppose the time divided into an infinite number of equal parts, each of which call 1, and let the times from the beginning be 0, 1, 2, 2, 4 .... to t. And let c be the velocity generated at the end of the time 1; then by Cor. 3. of the last Prop. the velocity generated in the same body, by the same force, in the times 0, 1, 2, 3, 4 .... t, will be respectively 0, c, $2c$, $3c$, $4c$ ... to tc. And by Prop III. the spaces described in each given part of time, will be as the time (1), and the velocity;

and therefore these spaces will be o, c, $2c$, $3c$ . . . tc; and the sum of all these is the whole space described by the body. But the sum of the arithmetic progression, o, c, $2c$, $3c$, $4c$ . . . tc where the number of terms is very great $\frac{o+tc}{2} \times t$, or $\frac{tc}{2} \times t$. but tc is the last velocity, and t the whole time, therefore the whole space is as $(tc \times t)$ the velocity and time conjunctly.

Cor. 1. *If a body moves uniformly forward with the velocity acquired by a uniform accelerative force; it will describe twice the space in the same or an equal time, that it described by the accelerative force.*

For here the spaces described in the several small parts of time, will be tc, tc, tc . . &c. to t terms. whose sum is $tc \times t$. And the sum of the former spaces, described by the accelerating force, was $\frac{tc}{2} \times t$. but $tc \times t$ is to $\frac{tc}{2} \times t$, as 2 to 1.

Cor. 2. *Since any active force is an endeavour of putting a body into motion; therefore the adequate and immediate effect of an accelerating force is either motion, or pressure, or both.*

SCHOL.

Let b = *body or quantity of matter.*
F = *accelerative force acting uniformly and equally on the body b.*
v = *velocity generated in b by the force F.*
m = *motion generated in b.*
s = *space described by b.*
t = *time of describing the space s.*

Then the two last propositions together with Prop. II. will resolve all questions relating to the times, forces, velocities, &c. in uniformly accelerated motions. Thus $m \propto bv$, $m \propto Ft$, and $s \propto tv$. whence we have in general,

$$b \propto \frac{m}{v}, \text{\&c.}$$

$$m \propto bv \propto Ft, \text{\&c.}$$

$$v \propto \frac{s}{t} \propto \frac{Ft}{b} \propto \sqrt{\frac{Fs}{b}} \propto \frac{m}{b},$$

$$s \propto tv \propto \frac{bvv}{F} \propto \frac{Ftt}{b},$$

$t \propto$

FIG.

$$t \propto \frac{s}{v} \propto \frac{bv}{F} \propto \sqrt{\frac{bs}{F}} \propto \frac{m}{F},$$

$$F \propto \frac{bv}{t} \propto \frac{bs}{tt} \propto \frac{bvv}{s} \propto \frac{m}{t}, \text{ \&c.}$$

Whence, if m, b, &c. be given; such quantities must be left out.

PROP. VII.

If any force acting on a body at A in direction AB, cause the body to pass through the space AB in any time; and another similar force acting in direction AC, would move it through the space AC in the same time: I say, by both forces acting together, in their proper directions, the body will, in the same time, be moved through the space AD, the diagonal of the parallelogram ABDC. 1.

CASE I.

Let the forces at A cause the body to move uniformly along the lines AB, AC. Then since the force acting in direction AC parallel to BD, by Ax. 2, will not alter the velocity towards the line BD; the body therefore will arrive at BD in the same time, whether the force in direction AC be impressed or not: Therefore, at the end of the time, it will be found somewhere in BD. By the same argument, it will be found somewhere in the line CD; therefore it will be found in D the point of intersection; and by Ax. 1, it will move in a right line from A to D.

Otherwise.

Suppose the line AC to move parallel to itself into the place BD, whilst A moves from A to C. Then since this line and the body are both equally moved towards BD, it is plain the body must be always in the moveable line AC. Therefore when AC comes to the position bg, let the body be arrived at d; then, since both the line AC moves uniformly along AB, and the body A along AC; therefore it will be as $Ab : bd :: AB : BD$, therefore AdD is a right line

CASE II.

Let the body be carried through AB, AC by an accelerative force. Then by Prop. VI. the space described will be as the time and velocity, and therefore the velocity will be as the space directly

FIG. 1. directly and time reciprocally. Alſo by Cor. 3. Prop. V. when the force and the body is given, the velocity is as the time. Whence the time will be as the ſpace directly and time reciprocally, and the ſpace as the ſquare of the time. That is, the ſame body acted on by the ſame force will deſcribe ſpaces which are as the ſquares of the times. Now let the time of deſcribing AB or AC be 1; and let the line AC move along with the body, always parallel to itſelf, and in the time t, let it arrive at bg, and the body at d, moving towards g. Then, from what has been ſaid, it is as $1 : tt :: AB : Ab$, and alſo as $1 : tt :: bg$ or $BD : bd$. whence $AB : Ab :: BD : bd$. therefore AdD is a right line.

And if you ſuppoſe the ſpace to be as the nth power of the time, it will ſtill be, $1 : t^n :: AB : Ab :: BD : bd$; and AdD is ſtill a right line for any ſimilar forces.

CASE III.

6. But if the body A be carried through AD by a uniform force, in the ſame time that it would be carried through AM by an accelerative force; then by both forces acting together, it will, at the end of that time, be found in the point H, of the parallelogram $MADH$; but then the line it deſcribes AGH, will not be a right line.

1. *Cor. 1. The forces, in the directions AB, AC, AD, are reſpectively proportional to the lines, AB, AC, AD; and in theſe directions.*

For by Cor. Prop. IV. the time and the quantity of matter being given; the force is directly as the ſpace deſcribed. And in accelerated motion, the ſame is true, by Prop. V. Cor. 3. and Prop. VI.

Cor. 2. The two oblique forces AB, AC is equivalent to the ſingle direct force AD, which may be compounded of theſe two, by drawing the diagonal of the parallelogram AD.

Cor. 3. Any ſingle direct force AD, may be reſolved into the two oblique forces whoſe quantities and directions are AB, AC, having the ſame effect; by deſcribing any parallelogram whoſe diagonal is AD.

Cor. 4. A body being agitated by two forces at once, will paſs through the ſame point, as it would do if the two forces were to act ſeparately and ſucceſſively. And if any new motion be impreſſed on

4 *a body*

a body already in motion, is does not alter its motion in lines parallel to its former direction. FIG.

Cor. 5. *If two forces as AB, AC, act in the directions AB, AC, respectively; draw AR to the middle of the right line BC, and 2 AR is the force compounded out of these, and AR its direction.* 2.

PROP. VIII.

Let there be three forces A, B, C of the same kind, acting against one another, at the point D, and whose directions are all in one plane; and if they keep one another in equilibrio, these forces will be to each other respectively, as the three sides of a triangle drawn parallel to their lines of direction, DI, CI, CD. 3.

Let *DC* represent the force *C*, and produce *AD*, *BD*, and complete the parallelogram *DICH*: And by the last Prop. the force *DC* is equivalent to the two forces *DH*, *DI*; put therefore, the forces *DH*, *DI* instead of *DC*, and all the forces will still be in equilibrio. Therefore by Ax. 11. *DI* is equal to its opposite force *A*, and *DH* or *CI* equal to its opposite force *B*. Therefore the three forces *A*, *B*, *C* are respectively as *DI*, *CI*, *CD*.

Cor. 1. *Hence the forces A, B, C are respectively as the three sides of a triangle, drawn perpendicular to their lines of direction, or in any given angle to them, on the same side.* For such a triangle will be similar to the former triangle.

Cor. 2. *The three forces ABC will be to each other as the sines of the angles through which their respective lines of direction do pass, when produced:*

For $DI : CI :: S.DCI$ or $CDB : S.DCI$ or CDA.

And $CI : CD :: S.CDI$ or $CDA : S.CHD$ or HDI or ADB.

Cor. 3. *If there be never so many forces acting against any point in one plane, and keep one another in equilibrio; they may be all reduced to the action of three, or even of two equal and opposite ones.*

For if *HD*, *ID* be two forces, they are equivalent to the single force *DC*. and in like manner *A* and *B* may be reduced to a single force.

Cor. 4. *And if ever so many forces in different planes, acting against one point, keep one another in equilibrio; they may be all reduced to*

FIG. *to the actions of several forces in one plane, and consequently to two equal and opposite ones.*

3. For if the four forces *A*, *B*, *H*, *I*, act against the point *D*; and *H*, *I*, be out of the plane *ABD*. Let *DC* be the common section of the planes *ADB*, *HDI*; then the forces *H*, *I*, are reduced to the force *C*, in the plane *ADB*.

SCHOL.

This Prop. holds true of all forces whatever; whether of impulse or percussion, thrusting, pulling, pressing; or whether instantaneous or continual; provided they be all of the same kind.

Hence if three forces act in one plane, their proportions are had; and if one force be given, the rest may be found. And if four forces act, and two be given, the other two may be found; but if only one be given, the rest cannot be found; for in the three forces, *A*, *B*, *C*, the force *C* may be divided into other two, an infinite number of ways, by drawing any parallelogram *DICH* about the diagonal *DC*. And in general if there be any number of forces acting at *D*, and all be given but two, these two may be found: otherwise not, though the positions of them all be given.

PROP. IX.

If one body acts against another body, by any kind of force whatever, it exerts that force in the direction of a line perpendicular to the surface whereon it acts.

4. Let the body *B* be acted on by the force *AB* in the direction *AB*. Let the body *C* and the obstacle *O*, hinder the body *B* from moving; divide the force *AB* into the two forces *AD*, *AE*, or *EB*, *DB*, by Cor. 5. Prop. VII. the one perpendicular, the other parallel to the surface *DB*; then the surface *DB* receives the perpendicular force *EB*, and the obstacle *O* the parallel force *DB*; take away the obstacle *O*, and the force *DB* will move the body *B* in a direction parallel to the surface, with no other effect than what arises from the friction of *B* against that surface, occasioned by the pressure of *B* against it by the force *EB*; which, if the surface be perfectly smooth and void of tenacity, will be nothing. The force *DB* therefore having no effect, the remaining force *EB* will be the only one, whereby the body *B* acts

against

againſt the ſurface *DB*, and that in direction *EB* perpendicular to it. FIG.

Cor. 1. *If a given body B ſtrike another body C obliquely, at any angle ABD, the magnitude of the ſtroke will be directly as the velocity and the ſine of the angle of incidence ABD; and the body C receives that ſtroke in the direction EB perpendicular to the ſurface DB.* 4.

For if the angle *ABD* be given, the ſtroke will be greater in proportion to the velocity; and if the velocity *AB* be given, the ſtroke will be as *AD*, or *S.* < *ABD*: Or the magnitude of the ſtroke is as the velocity wherewith the body approaches the plane.

Cor. 2. *If a perfectly elaſtic body A impinges on a hard or elaſtic body CB at B, it will be reflected from it, ſo that the angle of reflection will be equal to the angle of incidence.* 5.

For the motion at *B* parallel to the ſurface is not at all changed by the ſtroke. And becauſe the bodies are elaſtic they recover their figure, in the ſame time they loſe it by the ſtroke; therefore the velocity in direction *BE* is the ſame after as before the ſtroke. Let *AE*, *BE* repreſent the velocities before the ſtroke, and *ED* (= *AE*) and *BE* the reſpective velocities after the ſtroke; then in the two ſimilar and equal triangles *AEB*, *BED*, < *ABE* is equal to *EBD*.

But ſince no bodies in nature are perfectly elaſtic, they are ſomething longer of regaining their figure; and therefore the angle *DBF* will be ſomething more acute than the < *ABG*.

Cor. 3. *If one given body impinges upon another given body; the magnitude of the ſtroke will be as the relative velocity between the bodies.*

For the magnitude of the ſtroke is as the line *BE*, or the velocity wherewith the bodies approach each other; that is, as the relative velocity. 4.

Cor. 4. *And in any bodies whatever, if a body in motion ſtrike againſt another, the magnitude of the ſtroke will be as the motion loſt by the ſtriking body.*

For the motion impreſſed on the body that receives the ſtroke is equal to the magnitude of the ſtroke. And the ſame motion by Ax. 3. is equal to the motion loſt in the ſtriking body.

Cor.

FIG. *Cor.* 5. *A non-elastic body striking another non-elastic body, only loses half as much motion as if the bodies were perfectly elastic.*

For the non-elastic bodies only stop; but elastic bodies recede with the same velocity they meet with.

Cor. 6. *Hence also it follows, that if one body acts upon another, by striking, pressing, &c. the other re acts upon this in the direction of a line perpendicular to the surface whereon they act.* By Ax. 3.

SCHOL.

Though the momentum or quantity of motion in a moving body, is a quite distinct thing from the force that generates it; yet when it strikes another body and puts it into motion, it may with respect to that other body, be considered as a certain quantity of force proportional to the motion it generates in the other body.

Also, although the motion generated by the impulse of another body is considered as generated in an instant, upon account of the very small time it is performed in; yet in mathematical strictness it is absolutely impossible that any motion can be generated in an instant, by impulse or any sort of finite force whatever. For when we consider that the parts of the body which yield to the stroke, are forced into a new position; there will be required some time for the yielding parts to be moved through a certain space into this new position. Now during this time, the two bodies are acting upon each other with a certain accelerative force, which in that time generates that motion, which is the effect of their natural impulse. So that it is plain that this is an effect produced in time; and the less the time, the greater the force; and if the time be infinitely small, the force ought to be infinitely great, which is impossible. But by reason that this effect is produced in so small a time as to be utterly imperceptible, so that it cannot be brought to any calculation; upon this account the time is entirely set aside, and the whole effect imputed to the force only, which is therefore supposed to act but for a moment.

The quantity of motion in bodies has been proved to be as the velocity and quantity of matter. But the momentum or quantity of motion may be the same in different bodies, and yet may have very different effects upon other bodies, on which they impinge. For if a small body with a great velocity impinge upon another body; and if, by reason of its great velocity, it act more strongly upon the small part of the body upon which

which it impinges, than the force of cohesion of the parts of that body; then the part acted on, will, by this vigorous action, be separated from the rest; whilst, by reason of the very small time of acting, little or no motion is communicated to the rest of the body. But if a great body with a small velocity strike another body, and if by reason of its slow motion it does not act so vigorously as to exceed the force of cohesion, the part struck will communicate the motion to the rest of the body, and the whole body will be moved together. Thus if a bullet be shot out of a gun, the momentum of the bullet and piece are equal; but the bullet will shoot through a board, and the gun will only jump a little against him that discharges it. Therefore, small bodies with great velocity, are more proper to tear in pieces: and great bodies with small velocity, to shake or move the whole.

FIG.

PROP. X.

The sum of the motions of any two bodies in any one line of direction, towards the same part, cannot be changed by any action of the bodies upon each other, whatever forces these actions are caused by, or the bodies exert among themselves.

Here I esteem progressive motions, or motions towards the same part, affirmative; and regressive ones, negative.

CASE I.

Let two bodies move the same way, and strike one another directly. Now, since action and re-action (by Ax. 3.) are equal and contrary, and this action and re-action is the very force by which the new motions are generated in the bodies; therefore (by Ax. 2.) there will be produced equal changes towards contrary parts: And therefore whatever quantity of motion is gained by the preceding body will be lost by the following one; and consequently their sum is the same as before.

And if the bodies do not strike each other, but are supposed to act any other way, as by pressure, attraction, repulsion, &c. yet still, since action and re action are equal and contrary, there will be induced an equal change in the motion of the bodies, and in contrary directions; so that the sum of the motions will still remain the same.

FIG.

Case II.

Suppose the bodies to strike each other obliquely; then since (by the last Prop.) they act upon each other in a direction perpendicular to the surface in which they strike; the action and re-action in that direction being equal and contrary; the sum of the motions, the same way, in that line of direction, must remain the same as before. And since the bodies do not act upon each other in a direction parallel to the striking surface; therefore there is induced no change of motion in that direction. And therefore universally the sum of the motions will remain the same, considered in any one line of direction whatever. And if the bodies act upon one another by any other forces whatever, still (by Ax. 2. and 3.) the changes of motion will be equal and contrary, and their sum the same as before.

Cor. 1. *The sum of the motions of any system or number of bodies, in any one line of direction, taken the same way, remains always the same, whatever forces these bodies exert upon each other; esteeming contrary motions to be negative. And therefore,*

Cor. 2. *The sum of the motions of all the bodies in the world, estimated in one and the same line of direction, and always the same way, is eternally and invariably the same; esteeming these motions affirmative which are progressive, or directed the same way; and the regressive motions negative. And therefore in this sense motion can neither be increased nor diminished. But,*

Cor. 3. *If you reckon the motions in all directions to be affirmative, then the quantity of motion way be increased or decreased an infinite number of mays. As suppose two equal non-elastic bodies, to meet one another with equal velocities, they will both stop and lose all their motions.*

For let M be the motion of each, then before meeting the sum of their motions is $M+M$; and after their meeting, it is 0. But in the sense of this Prop. $M-M$ is the sum of the motions before they meet, because they move contrary ways, which is 0; and it is the same after they meet. And thus a man may put several bodies into motion with his hands, which had no motion before; and that in as many several directions as he will.

6 PROP.

FIG.

PROP. XI.

The motions of bodies included in a given ſpace, are the ſame among themſelves; whether that ſpace is at reſt, or moves uniformly forward in a right line.

For if a body be moving in any right line; and there be any force equally impreſſed, both upon the body and the right line in any direction. And in conſequence of this, they both move uniformly with the ſame velocity; now as there is no force to carry the body out of that line, it muſt ſtill continue in it as before; and as there is no force to alter the motion of the body in the right line; it will (by Ax. 1.) ſtill continue to move in it as before. For the ſame reaſon, the motions of any number of bodies moving in ſeveral directions, will ſtill continue the ſame; and their motions among themſelves will be the ſame, whether that ſpace be at reſt, or moves uniformly forward.

Likewiſe, ſince the relative velocities of bodies, (that is. the difference of the real velocities the ſame way; or their ſum, different ways) remain the ſame, whether that ſpace be at reſt, or it and the bodies move uniformly forward all together. Therefore their mutual impulſes, colliſions, and actions upon one another, being (by Cor. 3. Prop. IX.) as the relative velocities, muſt (by Ax. 3.) remain the ſame in both caſes.

SCHOL.

Before I end this ſection, it may not be amiſs to mention a certain kind of force, called by the foreigners, *vis viva*. This they term *a faculty of acting*; and diſtinguiſh it from the *vis mortua*, which with them ſignifies only a ſolicitation to motion, ſuch as preſſure, gravity, *&c.* concerning this *vis viva* they talk ſo obſcurely, that it is hard to know what they mean by it. But they meaſure its quantity by the number of ſprings which a moving body can bend to the ſame degree of tenſion, or break; whether it be a longer or ſhorter time in bending them. So that the *vis viva* is the total effect of a body in motion, acting till its motion be all ſpent. And according to this, they find, that the force (or *vis viva*) to overcome any number of ſprings, will always be as the body multiplied by the ſquare of the velocity.

FIG. Suppose any number of equal and similar springs placed at equal distances in a right line; and a body be moved in the same right line against these springs; then the number of springs which that body will break before it stop, will be as the square of its velocity; whatever be the law of the resistance of any spring in the several parts of its tension. For, from the foregoing Prop. it appears, that the swifter the body moves, so much the less time has any spring to act against it to destroy its motion: And therefore the motion destroyed by one spring will be as the time of its acting; and by several springs, as the whole time of their acting; and consequently the resistance is uniform. And since the resistance is uniform, the velocity lost will be as the time, that is as the space directly and velocity reciprocally; whence the space, and therefore the number of springs, is as the square of the velocity. And upon this account they measure the force of a body in motion, by the square of the velocity. So at last the *vis viva* seems to be the total space passed over, by a body meeting with a given resistance; which space is always as the square of the velocity. And this comes to the same thing as the force and time together, in the common mechanics.

Now it seems to be a necessary property of the *vis viva*, that the resistance is uniform. But there are infinite cases where this does not happen; and in such cases, this law of the *vis viva* must fail. And since it fails in so many cases, and is so obscure itself, it ought to be weeded out, and not to pass for a principle in mechanics.

Likewise, if bodies in motion impinge on one another, the conservation of the *vis viva* can only take place when the bodies are perfectly elastic. But as there are no bodies to be found in nature which are so; this law will never hold good in the motion of bodies after impulse; but, in this respect, it must eternally fail.

This notion of the *vis viva* was first introduced by M. *Leibnitz*, who believed that every particle of matter was endued with a living soul.

SECT.

SECT. II.

The laws of gravity, the descent of heavy Bodies, and the motion of projectiles, in free space.

FIG.

PROP. XII.

The same quantity of force is requisite to keep a body in any uniform motion, directly upwards, as is required to keep it suspended, or at rest.

And if a body descends uniformly, the same force that is sufficient to hinder its acceleration in descending, is equal to the weight of it.

For the force of gravity will act equally on the body in any state whether of motion or rest. Therefore if a body is projected directly upwards or downwards, with any degree of velocity; it would for ever retain its velocity if it were not for the force of gravity that draws it down, by Ax. 1. If therefore a force equal to its gravity were applied directly upwards; then (by Ax. 9.) these two forces destroy each other's effects; and it is the same thing as if the body was acted on by no force at all. And therefore (by Ax. 1.) it would retain its uniform motion.

Cor. But if a body be moved upwards with an accelerated motion; the force to cause that motion will be greater than its weight; and that in proportion to its acceleration (by Ax. 10.).

PROP. XIII.

The velocities of falling bodies are as the times of their falling from rest.

For by Postulate 3. the body is uniformly acted on by gravity, which is its accelerating force downwards; therefore, by Cor. 3. Prop. V. the velocity is as the force and time directly, and the matter reciprocally. But by Ax. 5. the force of gravity is

116. is as the quantity of matter; and consequently, the velocity will be as the time.

Cor. 1. *All bodies falling by their own weight, gain equal velocities in equal times.*

Cor. 2. *Whatever velocity a falling body gains in any time, if it be thrown directly upwards, it will lose as much in an equal time; by Ax.* 12. And therefore.

Cor. 3. *If a body be projected upwards with the velocity it acquired by falling in any time; it will, in the same time, lose all its motion.* Hence also,

Cor. 4. *Bodies thrown upwards lose equal velocities in equal times.*

PROP. XIV.

The spaces described by falling bodies are as the squares of the times of their falling from rest.

For by Postul. 3. gravity is a uniformly accelerating force; therefore by Prop. VI. the space described is as the time and velocity. But by the last Prop. the time is as the velocity; and therefore the space described is as the square of the time.

Cor. 1. *The spaces described by falling bodies are also as the squares of the velocities; or the velocity is as the square root of the height fallen.*

Cor. 2. *Taking any equal parts of time; then the spaces described by a falling body, in each successive part of time, will be as the odd numbers,* 1, 3, 5, 7, 9, 11, *&c.*

For in the times 1, 2, 3, 4, *&c.* the spaces described will be as their squares 1, 4, 9, 16, *&c.* And therefore in the differences of the times, or in these equal parts of time; the spaces described, will be as the differences of the squares, or as 1, 3, 5, 7, *&c.*

Cor. 3. *A body moving with the velocity acquired by falling through any space, will describe twice that space in the time of its fall.* By Cor. 1. Prop. 6.

Cor.

Cor. 4. *If a body be projected upwards with the velocity it acquired in falling, it will, in the same time, ascend to the same height it fell from; and describe equal spaces in equal times, both in rising and falling, but in an inverse order; and will have the same velocity at every point of the line described.* FIG.

For by Cor. 2. of the last Prop equal velocities will be gained or lost in equal times, (reckoning from the last moment of the descent). Therefore, since at the several correspondent points of time, the velocities will be equal, the spaces described in any given time will be equal, and the wholes equal.

Cor. 5. *If bodies be projected upwards with any velocities, the heights of their ascent will be as the squares of the velocities, or as the squares of the times of their ascending.*

For in descending bodies the spaces descended are the squares of the last velocities, by Cor. 1. And by Cor. 4. the spaces ascended will be equal to those descended.

Cor. 6. *If a body is projected upwards with any velocity, with the same velocity undiminished, it would describe twice the space of its whole ascent, in the same time.* By Cor. 3. and 4.

Cor. 7. *Hence also all bodies from equal altitudes descend to the surface of the earth in equal times.*

SCHOL.

It is known by experiments, that a heavy body falls $16\frac{1}{12}$ feet in a second of time, and acquires a velocity which will carry it over $32\frac{1}{6}$ feet in a second; which being known, the spaces described in any other times, and the velocities acquired, will be known by the foregoing propositions; and the contrary. These propositions are exactly true, where there is no resistance to hinder the motion; but because bodies are a little resisted by the air, descended bodies will be a little longer in falling; and a body projected upwards, will be something longer in descending than in ascending, and falls with a less velocity; and consequently a body projected upwards with the velocity it falls with, will not ascend quite to the same height; but these errors are so small, that in most cases they may safely be neglected.

If the force by which a body is accelerated in falling was directly as the height fallen from; it may be computed (by Cor. 2. Prop. V.) that the velocity acquired will also be as the height; or

FIG. or the ſpace deſcribed directly as the velocity. And therefore if bodies were projected upwards, they would in this caſe aſcend to heights, which are as the velocities with which they are projected. This being compared with Cor. 5. of the laſt Prop. it is eaſy to conclude that bodies projected upwards, and acted upon by a force which is neither of a given quantity, nor in proportion to the diſtance of the body from the top of the aſcent, but between them; that theſe bodies will then aſcend to heights which are between the ſimple and duplicate ratio of the velocities.

And as theſe propoſitions lead us to the knowledge of the relation between the velocities and ſpaces deſcribed, from the forces being given. So *vice verſa*, from that relation being given, the forces may be known. Whence if bodies are projected with any velocities into a reſiſting medium, and the ſpaces deſcribed within that body, meaſured; the conſtitution of that body, and the law of its reſiſtance will be found.

PROP. XV.

If a body be projected either parallel to the horizon, or in any oblique direction, it will, by its motion, deſcribe a parabola.

6, 7. Let *AD* be the direction of the motion, *AFG* the curve deſcribed; and let *AB*, *BC*, *CD*, &c. be all equal; draw *AM*, *BF*, *CG*, *DH*, &c. perpendicular to the horizon; and complete the parallelograms, *AF*, *AG*, *AH*, &c. then by Ax. 1. if the body were without gravity, it would move on in the line *AD*; and deſcribe the lines *AB*, *BC*, *CD*, &c. in equal times. Now ſince gravity acts in lines perpendicular to the horizon, it does not affect the motion in direction *AD*; but generates a motion in direction *AM*. So that the body, inſtead of being at *B*, *C*, *D*, &c. will at the ſame points of time be at *F*, *G*, *H*, &c. But in the time of deſcribing *AB*, *AC*, *AD*, the body, by the force of gravity, will deſcend through the ſpaces *BF*, *CG*, *DH*; which are as the ſquares of the times they are deſcribed in, (by Prop. XIV.) that is as the ſquares of the lines, *AB*. *AC*, *AD*. But *AB*, *AC*, *AD* are equal to *KF*, *LG*, *MH*; and *BF*, *CG*, *DH*, equal to *AK*, *AL*, *AM*. Therefore the parts of the axis of the curve, *AK*, *AL*, *AM*, &c. are reſpectively as the ſquares of the ordinates KF^2, LG^2, MH^2, &c. And therefore, by the conic ſections, the curve *AFG* is a parabola.

Cor. 1. *The line of direction AD is a tangent to the curve in A.* FIG. 7.
And the latus rectum to the point A is $\frac{KF^2}{AK}$ *or* $\frac{AB^2}{BF}$. *And in the oblique projection, the parameter is* $\frac{AO^2}{BF}$, *supposing AOP perpendicular to AM, and G the vertex.* For then $\frac{AO^2}{BF} = \frac{AP^2}{CG = GP}$.

Cor. 2. *If the horizontal velocities of projectiles be the same, whatever their elevations be, they will describe the same parabola.*

For if AB be the velocity and direction of the projectile, then AO is the horizontal velocity. When the body comes to the vertex G, its motion is then parallel to the horizon, which parallel motion remains the same as before, that is, equal to AO. Therefore it describes the same parabola, as a body projected from G with the velocity AO parallel to the horizon.

Cor. 3. *The velocity of a projectile in any point of the curve, is as the secant of the angle of its direction above the horizon.*

For AO the horizontal velocity is the same at all points of the curve; and the velocity AB at A, in the curve is the secant of the angle of elevation OAB.

Cor. 4. *The velocity at any point of the curve is the same that is acquired by falling through* ¼ *the parameter belonging to that point; or, which is the same, through* ¼ *of the principal latus rectum + the abscissa to that point.*

For let IA be the space fallen through to acquire the velocity in any point as A; then the space AD described in the same time with that velocity in direction AV, will be $2AI$ (by Cor 3. Prop. XIV.) but in the same time, by the same force of gravity, the body will descend through an equal space DH, therefore AD or $MH = 2DH$ or $2AM$; but the parameter $= \frac{MH^2}{AM} = \frac{4AM^2}{AM}$ or $4AM$. Therefore AM or $AI = \frac{1}{4}$ parameter

FIG.

PROP. XVI.

The horizontal distances of projections, made with any velocities, and at any elevations, are as the sines of the doubled angles of elevation, and the squares of the velocities conjunctly.

7. Let v = *velocity of the projectile; measured by the space it passes through in time 1″.*
f = *descent of a body by gravity in the same time.*
x = *AE the horizontal distance, or amplitude.*
$\left.\begin{array}{l} s = \textit{sine} \\ c = \textit{cosine} \end{array}\right\}$ *of the elevation VAE.*
$\left.\begin{array}{l} A = \textit{sine} \\ B = \textit{vers. sine} \end{array}\right\}$ *of twice the elevation.*

Then by trigonometry, $2sc = A$, and $2ss = B$, when the radius is 1. And in the right angled triangle AEV.

$$c : x :: (\text{rad.})\ 1 : \frac{x}{c} = AV, \text{ and}$$

$$c : x :: s : \frac{sx}{c} = VE.$$

And by Prop. III.

$$v : (\text{time})\ 1 :: (AV)\frac{x}{c} : \frac{x}{cv} = \text{time of describing } AV.$$

And by Prop XIV.

f : (time) 1 :: $\frac{sx}{c}(VE) : \frac{sx}{cf}$ = square of the time of describing VE. Now the times of describing AV, VE, and the curve AGE are all equal. Whence $\frac{sx}{cf} = \frac{xx}{vvcc}$. Therefore $x = \frac{vvsc}{f} = \frac{vvA}{2f}$. And therefore x or AE is as vvA.

Cor. Hence, the altitudes of projections, are as the squares of the sines of elevation, and the squares of the velocities; or as the versed sines of the doubled angles of elevation, and the squares of the velocities.

For if G be the vertex, $GP = \frac{1}{2} GP = \frac{1}{4} VE$, and $VE = \frac{sx}{c} = \frac{vvss}{f}$. Therefore $GP = \frac{vvss}{4f} = \frac{vvB}{8f}$. Therefore GP is as $vvss$ or as vvB.

FIG. 7.

Cor. 2. *The times of flight of projectiles, are as the velocities, and the sines of elevation.*

For the time $= \frac{x}{cv} = \frac{vs}{f}$.

Cor. 3. *The greatest random or horizontal projection, is at the elevation of* 45 *degrees. And the horizontal distances are equal, at elevations equally distant above or below* 45°.

SCOL.

Let h be the height of the perpendicular projection with the velocity v; then will $h = \frac{vv}{4f}$. Whence

Horizontal distance $= \frac{vvsc}{f} = \frac{vvA}{2f} = 4sch = 2Ah$.

Altitude of the projection $= \frac{vvss}{4f} = \frac{vvB}{8f} = ssh = \frac{1}{2}Bh$.

Time of flight $= \frac{vs}{f} = 2s\sqrt{\frac{h}{f}}$.

PROP. XVII.

The distances of projections made on any inclined planes, are in the complicate ratio of the sines of the angles which the lines of direction make with the plane and zenith, and the the squares of the velocities, directly; and the cosines squared of the plane's elevation reciprocally.

Let AE be the inclined plane, AV the direction of the projectile, SA, CP, VE perpendicular to the horizon; AGE the path of the projectile, and let 8.

v = *velocity of the projectile in* A, *measured by the space it describes in the times* 1.

f = *space described by a descending body in the same time.*

s = *sine of* VAE,

c = *sine of* VAS,

z = *sine of* SAE,

x = AE *the oblique distance, or random.*

Then

FIG. 8. Then by plane trigonometry,

$c : x\ (AE)\ :: z : AV = \frac{xz}{c}$, and

$c : x :: s : VE = \frac{sx}{c}$.

And by Prop. III.

Space v : time 1 :: $(AV)\ \frac{xz}{c} : \frac{xz}{cv}$ = time of defcribing AB.

And by Prop. XIV.

Space f : time 1 :: $(VE)\frac{sx}{c} : \frac{sx}{fc}$ = fquare of the time of defcending through VE.

But the times of defcribing AV, VE being equal, we have $\frac{sx}{fc} = \frac{xxzz}{ccvv}$, or $\frac{s}{f} = \frac{xzz}{cvv}$; whence $x = \frac{scvv}{fzz}$, and f being a given quantity, x or AE is as $\frac{scvv}{zz}$.

Cor 1. *The heights above the planes, are as the fquares of the velocities, the fquares of the fines of elevation above the plane, directly; and the fquares of the cofines of the plane's elevation, reciprocally.*

For if $AP = PE$, then G is the vertex of the parabola, in refpect of the plane AE. And $GP = \frac{1}{4}VE = \frac{sx}{4c} = \frac{ssvv}{4fzz}$.

Cor. 2. *The times of flight are are as the velocities and fines of elevation above the plane; and the cofines of the plane's elevation reciprocally.*

For the time is $= \frac{zx}{cv} = \frac{sv}{fz}$.

Cor. 3. *Hence alfo the altitude is as the fquare of the time of flight.*

For the altitude is $\frac{ssvv}{zz}$, and the time as $\frac{sv}{z}$.

Cor. 4. *The greateft projection upon an inclined plane, is when the line of direction bifects the angle between the plane and zenith. And the projections are equal at elevations equally diftant from this line, above and below.*

For upon the fame plane, AE is as sc, and sc is greateft when FIG.
$s=c$. And at equal diftances above and below sc is the fame. 8.

Cor. 5. *This Prop. holds true, whether the projections be made up, or down the planes*; or whether both planes be inclined, or one is inclined and the other horizontal.

SCHOL.

Let $h=$ height of the perpendicular projection as before, then $h=\frac{vv}{4f}$. Whence

$$\text{Length of the projection} = \frac{scvv}{fzz} = \frac{4sch}{zz}.$$

$$\text{Height of the projection} = \frac{ssvv}{4fzz} = \frac{ssh}{zz}.$$

$$\text{Time of flight} = \frac{sv}{fz} = \frac{2s}{z}\sqrt{\frac{h}{f}}.$$

And if $d = AE$ the length of the projection, then $\frac{scvv}{fzz} = d$, whence $v = z\sqrt{\frac{df}{sc}}$. And fupppofing the utmoft random of one of our greateft guns to be 5864 paces; then $v = 194$ paces $= 324$ yards, fo that a ball fhot out of her, moves at the rate of 324 yards in a fecond. All this fuppofes that there is no refiftance of the medium. But it may be noted, that by reafon of the air's refiftance, the upper randoms, being more refifted, fcarce go fo far as the under randoms; and the greateft random upon a horizontal plane, is therefore at fomething lefs elevation than 45 degrees.

SECT.

SECT. III.

The properties of the mechanical powers; the balance, the leaver, the wheel, the pulley, the scrue, and the wedge.

PROP. XVIII.

FIG. 9. *If at the ends of a balance AB, which are equally distant from the center of motion C, two equal weights be suspended; they will be in equilibrio.*

Here *AB* that represents the balance is supposed to be a right line, in which are the three points *A*, *B*, *C*. Now the weights *A*, *B*, cannot act upon one another any otherwise than by means of the balance *AB*, whose fixed point is *C*.

Suppose then that any force applied at *A* puts the body *A* into motion, and by means of the balance the body *B*; then since the brachia of the beam *CA* and *CB* are equal; the arches *Aa*, *Bb*, described by these bodies will be equal. Consequently the velocities and quantities of matter of *A*, *B*, being equal, their momenta or motions will be equal. And, because *ACB* is a right line, they move in a contrary direction; and therefore by Ax. 9. these bodies cannot of themselves raise one the other, but must remain in equilibrio.

Cor. Hence equal forces A, B, applied at equal distances from the center of motion C, will have the same effect in turning the balance.

PROP.

FIG.

PROP. XIX.

In any ſtreight leaver, if the power P be to the weight W, as the diſtance of the weight from the fulcrum C, to the diſtance of the power from the fulcrum; the power and weight (acting perpendicularly on the leaver) will be in equilibrio. 10. 19. 20. 12.

A leaver is any inflexible beam, ſtaff, or bar, whether of metal or wood, &c. that can any way be applied to move bodies. There are four kinds of leavers;

1. A leaver of the firſt kind is, that where the fulcrum is between the weight and the power, (fig. 10.)

2. A leaver of the ſecond kind is, where the weight is between the fulcrum and the power, (fig. 19.)

3. The leaver of the third kind is, where the power *P* is between the weight and the fulcrum, (fig. 20.)

4. The fourth kind is the bended leaver, (fig. 12.)

CASE I.

In the leaver of the firſt kind *WCP*, inſtead of the power *P*, apply a weight *P* to act at the end of *CP*. And let the leaver *WCP* be moved into the poſition *aCb*. Then will the arches *Wa*, *Pb* be as the radii *CW*, *CP*; that is as the velocities of the weight and power. Whence ſince $P : W :: CW ; CP$, therefore $P : W ::$ velocity of *W* : velocity of *P* : therefore $P \times$ velocity of $P = W \times$ velocity of *W*. Conſequently the momenta or motions of *P* and *W* are equal. And ſince they act in contrary directions, therefore by Ax. 9. neither of them can move the other, but they will remain in equilibrio. 10.

CASE II.

The leavers of the ſecond and third kind may be reduced to the firſt thus; make $Cp = CP$, and inſtead of the power *P*, apply a weight equal to it at *p*. Then by Caſe 1, the weight *W* and power *p* will keep one another in equilibrio; and (by Cor. Prop. 18.) the weight *p* and power *P* will have the ſame effect in turning the leaver about its center, therefore the power *P* and weight *W* will be in equilibrio. 19. 20.

Cor.

FIG. *Cor.* 1. *In any sort of leaver whether streight or bended, and whe-*
10. *ther moveable about a single point C, or an axis AB; or whether the*
11. *leaver be fixed to the axis and both together moveable about two cen-*
12. *ters A, B; or whatever form the leavers have; if AB be a right*
13. *line, and from the ends P, W, there be drawn lines to the center C,*
14. *or perpendiculars to the axis AB; and if the power and weight act*
15. *perpendicular to these lines, and be always reciprocally as these dis-*
16. *tances drawn to the center C or axis AB; then they will be in equi-*
librio.

17. *Cor.* 2. *In any sort of leaver WCP, and in whatever directions the*
18. *power and weight act on it; if their quantities be reciprocally as the*
perpendiculars to their several lines of direction, let fall from the center of motion, they will be in equilibrio. Or they will be in equilibrio, when the weight multiplied by its distance, and the S. angle of its direction, is equal to the power multiplied by its distance, and S. < of its direction. $W \times WC \times S.DWC = P \times PC \times S.EPC$.

For the power and weight will be in equilibrio if they be supposed to act at E and D; and by Ax. 14. it is the same thing whether they act at E and D, or at P and W. Also by trigonometry, $WC \times S.W = DC$, and $PC \times S.P = CE$.

Cor. 3. *Hence universally, if any force be applied to a leaver, its effect in moving the leaver, will be as that force multiplied by the distance of its line of direction from the center of motion. Or the effect is as the force* $\times$ *by its distance from the center, and by the sine of the angle of its direction,* $P \times PC \times S.P$.

Cor. 4. *If two bodies be in equilibrio on the leaver, each weight is reciprocally as its distance from the center.*

Cor. 5. *In the streight leaver when the weight and power are in equilibrio, and act perpendicularly on the leaver, or in parallel directions; then of these three the power, weight, and pressure upon the fulcrum, any one of them is as the distance of the other two.*

10. For if CP represent the weight W, then CW will represent the
19. power P. And in Fig. 10. C sustains both the weights, and
20. therefore the pressure is WP; and Fig. 19, 20. C sustains the
difference of the weights, and therefore the pressure will be WP.

PROP.

FIG.

PROP. XX.

If ſeveral weights be ſuſpended on a ſtreight leaver AB; and if the ſum of the products of each weight, multiplied by its diſtance from the center of motion C, on one ſide, be equal to the ſum of the like products on the other ſide; then they will be in equilibrio. And the contrary. 21.

For the force of each weight to move the leaver is as the weight multiplied by the diſtance (by Cor. 3. laſt Prop.); and the ſum of the products is as the whole forces; which if they be equal, the forces on both ſides are equal, and the leaver remains at reſt.

PROP. XXI.

If a bended leaver WCP be kept in equilibrio by two powers, acting in the directions PB, WA perpendicular to the ends of the leaver CP, CW; and if the lines of direction be produced till they meet in A, and AC be drawn, and CB parallel to WA. I ſay the power P, the weight or power W, and the force acting againſt the fulcrum C; will be reſpectively as AB, BC, AC; and in theſe very directions. 22.

Draw CB, CF parallel to WA, PA; then the angle $WFC = WAP = CBP$, and the right angled triangles WCF and BCP are ſimilar; whence $CF : CB :: CW : CP ::$ (by Cor. 2. Prop. XIX.) power P : power W. Now ſince (by Ax. 14.) it is the ſame thing to what points of the lines of direction PB, WF, the forces P, W be applied; let us ſuppoſe them both to act at the point of interſection A; then ſince the point A is acted on by two forces which are as CF and CB, or as AB and AF; and both theſe are equivalent to the ſingle force AC (by Cor. 2. Prop. 7.) Therefore the fulcrum C is acted on by the force AC, and in that direction, by Ax. 11.

Cor. 1. *Hence the power P, the weight W, and the preſſure the fulcrum C ſuſtains; are reſpectively as WC, PC, and PW. That is, any one is as the diſtance of the other two.*

For ſince the angles at P, W are right; CA is the diameter of a circle paſſing through the points A, P, C, W; therefore

F the

FIG. 22. the angle $WPC=WAC=ACB$, and the angle $CWP=CAP$; therefore the triangles ABC, WCP are similar; and $AB : BC :: WC : CP$, and $BC : AC :: CP : PW$. Therefore, &c.

Cor. 2. *In any leaver WCP, the lines of direction of the powers PW, WF, and of the pressure on the fulcrum C, all tend to one point A.*

For if not, the leaver would not remain in equilibrio.

PROP. XXII.

23. *If AB, CD be two leavers moveable about A and C, and some force acts upon the end B of the leaver AB, in a given direction BF; whilst the leaver AB acts upon CD at B. If BE be drawn perpendicular to CB, and AE parallel to BF: And if these leavers keep one another in equilibrio. Then I say, the force in direction BF, force against DC in direction EB, and the pressure against the center A, are respectively as AE, BE, AB.*

For since (by Prop. 9.) the leaver AB acts upon BC at B, in the direction EB perpendicular to BC; and the leaver CD re-acts in direction BE; and (by Ax. 19.) the point A is acted on in direction BA. Therefore the point B is acted on with three forces; BF the force applied at B, and BE the re-action of the leaver CD, and AB the re-action of the center A; and AE is parallel to BF; therefore (by Prop. VIII.) these forces are as AE, BE, and AB.

Cor. If two forces BF, BE acting perpendicular to the leavers AB, DC, keep these leavers in equilibrio: The force BE, force BF, and pressure at A, are respectively as radius, Cos. ABD, and S. ABD.

For then EAB is a right-angled triangle; and these forces are as BE, AE, and AB; that is, as radius, $S.ABE$, and $S.AEB$; that is, as radius, Cos. ABD, and $S.ABD$.

PROP.

FIG.

PROP. XXIII.

If AB, BC be two leavers moveable about the centers A and C; and if the circles KBM and DBE be described with the radii BC, BA; and upon the Circle BM as a base, with the generating circle DBE, the epicycloid BNE be described; and the leaver CB and epicycloid BNE be joined together in that very position, so as to make but one continued leaver CBNE. And if these leavers CBNE, and AB move about the centers C, A; so that the end D of the leaver AD be always in the curve of the epicycloid DK; I say, that two equal and contrary forces at D and K, acting perpendicular to the radii DA, KC, will always keep these leavers in equilibrio. 24.

For let the leavers *AB*, *CBNE* come into the position *AD*, *CKDF*; then since the epicycloid *KD* is described by the point *D*, whilst the arch *DB* rolled upon the equal arch *BK*, therefore the end *D* of the radius *AD* hath moved through an arch *BD* equal to the arch *BK*, through which *K* has moved. Therefore the points *D*, *K*, have equal velocities in any correspondent places *D*, *K*. Therefore equal weights *H*, *I*, applied to the circles *DB* and *BMG* will have equal quantities of motion; and will therefore keep each other in equilibrio, by Ax. 9.

Cor. 1. *Hence if one leaver AD move uniformly about the center A, the other CKD will also move uniformly about its center C. And the arch BD described by D will be equal to the arch BK described by K.*

Cor. 2. *It is the same thing whether the leaver AB act against the convex or concave side of the leaver CKD, provided the end D be always in the curve KD.*

Cor. 3. *After a like manner if BE be an epicycloid described within the circle BRM, by the generating circle BD. And the leaver CBE be compounded of the right line CB and the epicycloid BE; then the leavers CBE and AB, by equal forces acting at B, will keep one another in equilibrio in any position, as CKF and AD.* 25.

For when *AB* is come to *AD*, and *CBE* to *CKDF*; then the arch *BK* = arch *BD*. Whence the weight or forces acting at the distances *CB*, *AB*, have equal velocities; and therefore will sustain one another.

FIG. 26. *Cor. 4. If AC be infinite, or (which is the same thing) if BK be a right line perpendicular to AB; then BE or KD will be the common cycloid. Therefore, whilst the point D moves uniformly about the center A, the point K will move uniformly along the right line BK, and with equal velocities and forces: The point D in the mean time acting upon the cycloidal tooth KD. And any equal opposite forces will sustain one another*

27. *In like manner, if CA be infinite, or BD a right line perpendicular to BC; then BE or DK will be an epicycloid generated by the tangent DB revolving on the circle BK. And the velocities of K in the right line BK, and of D in the right line BD, will be always equal: And equal forces will be sustained at B, in all positions of the leaver CKD.*

28. *Cor. 5. If the figure of the tooth ef, at the end of the leaver AB, be given; and the epicycloid BE be described as before. And if the leavers AB, CB, be made to revolve about their centers A, C; so that the point B always move in the epicycloid BE or KD. And if the tract of the extreme points of the tooth be marked out upon the plane of the cycloidal tooth, as at fggggc, or at funnne. And if the part fgDaKf be cut away, if the tooth be to act on the concave side of the epicycloid; or fncDdKf, if on the convex side. Then if the leavers revolve so that the tooth move along the curve gg or nn; the points D and K of the leavers AB, CD, will move with equal velocities, in the arches BD, BK, as before.* For the fixed point *B* in the tooth will still describe the epicycloid.

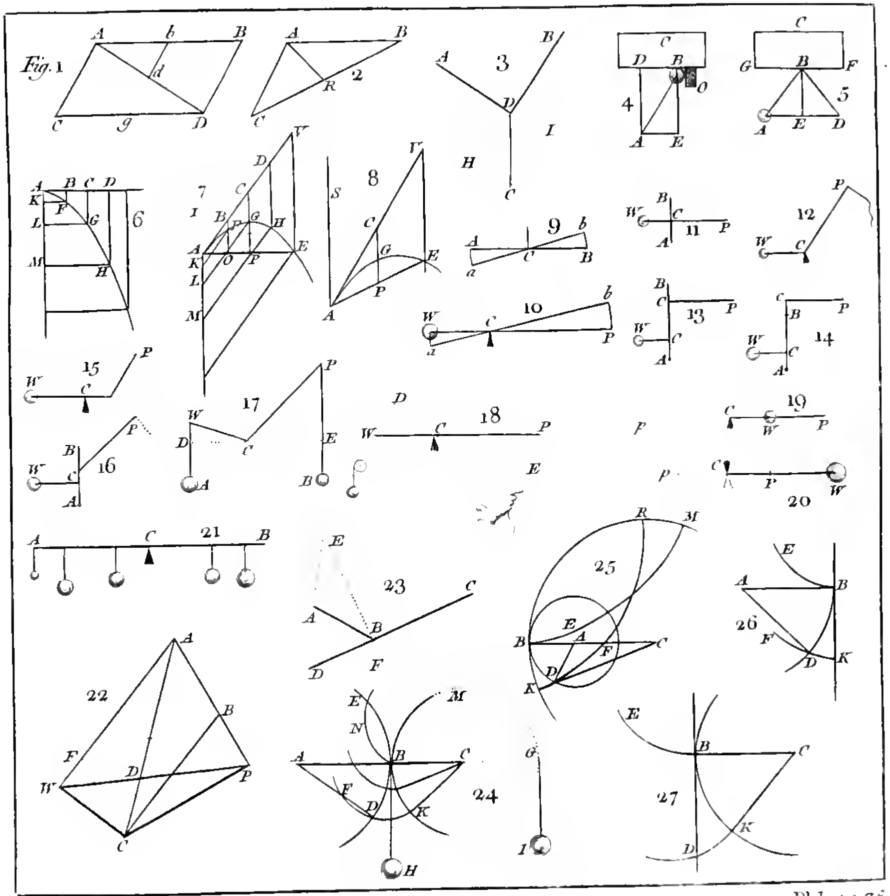
29. *Cor. 6. If the two epicycloids BE, BO be described upon BM, BL with the generating circles BD, BK; and the leavers AB, CB, revolve about the centers A, C; so that the point B or D of the leaver AB move along the epicycloid BE or KDS. Then the point B or K, of the leaver CB, will at the same time move along the epicycloid BO or DKT; and the points D, K, will describe the equal arches BD, BK. And therefore it is the same thing, on which leaver the cycloidal tooth be placed, or whether on one or both.*

For the epicycloid *DKT* generated on *DB*, will pass through *K*, if $BD = BK$. Also the epicycloid *KDS*, generated upon *KB*, will pass through *D*, when $BK = BD$.

SCHOL.

24. The leavers *AB*, *CB* are supposed only to act upon one ano-
25. ther, below the line *AC*; for was the action supposed to be continued

pa. 36



Mechanics.

Pl. 1. *pa.* 36

continued above the line *AC*, the point *B* would no longer act FIG.
on the fame, but on a different epicycloid; and the equality of 24.
motion would hold no longer. 25.

PROP. XXIV.

In the wheel and axel, if the power P be to the weight W, as the diameter of the axel EF where the weight acts, to the diameter of the wheel AB, where the power acts; then the power and weight will be in equilibrio. And the contrary. 30.

For let *AB* be the wheel, *CD* the axel; and fuppofe the wheel and axel to turn once round; then it is plain the power *P* will have defcended a fpace equal to the circumference of the wheel; and the weight *W* will have rifen a height equal to the circumference of the axis. Therefore, velocity of *P*, to velocity of *W* :: as circumference of the wheel, to circumference of the axis :: or as diameter of the wheel, to diameter of the axis :: that is (by fuppofition) as *W*, to *P*. Therefore the motions of *P* and *W* are equal; and have equal forces to move each other; and therefore (by Ax. 9.) will remain in equilibrio.

This Prop. will appear otherwife. For the wheel and axel may be reduced to a leaver of the firft kind: For the fulcrum will be in the middle of the axis *CD*. Therefore drawing lines from the middle of the axis to the power and weight, parallel to the horizon; and the radius of the wheel will be the diftance of the power, and the radius of the axel the diftance of the weight. And as their radii are reciprocally as the weight and power, therefore (by Prop. XIX.) they will be in equilibrio. And thus the wheel and axel is no more but a perpetual leaver.

Cor. 1. *If the rope have any fenfible thicknefs; then if the power P : weight :: diameter of the axel + the diameter of the rope : diameter of the wheel where the power acts; they will be in equilibrio.*

For the weight really hangs half the thicknefs of the rope, beyond the axel.

Cor. 2. *If the direction of the power is not a tangent to the wheel; fuppofe it to act at D or d, and let CA be perpendicular to the line of direction; then if P : W :: CB : CA, then they will be in equilibrio, by Cor.* 2. *Prop.* XIX. 31.

Cor.

FIG. 32. *Cor. 3. If the wheel and axel, one or both have teeth; then if the power P acting on the teeth at B, be to the weight W acting on the teeth at A; as the diameter of the axel at A, to the diameter of the wheel at B; then the wheel is in equilibrio.*

Cor. 4. And it is the same thing if instead of a wheel there be only spokes fixed in the axis, whose length is equal to the radius of the wheel: And any other equal force be applied for a power, instead of the weight P.

Cor. 5. The force of the weight is increased when one or more spires of the rope is folded about the axel. For that, in effect, augments the diameter of the axel.

Cor. 6. It matters not how low the weight hangs. For whilst the axel remains the same, the resistance of the weight remains the same, setting aside the weight of the rope.

PROP. XXV.

33. *Let NBD, MBK be two toothed wheels in the same plane, and if the teeth of the wheel BM be the epicycloids Be, kd, KD, described on the base KBM, with the generating circle BN, and these teeth all equidistant; and if B, d, D the ends of the teeth of the wheel NBD be also equidistant, and these distances Bd, dD equal to Bk, kK: Then, I say, the points of the teeth B, d, D will all act together, on the cycloidal teeth BE, kd, KD, as the wheels turn round. And any points D, K, will move through equal arches BD, BK in equal times.*

Draw the radii *AD*, *Ad*, *CK*, and *Ck*; then *AD* and *CKD* may be considered as two leavers moving about *A*, *C*, and acting on one another in *D*: And the same of *Ad*, *Ckd*, acting at *d*. But by the motion of the wheels *BD*, *BK*, suppose *D* always to be in the epicycloid *KD*; then (Cor. 1. Prop. XXIII.) will $BD = BK$, and since $Dd = Kk$, therefore Bd will be $= Bk$, and consequently (by Cor. 1. Prop. XXIII.) the point *d* will be in the epicycloid *kd*. And thus, if there be never so many teeth *B*, *d*, *D*, &c. they will always be in the curves of the epicycloids *BE*, *kd*, *KD*, &c. therefore the working teeth either act all at once

once upon one another, or they act not at all. And as the velocities of any points are equal in the two wheels *BD. BK*, when only one tooth acts upon one, they will still be equal, if never so many act together. FIG. 33.

Cor. 1. *Hence equal weights or forces applied to the circumferences of these two wheels, as at B, and acting one against the other, will keep these wheels in equilibrio. Likewise, it is the same thing whether the wheel AB drive the wheel CB, its teeth acting upon the concave side of the cycloids; or the wheel CB drive AB, the convex side of the cycloid acting against the teeth of AB.*

Cor. 2. *Hence, instead of the points B, or the infinitely small teeth of the wheel ABD; if any sort of a tooth rs be placed at B; and if the wheels be made to move about so that the given point B may describe the epicycloid BE or KD, whilst the track of the extreme points of the tooth is marked out as KeD, KfD; and the space KeDf be cut away; and the same be done for all the other teeth, being equidistant and of the same form and bigness. Then if one of these wheels is supposed to drive the other, by these teeth running in the spaces DfKe; I say, the circumferences of these wheels will move with equal velocities, and all the working teeth will act together.* This is evident, because the points *B*, *D*, will by this motion describe the epicycloids as before. 34.

Cor. 3. *If the epicycloid BV be described on the base KBH, with the generating circle BD; and a portion of the epicycloid be placed at equal distances B, L, K, for teeth; then the teeth of the wheel A acting against the cycloidal teeth, will make the motion equal in the two wheels. Where we may take as great a portion of the cycloid as we will; and the sides BO, LI, which act not, may be of any figure, not to hinder the motion of the teeth of A. And it is the same thing what part of the tooth LO, the tooth G acts against.* 35.

Cor. 4. *But the teeth ought not to act upon one another before they arrive at the line AC, which joins their centers. And though the side BO of the tooth may be of any form; yet it is better to make them both sides alike, which will serve to make the wheels turn backwards. Also a part as pqr may be cut away on the back of every tooth, to make way for those of A. And the more teeth that work together, the better; at least one tooth should always begin before the other hath done working: The teeth ought to be disposed in such manner as not to trouble or hinder one another, before they begin to work; and that there be a convenient length, depth and*

FIG. 35. *thickness given them, that they may more easily disengage themselves; as well as for strength.*

PROP. XXVI.

36. *In a combination of wheels with teeth, if the power P be to the weight W; as the product of the diameters of all the axels, pinions, or trundles; to the product of the diameters of all the wheels, they will be in equilibrio.*

For by Prop. XXIV.

The power *P* acting at *A* : Force on *B* :: diam. *B* : diam. *A*, and force on *B* or *C* : force on *D* :: diam. *D* : diam. *C*, and force on *D* or *E* : weight *W* at *F* :: diam. *F* : diam. *E*.

Therefore, ex equo.

Power *P* : weight *W* :: product of the diameters *B*, *D*, *F*: to the product of the diameters *A*, *C*, *E*.

38. *Cor.* 1. *In a combination of wheels going by cords, if the power P be to the weight W, as the product of all the diameters of the axels, B, D, F, to the product of all the diameters of the wheels, A, C, E; they will be in equilibrio.*

For the cords supply the place of teeth.

36. *Cor.* 2. *In any combination of wheels with teeth, if the power P be to the weight W; as the diameter of the axel F where the weight acts, multiplied into the product of the teeth in each pinion or spindle, is to the diameter of the wheel A, where the power acts, multiplied by the product of the teeth in each, of the wheels (that the pinions act against); the weight and power will be in equilibrio.*

For the number of teeth in each wheel and pinion that act against one another, are as the circumferences or as the diameters of that wheel and pinion.

Co. 3. *And hence also, if the power be to the weight, in a ratio compounded of the diameter of the axel F, where the weight acts to the diameter of the wheel A, where the power acts, and the ratio of the number of teeth in the first axel, (B), reckoning from the power; to the number of teeth in the second wheel (C), and of the number of teeth in the second axel (D), to the number in the third wheel (E); and so on till the last; then they will be in equilibrio.*

Cor.

Cor. 4. *In a combination of wheels, the number of revolutions of the wheel F where the weight acts, to the number of revolutions of the wheel A where the power acts, in the same time; is as the product of the teeth in the pinions, to the product of the teeth in the wheels which act in them; or as the product of the diameters of the pinions, to the product of the diameters of the wheels.* FIG. 36.

SCHOL.

Wheels with oblique teeth come under the same rules; but as they are related to the screw, we refer you thither for a farther account thereof.

In wheels whose teeth work together, they should not encounter before they come to the line joining their centers; because the rubbing is greater on that side; but being past the line, the teeth slip easily along one another, in making their escape, so that the friction is very inconsiderable.

PROP. XXVII.

If a power sustain a weight by means of a rope going over a fixed pulley; then the power is equal to the weight. But if the pulley be moveable together with the weight, and the other end of the rope fixed; then the power will be but half the weight. 39. 40.

For suppose a horizontal line *AB* drawn through the center of the pulley *C*; then that line will represent a leaver, and (in Fig. 39.) where the pulley is fixed, the center *C* being kept immoveable, represents the fulcrum; whilst the weight acts at *B*, and the power at *A*. And because $BC = CA$, therefore (by Prop. XIX.) the power *P* is equal to the weight *W*.

And (in Fig. 40.) the fixed point *B* is the fulcrum, and the weight acts at C, and the power at *A*; and since *BC* is half *AB*, therefore (by Prop. XIX.) the power at *P* is half the weight *W*.

Cor. Hence all fixed pulleys are equivalent to leavers of the first kind. And they add no new force to the power, but only serve to change the direction, and facilitate the motion of the rope: But a moveable pulley doubles the force. And if a rope go over several pulleys, A, B, C, whose blocks are all fixed; the power is neither increased nor diminished. 41.

FIG.

PROP. XXVIII.

42. *In a combination of pulleys all drawn by one running rope; if the power P be to the weight W; as 1 to the number of parts of the rope at the moveable block A; they will be in equilibrio.*

For (by Ax. 18.) all the parts of the rope *m, o, n, r, s, t, v*, are equally ſtretched; and the weight *W* is ſuſtained by the number of ropes that act againſt the moveable block; and the rope *v* or the power *P* acts with the force of one or unit. Therefore the power is to the weight, as 1 to the number of ropes pulling at the moveable block *A*.

Cor. Hence the power is to the force by which the immoveable block B is drawn; as 1 to the number of ropes acting againſt that immoveable block.

PROP. XXIX.

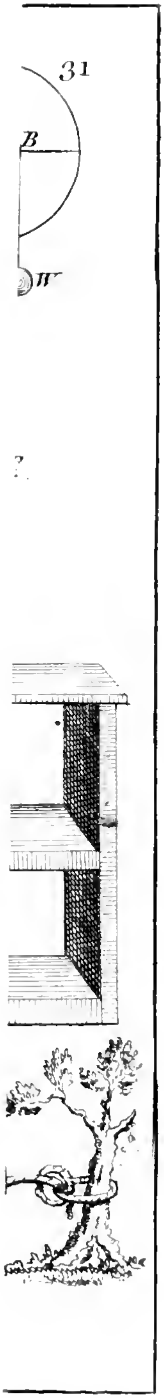
37. *In the ſcrew, if the power P be to the weight W, as the height of one thread (reckoned according to the length of the ſcrew) to the circumference deſcribed by one revolution of the power; then they will be in equilibrio.*

For the weight *W* raiſes the height of one thread, whilſt the power deſcribes the circumference whoſe radius is *PC*. Therefore the velocities of the power and weight are reciprocally as their quantities: Therefore their motions are equal, and they are in equilibrio.

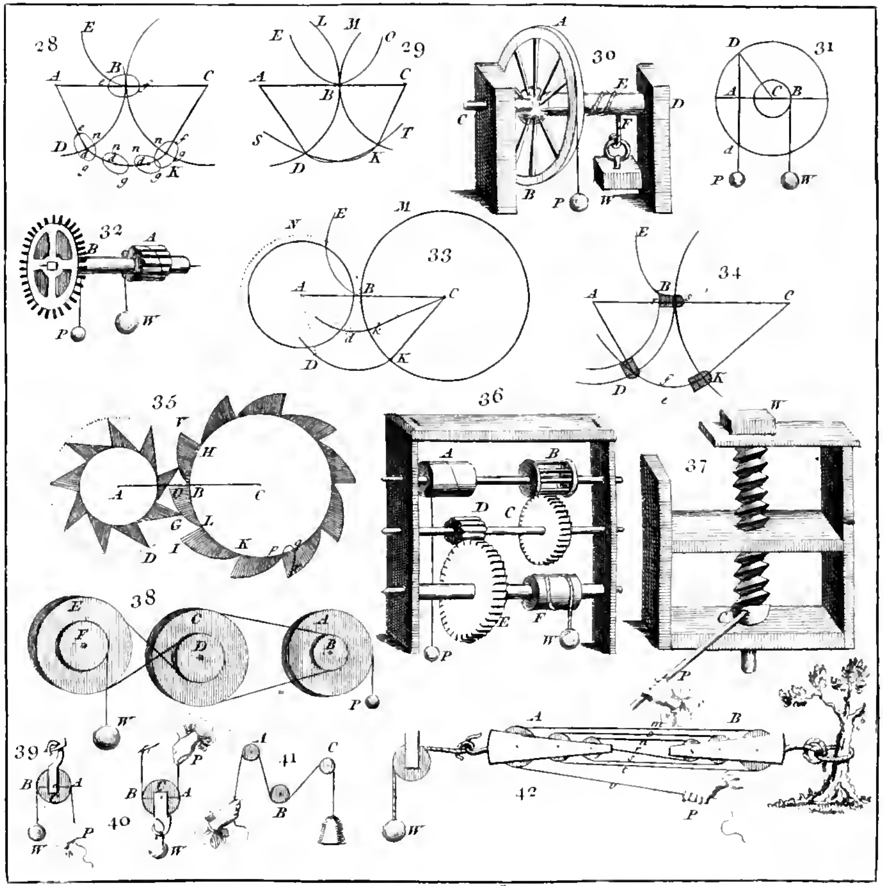
43. *Cor.* 1. *In the endleſs or perpetual ſcrew AB, having one worm, leaf, or tooth, which drives the teeth of the wheel CD. If you take the diſtance of two threads in the ſcrew AB, according to the length of the axis AB; or the diſtance of two teeth in the wheel CD, in direction of the circumference. And if a weight W act at the circumference of the wheel CD. Then if the power P be to the weight W, as that diſtance (of the teeth or threads) to the length deſcribed by the power P, in one revolution; then they are in equilibrio.*

For in one revolution of *P*, the wheel *DC* with the weight *W*, has moved the diſtance of one tooth.

Cor.



Pl.II. pa.42.



Pl.II pa.42.

Cor. 2. *And universally, if there be several worms or spiral leaves, upon the axis AB, and the weight G hangs upon the axel EF. Then if the power P, is to the weight G :: as the radius of the axel EF × number of worms in AB, to AP × number of teeth in CD. Then the power and weight are in equilibrio.* For by *Cor.* 3. *Prop.* XXVI. *if n be the number of worms, then P : G :: n × ½ EF : AP × teeth in CD.* FIG. 40.

Cor. 3. *And by reason of the obliquity of the teeth, the force acting perpendicular to the teeth, the lateral force perpendicular to the wheel, and the direct force in the plane of the wheel; will be respectively, as radius, the sine, and cosine of the obliquity of the teeth.*

For let *GD* be the side of a tooth acted on; *GE* parallel to the axis of the wheel, and *DE* perpendicular to it, or in the plane of the wheel. Now if *GD* represent the force acting perpendicular to the tooth. Then *DE*, *GE* will be the forces acting in the directions *GE*, *DE*, (by Cor. 1. Prop. VIII.) but if *GD* be radius, *DE* is the sine of the obliquity, and *GE* the cosine. 44.

Cor. 4. *In the common screw the less the distances of the threads are, and the longer the handle is, the easier any given weight is moved.*

Cor. 5. *What is here demonstrated, will hold equally true, if the wheel CD act upon another wheel with oblique teeth, instead of the worm AB.*

SCHOL.

The force of the screw resembles the force that drives a body up an inclined plane; the force acting parallel to the base of the plane.

All things here laid down relating to the perpetual screw, do suppose that the axis of the worm spindle lies in the plane of the wheel it works in, and that their axles are perpendicular to each other; but if they are in oblique position, and the teeth of one or both also oblique, they cannot work without loss of power; a part being lost proportional to the obliquity.

If any worm spindle contains one leaf or worm, then a spindle of twice the diameter will require two worms, and one of thrice the diameter, three worms, &c. to work in the same wheel; and the power is best estimated by the rise or fall of a tooth of *CD* (Fig. 43.) for a revolution of the power *P*.

FIG.

PROP. XXX.

45. *Let EFG be the back or base of a wedge in form of an isoceles triangle; then if the power acting perpendicular to the back FG, is to the force or resistance acting against either side, in a direction perpendicular to that side; as the back of the wedge FG, to either of the sides EF, EG: Then the wedge is in equilibrio, or, which is the same thing, the power is to the whole resistance against both sides, as the back FG, to the sum of the sides EF, EG.*

For draw the axis *ED* perpendicular to the base *FG*; and *CA*, *CB* perpendicular to the sides *EF*, *EG*; then *DC* is the direction of the power. And (by Prop. 9.) the impediment to be removed, acts against the wedge in the directions *AC*, *BC*; and therefore (by Cor. 1. Prop. VIII.) the power, and the actions of the impediment, are as *FG*, *FE*, *EG* respectively, when they are in equilibrio.

Cor. 1. *The power acting perpendicular to the base, is to the force acting against either side, in a direction parallel to the base FG, or perpendicular to the axis DE; as the base FG, to the height ED: When the wedge is in equilibrio, or the power is to the whole force against both sides (in direction parallel to FG) as the back FG, to twice the height DE.*

For the force *EG* may be divided into the two *ED*, *DG*, (by Cor. 3. Prop. VII.) Then since (by this Prop.) *EG* is the force acting in direction *CB*; *ED* will be the force acting in direction *DG*.

Cor. 2. *The sharper the wedge, or the more acute its angle, the easier it will divide any thing or overcome any resistance.*

SECT.

SECT. IV.

The descent of bodies upon inclined planes, and in curve surfaces. Also the motion of pendulums.

PROP. XXXI.

If a heavy body W, be sustained upon an inclined plane AC, by a power acting in a direction parallel to that plane. Then FIG. 46.

The weight of the body,	*the length AC,*
The power that sustains it,	*the height CB,*
And its pressure against the plane:	*and the base AB,*
Are respectively, as	*of the plane.*

Draw BD perpendicular to AC; then the force of gravity tends perpendicular to the horizon, or parallel to CB; and the direction of the power is parallel to DC; and the pressure against the plane is (by Prop. IX.) parallel to DB. And therefore their quantities are respectively as the three lines CB, CD, BD, (by Prop. VIII.) that is, by similar triangles, as AC, CB, and AB.

Cor. 1. *The weight, power, and pressure on the plane, are respectively, as radius, the sine and cosine of the plane's elevation.* For the sides of a triangle are as the sines of the opposite angles.

Cor. 2. *The relative weight of a body, to make it run down an inclined plane, is as the height directly, and length reciprocally.* $\frac{BC}{AC}$; *or it is as the sine of the plane's elevation.*

Cor. 3. *If a cylinder be sustained upon an inclined plane, by a power drawing one end of a rope parallel to the plane, whilst the other end is fixed. This power is to the weight of the cylinder, as half the height to the length of the plane.* 47.

For half the relative weight of the cylinder is sustained by the other end of the rope which is fixed.

FIG.

SCHOL.

46. If it be required to find the position of the plane AC, whose height BC is given, so that the given weight W may be raised through the length of the plane AC, in the least time possible, by any given power P, acting in the direction DC. Make $AC = \frac{2W}{P} \times BC$, and you have your desire.

PROP. XXXII.

46. *If a heavy body W be sustained upon an inclined plane AC, by a power acting parallel to the horizon. Then,*

The weight of the body,
The power that sustains it,
The pressure against the plane,
Are respectively as
the base AB, the height CB, and the length AC, of the plane.

For the body is sustained by three forces, the power, the gravity, and re-action of the plane; the weight is perpendicular to AB, the power is perpendicular to CB, and the pressure is perpendicular to AC. Therefore (by Cor 1. Prop. VIII.) these forces are as AB, CB, AC.

Cor. Hence the pressure on the plane, the power, and the weight, are respectively, as radius, the sine and cosine of the plane's elevation.

PROP. XXXIII.

48. *If a heavy body W be sustained upon an inclined plane AC, by a power*
49. *P acting in any given direction WP. And if BED be let fall perpendicular on WP. Then,*

Power P,
Weight of the body W,
Pressure upon the plane,
Will be respectively, as
DB, AB, AD.

For

For since BD is perpendicular to the direction of the power, FIG.
AB to the direction of gravity, and AD to the direction of the 48.
pressure on the plane. Therefore (by Cor. 1. Prop. VIII.) these 49.
forces will be respectively as BD, AB, AD, when they are in equilibrio.

Cor. 1. *The power, weight, and pressure against the plane, are respectively as the sine of the planes elevation, cosine of the angle of traction CWP, and the cosine of the angle of direction of the power above the horizon.*

The angle of traction is the angle that the direction of the power makes with the plane. And in the triangle ABD, the sides are as the sines of the opposite angles, where $< D =$ complement of DWP.

Cor. 2. *Hence whether the line of direction of the power be elevated above or depressed below the plane; if the angles of traction be equal, equal powers will sustain the weight; but the pressure is greater when the line of direction runs below the plane.*

Cor. 3. *The power P is least when the line of direction is parallel to the plane; and infinite when perpendicular to it; and equal to the weight, when perpendicular to the horizon.*

Cor. 4. *If a weight upon an inclined plane be in equilibrio with* 50.
another hanging freely, their perpendicular velocities will be reciprocally as their quantities of matter.

For let the weight at W be made to descend to A, and draw Wr perpendicular to AE, and Wt, Dv to AB; then the weight P will have ascended a height $= Ar$, which is its perpendicular ascent; and Wt is the perpendicular descent of W. The figures $Arwt$ and $AEDv$ are similar, as are also the triangles AEB, DvB. Whence $Wt : Ar :: Dv : AE :: DB : AB =$ (by this Prop.) $P : W$.

Cor. 5. *And therefore if any two bodies be in equilibrio upon two inclined planes, their perpendicular velocities will be reciprocally as their quantities of matter.*

PROP.

FIG.

PROP. XXXIV.

51. *The space which a body (descending from rest) describes upon an inclined plane, is to the space which a body falling perpendicularly, describes in the same time, as the height of the plane CB, to its length AC.*

The force wherewith a body endeavours to descend upon an inclined plane, is equal to the power that sustains it; and (by Prop. XXXI.) that power is to the weight of the body as *CB* to *CA*. Therefore the body is urged upon the plane, by an uniformly accelerating force, which is to the force of gravity as *CB* to *CA*. But (by Prop V.) the motion generated in the same time, and in the same body, is as the force, that is (since the body is given) the velocity is as the force. And (by Prop. III.) the spaces uniformly described with the last velocities will be as these velocities; and (by Cor. 1. Prop. VI.) these spaces are double the spaces described by the accelerating forces. Therefore the spaces described on the plane and in the perpendicular, are as the last velocities, or as the forces, that is as *CB* to *CA*.

Cor. 1. *Hence if BD be let fall perpendicular to AC; then in the time a body falls through the height CB; another body, descending along the inclined plane, will run through the space CD.*

For these spaces are as *CA* to *CB*, that is, as *CB* to *CD*, by similar triangles.

Cor. 2. *The velocity acquired upon an inclined plane, is to the velocity acquired in the same time by falling perpendicularly, as CB to CA, or as CD to CB.*

Cor. 3. *The space described by a body moving down any plane in a given time, is as the sine of the plane's elevation.*

For if *CB* be given, *CD* is as the sine of *CBD* or *CAB*.

Cor. 4. *The spaces described by a body descending on any given plane, are as the squares of the times.*

PROP.

FIG.

PROP. XXXV.

The time of a body's descending through the plane CD, is to the time of falling through the perpendicular height CE, as the length of the plane DC, to the height CE. 51.

For *DE*, *AB* being perpendicular to *CB*, and *BD* to *AC*. The time of descending through *CD* or the perpendicular *CB* : time of descending through *CE* :: (by Prop. XIV.) $\surd CB$: $\surd CE$ :: *CD* : *CE*.

Cor. If the body be made to move back again with the velocity acquired in descending, it will ascend to the same height on the plane, and in the same time.

For it will be uniformly retarded in ascending; and in all points will have the same velocity in ascending as descending.

SCHOL.

Since the force by which bodies descend down an inclined plane, is a uniformly accelerative force; therefore whatever is demonstrated of falling bodies in Sect. II. holds equally true, in regard to the motion of bodies upon an inclined plane; substituting the relative weight upon the plane, instead of the absolute weight of the body.

Hence therefore a body projected on an inclined plane, will describe a parabola. And if the velocity of projection upon the plane, be to the velocity of a projectile in the air; as the relative gravity on the plane, to the absolute gravity. And both projected at the same obliquity; the same parabola will be described in both cases.

PROP. XXXVI.

A body acquires the same velocity in descending down an inclined plane CD, as by falling perpendicularly through the height of that plane CE. 51.

For draw *DB* perpendicular to *CD*, and the bodies will descend through *CD*, *CB* in the same time; then (by Cor. 2.

FIG. Prop. XXXIV.) velocity in D : velocity in B :: CE : CD, and (Cor. 1. Prop. XIV.) velocity in B : velocity in E :: $\surd CB$:
51. $\surd CE$ :: CD : CE. Therefore velocity in D : velocity in E :: CE : CE, and therefore the velocities in D and E are equal.

Cor. 1. A body acquires the same velocity in falling from any height, whether it falls perpendicularly, or down an inclined plane of equal height.

Cor. 2. Hence the velocities acquired by heavy bodies falling from the same height, to the same horizontal right line, on any planes whatever, are equal among themselves.

Cor. 3. If the velocities be equal at any two equal altitudes D, E; they will be equal at any other two equal altitudes A, B: And acquire equal increases of velocity, in passing through EB, DA of equal heights.

Cor. 4. The velocities acquired by descending down any planes whatever, are as the square roots of the heights.

PROP. XXXVII.

52. *In a circle whose diameter CB is perpendicular to the horizon, a body will descend through any cord CD or DB, in the same time as it will descend perpendicularly through the diameter CB.*

For the angle at D is right, therefore (by Cor. 1. Prop. XXXIV.) the time of descending through CD will be equal to the time of descending perpendicularly through CB. Draw CE parallel to DB, then will CE be equal to DB; and a body will descend through the cords CE, DB in the same time. But the time of descending through CE is the same as falling through the diameter. Therefore the time of descending through any cord CD, DB, is the same as falling through the diameter CB.

Cor. 1. The times of descending through all the cords of a circle, drawn from either point C or B, are equal among themselves.

Cor. 2. The velocity acquired by descending through any cord CD, or DB, is as the length of the cord.

For

For draw DF perpendicular to CB, then $CD = \sqrt{CB \times CF}$, FIG.
and $DB = \sqrt{CB \times BF}$; and (by Prop. XXXVI.) a body ac- 52.
quires the ſame velocity in deſcending through CD, as in falling through CF, but this (by Cor. 1. Prop. XIV.) is as $\sqrt{CF}$, that is as CD. Alſo a body acquires the ſame velocity through DB as FB, and that is as $\sqrt{BF}$, or as DB.

Cor. 3. But a body will deſcend ſooner through the ſmall arch of a circle, than through its cord TB.

For if BG, TG be two tangents, then the relative gravity at T in the arch and cord, will (by Cor. 1. Prop. XXXI.) be as the ſines of the angles TGO, TBO, or as BT and TG, or BG, that is nearly as two to one when the arch is very ſmall. And the accelerative force in the circle being double to that in the cord; therefore the velocity will be greater in the arch, and the time of deſcription ſhorter, though their lengths are nearly the ſame.

PROP. XXXVIII.

If a body deſcends freely along any curve ſurface, and another body deſcends from the ſame height in a perpendicular right line; their velocities will be equal at all equal altitudes. 53.

Let a body deſcend from A towards C perpendicular to the horizon BC; and another deſcend through the curve ſurface AKB. Divide AC into an infinite number of equal parts, at the points D, E, F, &c. to which draw lines parallel to BC, interſecting the curve in I, K, G, &c. then the curve line AKB will be divided into an infinite number of parts, IK, KG, &c. which may be taken for right lines; or the curve ſurface into an infinite number of planes, joining at I, K, G, &c.

Now if the velocities be ſuppoſed to be equal in any correſpondent points as I and D, then (by Cor. 3. Prop. XXXVI.) they will be equal in K and E, after the deſcent through IK; and being equal in K and E, they will alſo be equal in G and F, after the deſcent through KG, and ſo on. Therefore ſince the motion begins in A, they will acquire equal velocities in deſcending through the firſt plane, and likewiſe through the 2d, 3d, 4th, &c. And therefore the velocities will be equal in all correſpondent points I and D, K and E, G and F, &c. and at B and C.

FIG. 53. *Cor. 1. Therefore if a body be suspended by a string, and by oscillating describes any curve AB; or if it is any way forced to move in any polished, and perfectly smooth surface AB; whilst another body ascends or descends in a right line. Then if their velocities be equal at any one equal altitude; they will be equal at all other equal altitudes.*

For the same thing is effected by the string of the pendulous body, as by the smooth surface of a polished body.

Cor. 2. Hence a body oscillating in any curve line whatever, acquires the same velocity in the curve; as if it had fallen perpendicularly from the same height. And therefore the velocity in any point of the curve, is as the square root of the height descended.

Cor. 3. And a body after its descent through any curve, will ascend to the same height in a similar and equal curve, or even in any curve whatever. And the velocities will be equal at all equal altitudes. And the ascent and descent will be in the same time, if the curves are the same.

For the forces that generated the motion in descending, will equally destroy it in ascending, and therefore they will lose equal velocities by ascending equal heights. And if the curves are similar and equal, every particle of the curve will be described with the same velocity, and therefore in the same time, whether ascending or descending.

Cor. 4. This Prop. is equally true, whether the curve AKB be in one plane perpendicular to the horizon, or in several planes IK, KG, &c. winding about in nature of a spiral.

PROP. XXXIX.

54. *The times of descent through two similar parts of similar curves, are in the subduplicate ratio of their lengths, ab, AB.*

Divide both curves into an equal number of infinitely small parts, similar to each other; and let *bc*, *BC*, be two of them, similarly posited; and draw *rb*, *RB*, perpendicular to *ab*, *AH*. By Prop. III. the space described is as the time and velocity, and the time of describing any space, is as the space directly and velocity reciprocally. By Cor. 2. Prop. XXXVIII. the velocities in

in b and B are as $\sqrt{rb}$ and $\sqrt{RB}$, that is because arb, ARB FIG.
are similar, as $\sqrt{ab}$ and $\sqrt{AB}$. Therefore the time of de- 54.
scribing bc : to time of describing BC :: $\frac{bc}{\sqrt{ab}} : \frac{BC}{\sqrt{AB}}$ :: $\frac{ab}{\sqrt{ab}} : \frac{AB}{\sqrt{AB}}$ :: $\sqrt{ab} : \sqrt{AB}$ :: $\sqrt{ad} : \sqrt{AD}$, because the curves are similarly divided. Whence, by composition, the whole time of describing ab : whole time of describing AB :: is in the same given ratio of $\sqrt{ab} : \sqrt{AB}$, or $\sqrt{ad} : \sqrt{AD}$.

Cor. 1. *Hence if two pendulums describe similar arches, the times of their vibrations, are as the square roots of their lengths : Or the lengths as the squares of the times of vibration.*

For let bd, Hd, be the lengths of the pendulums; then because the figures are similar, it is $ad : AD :: bd : HD$.

Cor. 2. *If a pendulum vibrates in a circle, the velocity in the lowest point, is as the cord of the arch it described in descending.*

For (by Cor. 2. Prop. XXXVIII.) it acquires the same velocity in the arch as in the cord; and (by Cor. 2. Prop. XXXVII.) the velocity in the cord, is as the cord.

Cor. 3. *The lengths of pendulums vibrating in similar arches, are reciprocally proportional to the squares of the number of their vibrations, in a given time.*

PROP. XL.

If a pendulum vibrates in a cycloid, the time of one vibration, is to 55.
the time of a body's falling perpendicularly through half the length of the pendulum, as the circumference of a circle to the diameter.

Let ADa be the cycloid, FD its axis, FGD the generating circle. Let the body descend from H, and in vibrating describe the arch Hdb. Divide HD into innumerable small parts, and let Bb be one of them. Through H, B, b, draw HMh, BL, bl, perpendicular to the axis FD. About the diameter MD describe the semi-circle MLD; and from its center Q, draw QL, also draw LP parallel to MD, and DE, DG, GE.

The triangles CDG, GDE are similar, and $CD \times DE = GD^2$. Also the triangles QLN and lLP are similar, and $NL : PL$ :: $QL : Ll$, and $2Nl : Nn :: MD : Ll$. And since by the nature of the cycloid, the tangent in B is parallel to the arch GD, therefore Gg is equal and parallel to Bb.

Now suppose a body to descend from E through the inclined plane ED, since this is a motion uniformly accelerated, therefore

(by

FIG. 55. (by Cor. 1. Prop. VI.) it would, in the time of its fall, describe $2ED$, with the velocity acquired in D. And since (by Cor. Prop. III.) the times are as the spaces directly, and velocities reciprocally; and (by Cor. 2. Prop. XXXVIII.) the velocities are as the square roots of the heights; therefore it will be, as time of describing ED : time in Cc :: $\frac{2ED}{\sqrt{MD}} : \frac{C}{\sqrt{MN}}$ :: $\frac{2MD}{\sqrt{MD}}$ or $2\sqrt{MD} : \frac{Nn}{\sqrt{MN}} :: 2\sqrt{MD \times MN} : Nn$; by similar triangles.

Again, when the velocity is given, the time is as the space described. Therefore it will be, as time in Cc : time in Bb :: $Cc : Bb$ or $Gg :: CD : GD$ or $\sqrt{CD \times DE} :: \sqrt{CD} : \sqrt{DE} :: \sqrt{DN} : \sqrt{DM}$; by similar triangles. Therefore ex equo, time in ED : time in $Bb :: 2\sqrt{MD \times MN \times DN} : Nn\sqrt{DM} :: 2\sqrt{MN \times DN}$ or $2NL : Nn :: MD : Ll$. Therefore, by composition it is, as time in ED : time in the arch $Hb :: MD$: arch Ml. And as the time in ED : whole time in $HD :: MD$: semi-circumference MLD.

And since the time of descending through HD is equal to the time of descending through Db: and (by Prop. XXXVII.) the time of descending through ED is equal to the time in the diameter FD. And $2FD$ is $= DV$, the length of the pendulum. (being the radius of curvature in D); therefore as the time of falling through half the length of the pendulum FD : time in HDb, or time of one vibration :: diameter MD : circumference $2MLD$.

Cor. 1. Hence all vibrations great and small, are performed in equal times. And in unequal arches the velocities generated, and the parts described, and those to be described, in the same time, will always be as the whole arches; and in any arch HD, the accelerative force at any point B, will be as the length BD from the bottom.

For the descent through HD is always the same, wherever the point H is taken. Also (by the nature of the cycloid) the tangent at B is parallel to GD, and $BD = 2DG$. And (by Cor. 2. Prop. XXXI.) the relative weight on GD (which is the accelerating force along GD, or the tangent at B) is as $\frac{ND}{GD}$, that is as $\frac{GD}{FD}$, or $\frac{2GD}{FD}$, or as $\frac{BD}{FD}$, or as BD, because FD is given. Whence the accelerating force being always as the distance from the bottom, therefore, in any two arches, the ve-

 locities

locities generated every moment, and the parts continually described, will be as these forces; that is, as the whole arches. And consequently the spaces described, and the velocities generated, in any time, will be as the whole arches; and therefore the parts to be described will also be as the wholes. FIG. 55.

Cor. 2. *The time of descent in HB, to the time of descent in HD, is as the arch ML, to the semi-circumference MLD.*

Cor. 3. *The velocity of the pendulum in any point B, is as* $\sqrt{DH^2-DB^2}$, *or* $\sqrt{HB\times BDh}$.

For (by Cor. 1. Prop. XIV.) the square of the velocity in B is as MN, that is as $MD-ND$, or $\frac{ED^2-GD^2}{DF}$ or DH^2-DB^2; because $DH=2DE$, and $DB=2GD$, by the nature of the cycloid; and DF is given.

Cor. 4. *If the length of the pendulum VD be made double the axis FD; and ARV, arV, be two semi-cycloids equal to AHD, and so placed that the vertex (as D) be at A and a. Then the pendulum VH vibrating between the cycloidal checks ARV, arV; the point H will describe the cycloid AHDha; and the time of its vibration will be* 3.1416 × *time of falling through FD, half the length of the pendulum.* 56.

All this follows from the nature of the cycloid.

Cor. 5. *Hence also it appears from experiments on pendulums, that at the surface of the earth, a heavy body will descend through a space of* 16 $\frac{1}{12}$ *English feet nearly, in one second of time.*

For it is found by observations upon clocks. that a pendulum 39, 13 inches long, vibrates once in a second; therefore $\frac{1}{3.1416}$ = time of a body's falling through FD or $\frac{39.13}{2}$ inches. And consequently (by Prop. XIV.) the space fallen through in one second will be $\frac{39.13}{2}\times\overline{3.1416}^2=193{,}096$ inches = 16.0913 feet. Yet a pendulum vibrating freely will be something longer in vibrating than a clock, because the palate wheel of the clock acting against it, takes off something from its ascent, and makes it return sooner, or shortens the time.

Cor. 6. *Hence also if pendulums of the same quantity of matter and any lengths, be acted on by different forces of gravity, their lengths will be as the forces of gravity, and the squares of the times of vibration.*

For the times of vibration are in a given ratio to the times of descent through half the lengths of the pendulums. And (by Cor. 3. Prop. V.) if the matter be given, the velocity generated

FIG. nerated in defcending bodies is as the force and time; and (by
56. Prop. VI.) the fpace defcended is as the velocity and time, that is as the force and fquare of the time. Therefore half the length of the pendulum, is as the force, and fquare of the time of defcending half its length; whence the length is as the force and fquare of the time of vibration.

Cor. 7. From the motion of pendulums it alfo follows, that in any one place, the quantity of matter in any body is proportional to its weight. For it is certain from experience that pendulums of equal length, whatever quantities of matter they contain, vibrate in the fame time. Therefore they will defcend through half the length of the pendulum in the fame time; and confequently would acquire equal velocities in the fame time. Therefore (by Prop. V.) the velocity and time being given, the quantity of matter is as the force of gravity.

Cor. 8. Hence it alfo follows, that there are vacuities or empty fpaces in bodies. For fince (by Cor. 7.) the quantity of matter is as the weight of the body, if it were true that there is an abfolute *plenum*, all bodies of the fame bulk muft be of equal weight; which is contrary to all experience.

PROP. XLI.

57. *If a pendulum AT ofcillates in a circle TRQ, and in the mean time be acted on in the feveral points T, by a force tending perpendicular to the horizon, which is to the uniform force of gravity; as the arch TR, is to the fine TN: The times of all vibrations will be equal, whether great or lefs.*

For from any point T draw TZ perpendicular to the horizon, and TY a tangent to the circle in T; and let AT exprefs the uniform force of gravity, TZ the variable force at T; draw ZY perpendicular to TY. Then the force TZ will be refolved into the two TY, YZ. Of which YZ, acting in direction AT, does not at all change the motion of the body. But the force TY directly accelerates its motion in the circle TR. The triangles ATN, ZTY are fimilar, and $TZ : TA :: TY : TN$; but (by fuppofition) $TZ : TA ::$ arch $TR : TN$; therefore $TY =$ arch TR: that is, the force TY is as the arch to be defcribed TR. Therefore if AT, At, be let fall together from the points T, t; the velocities generated in equal times, will be as the forces TY, ty; that is, as the arches TR, tR, to be defcribed. But the parts defcribed at the beginning of the motion, are as the velocities; that is, as the wholes to be defcribed

at

at the beginning; and therefore the parts which remain to be deſcribed, and the ſubſequent accelerations proportional to theſe parts, are alſo as the wholes, &c. Therefore the velocities generated, and the parts deſcribed with theſe velocities, and the parts to be deſcribed, are always as the wholes. And therefore the parts to be deſcribed, being every where as the velocities they are deſcribed with, will be deſcribed in equal times and vaniſh together; that is, the two bodies oſcillating will arrive at the perpendicular AR together. FIG. 57.

Cor. 1. *Hence that the vibrations in a circle may be iſocronal; the force* TZ *muſt be* $= \frac{TR}{TN} \times$ *gravity.*

Cor. 2. *Hence if a pendulum vibrates by the force of gravity only, the times of vibration, in very ſmall different arches, will be very nearly equal.*

For in ſmall arches the ratio of the arch to the cord is nearly a ratio of equality.

Cor. 3. *But the time of vibration in larger arches, is greater than the time in leſs arches of a circle.*

For the gravity at T being leſs than the iſocronal force, the body will be longer in deſcribing that arch.

Cor. 4. *Hence alſo if a pendulum vibrates in the ſmall arch of a circle, the time of one vibration is to the time of a body's falling through twice the length of the pendulum, as half the circumference of a circle to the diameter.*

For AR is the radius of curvature of a cycloid, whoſe axis is $\frac{1}{2}$ AR. Therefore the circle and cycloid coincide at R, and the ſmall arches of both will be deſcribed in the ſame time; that is, as expreſſed by Prop. XL. only here we take twice the length of the pendulum and half the circumference, which comes to the ſame thing, by Cor. 1. Prop. XXXIX.

SCHOL.

In theſe propoſitions, the vibrating body is ſuppoſed to be very ſmall, and is therefore conſidered only as a point. But if it be of any determinate bigneſs, the point to which the length of the pendulum is meaſured, is not in the middle or center of gravity of the body; but in another place, and is called the center of oſcillation, as will appear in the VI. Section.

FIG. 57. It has been proved, that the same pendulum is longer in vibrating in a large arch of a circle than in a small one. And it may be computed, that if a pendulum vibrates seconds in an extremely small arch; and C be the length of inches of the cord of any arch A; then $3\frac{1}{2}CC$ will be the seconds lost in 24 hours by vibrating in the arch $2A$.

And if a pendulum vibrates seconds in an arch $2a$, and c be the cord of a, or of half the whole arch. Then $3\frac{1}{2} \times \overline{CC-cc}$ will be the seconds lost in 24 hours, by vibrating in the arch, the cord of whose half is C.

Also if the bob of such a pendulum can be screwed up or down; and you put n=number of threads of the screw contained in an inch, y=time in minutes that the clock gains or loses in 24 hours. Then it follows, by the theory of pendulums, that $\frac{2}{37}ny$ will be the number of threads or revolutions of the nut, that the bob is to be let down or raised up, to beat seconds.

SECT.

SECT. V.

Of the center of gravity and its properties.

PROP. XLII.

If a line be drawn from the center of gravity of a body perpendicular to the horizon; and this perpendicular falls within the base upon which the body rests, the body will stand; but if it falls without the base, it will fall down.

CASE I.

Let *C* be the center of gravity, *CD* perpendicular to the horizon, falling within the base *BEFG*; draw *RC*, and suppose the whole body suspended at the point *C*; then (by Def. 12.), the body will be in equilibrio, and remain at rest upon *DC*. Now take away *DC*, and suppose the body to be supported only upon the line *RC*, moveable about *R*; then (by Ax. 8) the body *AB*, together with the line *RC* will endeavour to descend from the position *RC* towards *D*. Also, for the same reason, the body and the line *CS* will endeavour to descend from the position *CS* towards *D*; but as these two motions oppose each other, the body will be sustained by the points *R*, *S*, and therefore it will stand. And the same is true of every two opposite points *R*, *S*. FIG. 58.

CASE II.

But if *CD* fall without the base, then the line *RC* and the body at *C* will endeavour to descend towards *D*; also the body *C* and line *CS* will endeavour to descend towards *D* likewise; and as this motion does not oppose the other, there will be nothing to support the body; therefore it must necessarily fall towards *D*. 59.

FIG. 59. *Cor.* 1. *Hence it follows, that if the center of gravity of a body be ſupported, the whole body is ſupported. And the center of gravity of the body muſt be eſteemed the place of the body. And if it be ſuſtained by any leaver or beam, its place is at the point where the beam is cut by a line drawn from the center of gravity perpendicular to the horizon.*

Cor. 2. *All the gravity of a body, or the force it endeavours to deſcend with, is collected into the center of gravity; and therefore whatever ſuſtains the center of gravity, ſuſtains the whole weight. And the deſcent of a body muſt be eſtimated by the deſcent of its center of gravity.*

Cor. 3. *Hence alſo the larger the baſe is, upon which a body ſtands, and the farther within it the center of gravity lies, the firmer the body will ſtand, and the more difficult to be removed. On the contrary, the leſs the baſe, or the leſs the center of gravity falls within it, ſo much the eaſier it is to be moved out of its place.*

60. *Cor.* 4. *If a body be laid upon a plane GF, and one end F gradually raiſed up, the body will ſlide down the plane, if the perpendicular CD fall within the baſe; but if it fall without, it will roll down.*

PROP. XLIII.

61. *The common center of gravity C of two bodies A, B, is in the right line joining their centers of gravity; and the diſtance of either body from the common center of gravity, is reciprocally as the quantity of matter in it.*

Let *A*, *B* be the centers of gravity of *A* and *B*, and ſuppoſe *AB* to be an inflexible right line, or leaver; and *C* the fulcrum. Then if *C* be the center of gravity of the bodies *A*, *B*; thoſe bodies (by def. 12.) will be in equilibrio. And conſequently (by Cor. 4. Prop. XIX.) $AC : CB :: B : A$.

Cor. 1. *If there be never ſo many bodies, the common center of gravity of them all, is in the right line drawn from the center of gravity of any one, to the common center of gravity of all the reſt; and it divides*

divides this line into two parts, reciprocally as that body to the sum of all the rest of the bodies. FIG. 61.

For let D be another body, and let B and A be placed in C, then will $C : D :: DE : CE$. And so on for more bodies.

Cor. 2. *If several bodies A, B, D, E, F, be in equilibrio upon a streight leaver AF, then the fulcrum C is at the common center of gravity of all those bodies.* 62.

PROP. XLIV.

If there be several bodies, A, B, D, E, F; and if any plane PQ be drawn perpendicular to the horizon; the sum of the products of each body multiplied by its distance from that plane, if they are all on one side; or their difference, if on contrary sides; is equal to the sum of all the bodies multiplied by the distance of their common center of gravity from that plane. 63.

Draw lines perpendicular and parallel to the plane PQ as in the fig. and let C be the center of gravity. Then (by Cor. 3. Prop. XIX.) the force of all the bodies to move the plane PQ about R, will be $mD\times D+oE\times E+rF\times F-Ak\times A-Bl\times B$. That is $\overline{dC+RC}\times D+\overline{eC+RC}\times E+\overline{RC-fC}\times F-\overline{aC-RC}\times A-\overline{bC-RC}\times B$, or $dC\times D+eC\times E-fC\times F-aC\times A-bC\times B+RC\times\overline{D+E+F+A+B}$. But because C is the center of gravity of the bodies, therefore (By Prop. XX.) $dC\times D+eC\times E=fC\times F+aC\times A+bC\times B$; therefore we have $mD\times D+oE\times E+rF\times F-Ak\times A-Bl\times B=RC\times\overline{A+B+D+E+F}$.

Cor. 1. *This Prop. is equally true for any plane whatever.* For suppose the plane and the bodies to be put into any oblique position, all the distances will remain the same as before.

Cor. 2. *If any plane be drawn through the common center of gravity C, of any number of bodies A, B, D, &c. and each body be multiplied by the distance of its center of gravity from that plane; the sum of the products on each side are equal: $A\times aC+B\times bC+F\times fC=D\times dC+E\times eC$.*

For the distance of a body must be estimated by the distance of its center of gravity.

FIG. 63. *Cor.* 3. *Hence also the sum (or difference) of the products of each particle of a body multiplied by its distance from any plane whatever, is equal to the whole body multiplied by the distance of its center of gravity from that plane. And if the plane pass through the center of gravity, the sums of the products on each side are equal.*

Cor. 4. *The sum of the forces of a system of bodies is the very same, as if all the bodies were collected into their common center of gravity, and exerted their several forces there.*

For the sum of all the forces are $mD \times D + oE \times E$, &c. or $RC \times \overline{A+B+D+E+F}$.

Cor. 5. *And the same is true of any forces whatever, with regard to the center of gravity of those forces. And therefore if several forces act in parallel directions; the sum of all these forces will be equivalent to one single force; and their common center of gravity, the place where it acts.*

80. *Cor.* 6. *If a circle be described about the center of gravity G, of a system of bodies A, B, C; and any point S be taken at pleasure in the circumference; then $SA^2 \times A + SB^2 \times B + SC^2 \times C$, is a given quantity. And the same holds true for the surface of a sphere, and the bodies not all in one plane.*

For draw SG, on which let fall the perpendiculars Aa, Bb, Cc. Then (by Eucl. II. 12, 13.) $SA^2 \times A + SB^2 \times B + SC^2 \times C = \overline{SG^2 + GA^2 + 2SG \times Ga} \times A + \overline{SG^2 + GB^2 - 2SG \times Gb} \times B + \overline{SG^2 + GC^2 + 2SG \times Gc} \times C$. But (by Cor. 2.) $Ga \times A - Gb \times B + Gc \times C = 0$, and all the rest are given quantities.

PROP. XLV.

64. *If there be several forces in one plane, acting against one another in the point C, whose quantities and directions are CA, CB, CD, CE, CF; and if they keep one another in equilibrio: I say C is the common center of gravity of all the points A, B, C, D, E. And anyone of them as EC being produced, will pass through the center of gravity G of all the rest.*

Since

Since all the forces are in equilibrio, the sum of the forces acting against EC will destroy its effects, and act against it in the same line of direction. FIG. 64.

Upon EC let fall the perpendiculars Aa, Bb, Dd, Ff. Then any force AC is divided into two Aa, aC. Now as the point C is in equilibrio, all the perpendicular forces Aa, Bb, on one side, are equal to all those Dd, Ff, on the other, by Ax. 11. And if the body 1 be supposed to be suspended at A, B, D, E, F. Then since $Aa \times 1 + Bb \times 1 = Dd \times 1 + Ff \times 1$; the center of gravity of the bodies A, B, C, D, (and also of all the bodies) is in the line EC. Again it follows from the equilibrium of the forces, that $EC + aC = Cb + Cd + Cf$, by Ax. 11. And therefore if the body 1 be suspended at the points E, a, d, b, f; C is their center of gravity. That is, C is the center of gravity of E, A, B, D, F.

Cor. 1. *If G be the center of gravity of A, B, D, F; then $EC = CG \times$ number of points A, B, D, F.*

For $EC = Cb + Cd + Cf - Ca = CG \times \overline{A + B + D + F}$, or $4CG$, by Prop. XLIV.

Cor. 2. The sum of all the perpendiculars on one side, Aa, Bb = sum Dd, Ff, on the other side of EC. And the sum of their distances CE, Ca, on one side = sum Cd, Cb, Cf, on the other side of C.

PROP. XLVI.

If a body be acted on by several forces A, B, C, D, E, in the parallel directions Aa, Bb, &c. and kept in equilibrio; and if any plane RN be drawn from any point R: The sum of the forces on each side are equal, $A + D = B + C + E$; and the sum of the products on the other side, $Ra \times A + Rd \times D = Rb \times B + Rc \times C - Re \times E$; and the contrary: Where any product lying the contrary way from R, must be taken negative. 65.

For suppose RN to be the plane, acted on by these forces; then (by Cor. 5. Prop. XLIV.) the effect of the forces A and D acting at a and d, is the same as if they both acted at O, their center

FIG. 65. center of gravity. And the effect of B, C, E, acting at b, c, e, is the fame as if they all acted at their center of gravity; which, becaufe the body is unmoved, is the fame point O. And therefore, becaufe of the equilibrium; $A+D=B+C+E$, for the quantity of force.

In refpect to their places, becaufe O is the center of gravity of A and D, as well as of B, C, and E; therefore (by Prop. XLIV.) $Ra\times A+Rd\times D=RO\times\overline{A+D}=RO\times\overline{B+C+E}=Rb\times B+Rc\times C-Re\times E$. And on the contrary, if thefe forces be equal, the body will be in equilibrio, by Ax. 9.

66. *Cor.* 1. *If a body FGHI be at reft whilft it is acted upon by feveral forces, in the fame plane, whofe quantities and directions are pA, qB, rC, sD, tE, cutting any line RN drawn in the body at p, q, &c. and the perpendiculars Aa, Bb, &c. are drawn; then I fay,* 1. *The fum of the perpendicular forces on each fide are equal,* $Aa+Dd=Bb+Cc+Ee$. 2. *The fums of the contrary forces in direction of the line RN are equal,* $pa+qb=rc+sd+te$. 3. *The fum of the rectangles on each fide, from any point R, are equal,* $Rp\times Aa+Rs\times Dd=Rq\times Bb+Rr\times Cc+Rt\times Ee$. *But where the points lie the contrary way from R, the rectangles muft be negative. And when all thefe are equal, the body is at reft.*

For fince it is the fame thing whether any force A act, at A at F, or at p, we will fuppofe it to act at p; then if the oblique force pA be divided into the two pa, aA; and the fame for the reft: then the fum of all the forces pa muft be equal to the fum of all the contrary forces cr, by Ax. 11. The reft follows from this Prop.

Cor. 2. *And if a body be kept in equilibrio by feveral forces acting at different points, and in different directions, either in the fame plane, or in different planes; it will ftill be in equilibrio, by the fame forces, acting from any one point, and in directions refpectively parallel to the former.*

For, in the fame plane, the forces parallel and perpendicular to RN, will remain the fame as before. And when the directions of any of the forces are out of this plane, all thefe extraneous forces may be reduced to others, one part acting in the plane, the other perpendicular to it; and both thefe remain the fame in quantity as before. And fince the forces acting in the plane, kept one another in equilibrio at firft, they will do the fame ftill. And as the parts perpendicular to this plane, alfo kept one another in equilibrio at firft, they will

do

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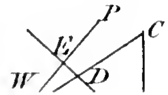

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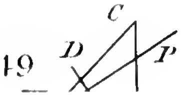
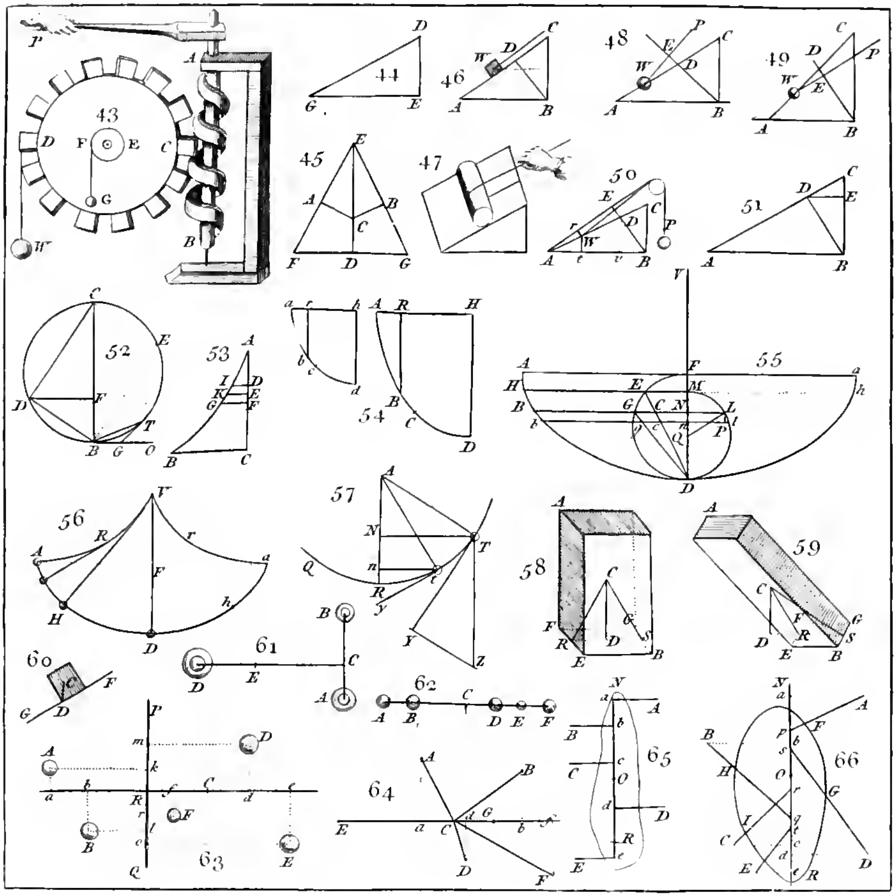




Pl III pa 64.

do the same when applied to their common center of gravity, FIG.
or to any other point. 66.

Cor. 3. *If several forces acting after any manner keep a body unmoved; and any plane whatever be drawn; and the vagrant forces be all reduced to that plane; then all the perpendicular forces on one side, are equal to those on the other; and their centers of gravity fall in the same point. When this does not happen in all planes, the body will be moved some way or other.*

PROP. XLVII.

To find the center of gravity of a system of bodies, A, B, C.

Draw any plane ST, and from the centers of gravity of all the bodies, draw perpendiculars to this plane, Aa, Bb, Cc; then (by Cor. 3. Prop. XIX.) the forces of A, B, C, at the distances Aa, Bb, Cc, from the plane, will be $A\times Aa$, $B\times Bb$, $C\times Cc$. Let G be the center of gravity, then the sum of the forces $A\times Aa+B\times Bb+C\times Cc$ must be $=\overline{A+B+C}\times Gg$, the power of all the bodies situated in G (by Prop. XLIV.) whence the distance of the center of gravity from the plane, that is $Gg=\dfrac{Aa\times A+Bb\times B+Cc\times C}{A+B+C}$: where if an of the bodies be situate on the other side of the plane, the correspondent rectangles will be negative. 67.

And if the distance be in like manner found from the plane TV, perpendicular to ST, the point G will be determined by making the parallelogram TG with the respective distances from those planes.

Cor. 1. *Let b be any body, p any particle in it, d its distance from a given plane; then the distance of its center of gravity from that plane is* $=\dfrac{\textit{Sum of all the } dp^s}{b}$.

68.

Cor. 2. *To find the center of gravity of an irregular plane figure. Suspend it by the string AEB, at E; and draw the plumb line ECF. Then suspend it by another point of the string as D, and draw another plumb line through E, to intersect CF; and the point of intersection is the center of gravity.*

FIG. 68. *Cor. 3. To find the center of gravity of a flexible body: lay it upon a board whose center of gravity is known; lay the center of gravity of the board upon the edge of a prism; and lay the body upon it, and remove it back or forwards, till it be in equilibrio upon the board.*

SCHOL.

The centers of gravity of several planes and solids have been determined to be as follows.

69. 1. If two lines be drawn from two angles of a *triangle*, to the middle of the opposite sides, the point of intersection is the center of gravity. Therefore the distance of the center of gravity from the vertex, is $\frac{2}{3}$ of the line bisecting the opposite side.

2. In a *Trapezium ABCD*, the center of gravity is found by dividing it into triangles. Find *E*, *G*, the centers of gravity of the triangles *ADB*, *CDB*; and *F*, *H*, the centers of gravity of *ABC*, *ADC*. Then draw *EG*, *FH*, to intersect in *O*, the center of gravity of the Trapezium.

3. The center of gravity of a *right line*, *parallelogram*, *cylinder*, and *prism*, is in the middle.

4. For the *arch of a circle*, as $\frac{1}{2}$ arch : sine of $\frac{1}{2}$ arch :: radius : distance of its center of gravity from the center.

5. For the *sector of a circle*, as arch : cord :: $\frac{2}{3}$ radius : distance of its center of gravity from the center.

6. For the *parabolic space*, the distance of the center of gravity from the vertex is $\frac{3}{5}$ of the axis.

7. In the *cone and pyramid*, the distance of the center of gravity from the vertex is $\frac{3}{4}$ of the axis.

8. In a *paraboloid*, the distance of the center of gravity from the vertex is $\frac{2}{3}$ of the axis.

9. For the *segment of a sphere*, let $r =$ radius, $x =$ height of the segment; then the distance of the center of gravity from the vertex is $\frac{8r-3x}{12r-4x}x$.

PROP. XLVIII.

If two or more bodies move uniformly in any given directions, their common center of gravity will either be at rest, or move uniformly in a right line,

Case

Case 1. Let one body ſtand ſtill, and the other move directly to or from it in a right line. Then ſince the center of gravity divides the diſtance, in a given ratio; and the diſtance increaſes uniformly, therefore that center moves uniformly. Now ſuppoſe the other body likewiſe to move in the ſame right line, and any quantity of ſpace to move along with it; then ſince the body is relatively at reſt in this ſpace, the center of gravity, in regard to that ſpace, moves uniformly; to which adding or ſubtracting the uniform motion of that ſpace; the center of gravity will ſtill move uniformly. FIG. 69.

Case 2. Let the bodies move in one plane, in the directions DE, AB. Produce their lines of direction till they meet in D. And when one body is in D and E, let the other be in A and B reſpectively. Let H be their center of gravity, when in D and A, and K when in E and B, and draw HK, and make $BP = AD$, and draw EP, and KL parallel to AB. Then DE is to AB or DP in the given ratio of the motion of the bodies; and ſince the $\angle EDP$ is given, therefore all the angles of the triangle EDP are given, and DP will be to PE in a given ratio. But by ſimilar triangles PE is to PL in the given ratio of BE to BK, by the property of the center of gravity: therefore DP is to PL in a given ratio. And all the angles in the triangle DPL are given, and therefore the angle PDL. Therefore the point L is always in the line DL given in poſition. And by the nature of the center of gravity, $DA : DH :: EB : EK :: PB$ or $DA : LK$. Therefore $DH = LK$, whence $DHKL$ is a parallelogram, and HK parallel to DL, and therefore the angle BHK is given; and the center of gravity K is always in the right line HK given by poſition. And becauſe all the angles of the triangles DPL, and DLE are given; therefore the lines DP, DE, DL, that is, AB, DE, HK are in a given ratio; and conſequently the point K moves uniformly along the right line HK. And the demonſtration is in the ſame manner, if one of the bodies B moves from B towards A. 70.

Case 3. Let the paths of the bodies AB, DE be in different planes. Through the path AB draw a plane Bde parallel to the path DE, and through DE draw the plane $DdeE$ perpendicular to Bde, produce AB to d, and let Dd, Ee be perpendicular to de. Then the planes DdA, EeB will be perpendicular to the plane edB. Let one body be in A and B, when the other is in D and E reſpectively. Now if the body at D were to move in de, then 71.

 by

FIG. 71. by Cafe 2d, the center of gravity would move uniformly along fome right line *HK*; through *HK* erect the plane *HKkb* perpendicular to *HBK*. Then by fimilar triangles, and the nature of the center of gravity, $Ab : bD : : (AH : Hd : : BK : Ke : :) Bk : kE$. And *bk* is the path of the center of gravity of the bodies moving in *AB*, *DE*. Likewife $Dd : Hb : : Ad : AH : : bE : BK : : eE$ or $Dd : Kk$; therefore $Hb = Kk$, and *bk* is equal and parallel to *Hk*; therefore the center of gravity of the bodies (moving in *AB*, *DE*) moves uniformly through the right line *bk*.

Cafe 4. The common center of gravity of two bodies, and a third is either at reft, or moves uniformly in a right line; for thefe two may be put into the place of their center of gravity, which before moved uniformly; and then the center of gravity of the three will move uniformly. Likewife the common center of gravity of three bodies and a fourth, will move uniformly in a right line; and fo on.

PROP. XLIX.

The common center of gravity of two or more bodies, does not change its ftate of motion or reft, by any actions of the bodies among themfelves, or by any forces they exert upon one another.

Suppofe any fpace in which the bodies are inclofed, to move uniformly along with the center of gravity of the bodies, before the actions of the bodies upon one another; then the center of gravity is at reft in that fpace. Now, if two bodies mutually act upon one another, fince their diftances from their center of gravity are reciprocally as the bodies; and as action and reaction are equal, the bodies will approach or recede from that center by fpaces which are in the fame ratio; therefore the center of gravity will ftill remain at reft. And in a fyftem of feveral bodies, becaufe the common center of gravity of any two acting mutually upon each other, is at reft: and the actions of all the bodies being the fum of the actions of every two, it is evident the center of gravity of all the bodies remains the fame, as if they did not act at all upon one another; and therefore is at reft in this fpace, or moves uniformly forward along with it.

Cor.

Cor. 1. *Hence if a body be projected into free space, if it have any circular motion, this motion will be performed uniformly about an axis passing through the center of gravity.* FIG. 71.

For if every particle of the body retained the distinct motion first impressed on it; the common center of gravity of the whole would move in a right line, by the last Prop. And since the cohesion of the parts of the body retains the particles in one mass, therefore (by this Prop.) the motion of the center of gravity is not altered, Which it would be if the axis of circular motion did not pass through the center of gravity, but through some other point.

Cor. 2. *And if a body be hurled into the air, its center of gravity will either move in a right line, or describe a parabola; whilst that body revolves about an axis passing through the center of gravity, if it have any circular motion.*

PROP. L.

The sum of the motions of several bodies in any given direction, is the same as the motion of all the bodies in the same direction, moved with the velocity of their common center of gravity.

Let the bodies, A, B, move round the center of gravity C at rest, to the places a, b; draw BCA, bCa. Then since $A : B :: BC : AC :: bc : ac$; therefore the triangles ACa, BCb, are similar, and $< bBc = CAa$, therefore Bb is parallel to Aa, and the bodies move in contrary directions. Also since $Aa : Bb :: AC : CB :: B : A$, or $Aa \times A = Bb \times B$. Therefore the motions of A, B, in contrary directions are equal, or their motion the same way is 0. Now let the space and bodies moving in it, be moved in any direction with any velocity v; it is manifest, the motion of each body in that direction will be greater than before, by the quantity of matter $\times$ velocity. Therefore the sum of the motions is now $vA + vB$ or $v \times \overline{A+B}$, that is equal to the sum of the bodies $\times$ velocity of the center of gravity. 72.

After the same manner, the motion of three bodies is the same as the motion of two of them, moved with the velocity of their common center of gravity, together with the motion of the third;

FIG. 72. third; that is (by what has been ſhown) equal to the ſum of all the three, moved with the velocity of the center of gravity of all the three. And ſo for more bodies.

Cor. The center of gravity of a body muſt be taken for the place of the body. And the motion of any body, or of any ſyſtem of bodies, muſt be eſtimated by the motion of the center of gravity.

PROP. LI.

If two weights on any machine keep one another in equilibrio, if they be any how raiſed or moved by help of the machine, the center of gravity of the weight and power, will always be in the ſame horizontal right line.

For in the leaver, the center of gravity is at the fulcrum, and therefore it neither aſcends nor deſcends. In the wheel and axel, and in the pulley or any combination of pulleys, the weight and power approach or recede from each other, by ſpaces which are reciprocally as the bodies; and therefore their center of gravity is at reſt. And upon any inclined plane, the perpendicular velocities of the power and weight (by Cor. 4. Prop. XXXIII.) are reciprocally as their quantities: and the diſtance of the center of gravity from each, being in the ſame ratio, is alſo at reſt. And univerſally in any combination of theſe, or any machine whatever, where the equilibrium continues; the aſcent and deſcent of the power and weight being reciprocally as their quantities; the center of gravity neither aſcends nor deſcends.

PROP. LII.

73. *If a heavy body AB be ſuſpended by two ropes AC, BD; a right line perpendicular to the horizon, paſſing through the interſection F, of the ropes, will alſo paſs through the center of gravity G, of the body.*

For continue the lines *AC*, *BD* to *F*; then it is the ſame thing whether the lines that ſuſtain the body, act at *C* and *D*, or

or at *F*, in the same directions; suppose therefore that the body *AFB* is suspended at *F*; then since (by Ax. 7.) the body will descend as low as it can get; and (by Cor. Prop. L.) the center of gravity must be taken for the place of the body; therefore the center of gravity *G* will be in the line *FG* perpendicular to the horizon. And it is the same thing if *AC*, *BD*, intersect in a point *F* below the body; for the body cannot be supported except the center of gravity *G* be in the perpendicular *GF*. FIG. 73.

Cor. Hence, if GN be drawn parallel to AC, the weight of the body, the forces acting at C and D, are respectively as FG, GN, and FN; or as the sines of the angles AFB, GFB, and GFA.

Cor. 2. The lines AC, DB and FG, are all in one plane perpendicular to the horizon.

Cor. 3. If the center of gravity fall not in the line FG, the body will not rest till it fall in that line.

PROP. LIII.

If any body whatever, as BC, or any beam loaded with a weight, be supported by two planes AB, CD, at C and B; and from the points C, B, the lines CF, BF, be drawn perpendicular to these planes; and from the intersection F, the line FH be drawn perpendicular to the horizon, it will pass through the center of gravity G, of the body. 74.

For since the body is sustained by the planes at *B*, *C*, and these planes re-act against the body in the perpendicular directions *BF*, *CF*; therefore it is the same thing as if the body was sustained by the two ropes *BF*, *CF*; and consequently (by Prop. last) *FH* will pass through *G* the center of gravity of the whole weight.

Cor. 1. If EG be drawn parallel to CF, then the whole weight, the pressure upon the planes CD, AB; are respectively as FG, EG, EF; and in these very directions; or as the sines of the angles BFC, BFG, and CFG.

Cor. 2. If the line FG drawn (from the intersection of the perpendiculars FC, FB) perpendicular to the horizon, does not pass through the center of gravity; the body will not be sustained, but will move till the center of gravity fall in that line.

FIG. 74. *Cor. 3. Hence if the position of one plane CD be given, and the position of the body CB, and its center of gravity G. The position of the other plane AB may be found, by which the body will be supported; by drawing CF perpendicular to CD, and GF perpendicular to the horizon; and from F drawing FB; then BA perpendicular to it, is the other plane.*

SCHOL.

Some people have objected against the truth of the two last Propositions, as well as some others, though demonstrably proved. But this arises only from their own ignorance of the principles. They that have a mind may see this very Proposition demonstrated five or six different ways in Prop. XXIX. of the small Treatise of Mechanics, published in Vol. VII. of the Cyclomathesis.

PROP. LIV.

75. *If a heavy body HD, whose center of gravity is G, be sustained by three forces A, B, C, in one plane, acting in directions AH, BI, CD. And if FGP be drawn perpendicular to the horizon, and CD produced to cut it in P; and if AH, BI produced, intersect in O; then if OP be drawn; and if EP, OF, be drawn parallel to AO, PC; then I say the weight of the body, the three forces A, B, C, are respectively as FP, EP, EO, OF.*

Because the line *OP* is unmoved, the point *O* is sustained by three forces in directions *OP*, *OA*, *OB*; which therefore, are as the lines *OP*, *EP*, *OE*. Also the point *P* is sustained by three forces in the directions *PO*, *PC*, *GP*; which therefore, are as the lines *OP*, *OF*, *FP*: of which that in direction *FP* is the weight of the body, at *G* the center of gravity. And the forces at *O* and *P*, in the direction *OP* and *PO*, are equal and contrary.

Cor. Hence if any other force instead of the weight act at G, in direction GP; then the forces at P, A, B, C, will be respectively as FP, EP, EO, OF.

SCHOL.

If one of the forces be given, all the rest may be found, if they act two and two at different points *O*, *P*. But if five forces act in one plane, two of them must be given.

PROP.

FIG.

PROP. LV.

If EBDF be any prismatic solid erected upon a plane AD; and if it be cut by any plane AGH. I say the surface, or solid GBDH, cut off by this plane, is respectively equal to the surface or solid EBDF, whose altitude is CI, the line passing through the center of gravity of the base, and parallel to the axis of the solid. 76.

I shall not demonstrate this geometrically by measuring, but mechanically by weighing them. Suppose the periphery, or the base, *BD*, to be divided into an infinite number of equal parts, by planes perpendicular to the horizon, and parallel to the axis of the solid, and to one another. And imagine *AD* to be a leaver, and let each particle be placed on *AD* where its plane cuts it. Then since the force of any particle to move the leaver *AD*, is as that particle multiplied by its distance from *A*, (by Cor. 3. Prop. XIX.) Therefore the forces of the equal particles at *B*, *C*, *D*, &c. will be as *AB*, *AC*, *AD*, &c. and the sum of all, as the sum of these lines. And because *C* is the center of gravity of all the particles. Therefore the sum of all, *AB*, *AC*, *AD*, &c. = sum of as many times *AC*; that is, (because the parts of the base are given) = *AC* × base. But *GB*, *IC*, *HD*, &c are as *AB*, *AC*, *AD*. Therefore all the *GB*'s, *IC*'s, *HD*'s, &c. = whole base × *IC*. That is the whole surface or solid *GBDH* = whole surface or solid *BEFD*.

Cor. 1. *If a line right or curve, or any plain figure whether right-lined or curve-lined, revolve about an axis in the plane of the figure; the surface or solid generated, is respectively equal to the surface or solid, whose base is the line or figure given, and height equal to the arch described by the center of gravity.*

Let *BDdb* be the figure generated. On the base *BCD* erect the surface or solid *BDFE*, and let *C* be the center of gravity. Since the arches *Bb*, *Cc*, *Dd*, are as the radii, *AB*, *AC*, *AD*, that is as *BG*, *CI*, *DH*: therefore if *CI*=*Cc*, then will all the lines *BG*, *CI*, *DH*. &c. = all the arches *bb*, *Cc*, *Dd*, &c. that is, the surface or solid *BDdb* = *BDHG*, that is (by this Prop.) =*BDFE*. 77.

Cor. 2. *Also if a curve revolves about any right line drawn through its center of gravity: the surfaces generated (either by a partial or total revolution) on opposite sides of the line, will be equal.*

FIG. 77. For by Cor. 2. Prop. XLIV. each part of the curve multiplied by the distance of its center of gravity from this line, must be equal on both sides. And by Cor. 2. each surface generated, is equal to the curve multiplied by the arch described at that distance; and these arches (being similar) are as these distances. Whence each surface is as the curve multiplied by the distance of its center of gravity: and therefore they are equal.

SECT.

SECT. VI.

Of the centers of percussion, oscillation, and gyration.

PROP. LVI.

Let there be any system of bodies A, B, C, considered without weight, and moveable about an axis passing through S; and if any force f can generate the absolute motion m in a given time; if the same force act at P, perpendicular to PS; the motion generated in the system, in the same time, revolving about the axis at S, will be $\frac{A\times SA + B\times SB + C\times SC}{A\times SA^2 + B\times SB^2 + C\times SC^2}\times SP\times m$. FIG. 78.

For suppose PS perpendicular to the axis at S, and to the line of direction PQ. And SA, SB, SC, perpendicular to the axis at S. And suppose the force f divided into the parts p, q, r, acting separately at P to move A, B, C. Then (by Cor. 3. Prop. XIX.) the bodies A, B, C will be acted on respectively with the forces $\frac{SP}{SA}p$, $\frac{SP}{SB}q$, $\frac{SP}{SC}r$.

Since the angular motion of the whole system is the same, the velocities of A, B, C, are as SA, SB, SC; and their motions as $A\times SA$, $B\times SB$, $C\times SC$; and these motions are as their generating forces $\frac{SP}{SA}p$, $\frac{SP}{SB}q$, $\frac{SP}{SC}r$. Whence p, q, r, are as $\frac{A\times SA^2}{SP}$, $\frac{B\times SB^2}{SP}$, $\frac{C\times SC^2}{SP}$: put the sum of these $= s$, and since $p+q+r=f$. Therefore $s : f :: \frac{A\times SA^2}{SP} : \frac{f\times A\times SA^2}{s\cdot SP}=p$. And $\frac{SP}{SA}p = \frac{f\times A\times SA}{s}$ = force acting at A. Then $f : m :: \frac{f\times A\times SA}{s} : \frac{m\times A\times SA}{s}$ = motion of A. After the same manner

 $m\times\ \times SB$,

FIG. 78. $\frac{m\times B\times SB}{s}$, $\frac{m\times C\times SC}{s}$ are the motions of B and C. Therefore the whole motion generated in the syſtem is $\frac{A\times SA+B\times SB+C\times SC}{s}\times m$.

Cor. 1. *If you make* $SO=\frac{A\times SA^2+B\times SB^2+C\times SC^2}{A\times SA+B\times SB+C\times SC}$, *then if all the bodies be placed in O, the motion generated in the ſyſtem, will be the ſame as before, as to the quantity of motion or the ſum of all the abſolute motions; but the angular velocity will be different.*

For the motion generated in theſe two caſes, will be $\frac{A\times SA+B\times SB+C\times SC}{A\times SA^2+B\times SB^2+C\times SC^2}\times SP\times m$, and $\frac{\overline{A+B+C}\times SO}{\overline{A+B+C}\times SO^2}\times SP\times m$; and if theſe be ſuppoſed to be equal, there comes out $SO=\frac{A\times SA^2+B\times SB^2+C\times SC^2}{A\times SA+B\times SB+C\times SC}$.

Cor. 2. *The angular velocity of any ſyſtem A, B, C, generated in a given time, by any force f, acting at P, perpendicular to PS, is as* $\frac{SP\times f}{A\times SA^2+B\times SB^2+C\times SC^2}$.

For the angular velocity of the whole ſyſtem is the ſame as of one of the bodies A. But the abſolute motion of A is $=\frac{m\times A\times SA}{s}$, and the abſolute velocity of $A=\frac{m\times SA}{s}$; but the angular velocity is as the abſolute velocity directly, and the radius or diſtance reciprocally; therefore the angular velocity of A, and conſequently of the whole ſyſtem, is as $\frac{m}{s}$ or $\frac{m\times SP}{A\times SA^2+,\ \&c.}$ that is (becauſe m is as the force f), as $\frac{f\times SP}{A\times SA^2+B\times SB^2,\ \&c.}$

Cor. 3. *Hence there will be the ſame angular velocity generated in the ſyſtem, and with the ſame force, as there would be in a ſingle body placed at P, and whoſe quantity of matter is* $\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP^2}$.

For let $P=$ that body, then (by Cor. 2. ſince f and SP are given) the angular velocities of the ſyſtem and body P, will be

be to one another, as $\frac{1}{A\times SA^2+B\times SB^2 \&c.}$ to $\frac{1}{P\times SP^2}$. Which be- FIG. 78.
ing supposed equal, we shall have $P=\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP^2}$.

Cor. 4. *The angular motion of any system, generated by a uniform force, will be a motion uniformly accelerated.*

PROP. LVII.

To find the center of percussion of a system of bodies, or the point, which striking an immoveable object; the system shall incline to neither side, but rest as it were in equilibrio.

Through the center of gravity G of the system, draw a plane perpendicular to the axis of motion in S. And if the bodies are not all situated in that plane, draw lines perpendicular to it from the bodies, and let A, B, C, be the places of these bodies in the plane. Draw SGO, and let O be the center of percussion. Draw Af, Bg, Ch, perpendicular to SO, and Aad to SA, and make $ad=SA$, and draw ea perpendicular, and de parallel to SO. Then aA will be the direction of A's motion, as it revolves about S. And the system being stopt at O; the body A will urge the point a forward, with a force proportional to its matter and velocity; that is as $A\times SA$ or $A\times ad$. And the force wherewith A acts at a in direction ea, is $A\times ea$ or $A\times Sf$. And the force of A to turn the system about O, is $A\times Sf\times aO$ (by Cor. 3. Prop. XIX.) $=A\times Sf\times\overline{SO-Sa}=A\times Sf\times SO-A\times SA^2$. Likewise the forces of B and C to turn the system about O, is as $B\times Sg\times SO-B\times SB^2$, and $C\times Sh\times SO-C\times SC^2$. And since the forces on the contrary sides of O destroy one another; therefore $A\times Sf\times SO-A\times SA^2+B\times Sg\times SO-B\times SB^2+C\times Sh\times SO-C\times SC^2=0$. Therefore $SO=\frac{A\times SA^2+B\times SB^2+C\times SC^2 \&c}{A\times Sf+B\times Sg+C\times Sh \&c}$, the distance of the center of percussion, from the axis of motion. Where note, if any points f, g, h, fall on the contrary side of S; the correspondent rectangles must be negative, $-A\times Sf$, $-B\times Sg$, &c. 79.

Cor. 1. *If G be the center of gravity of a system of bodies A, B, C; the distance of the center of percussion from the axis of motion, that is,* $SO=\frac{A\times SA^2+B\times SB^2+C\times SC^2 \&c}{SG\times\overline{A+B+C} \&c.}$.

For by Prop. XLIV. $A\times Sf+B\times Sg+C\times Sh=\overline{A+B+C}\times SG$.

Cor. 2. *The distance of the center of percussion from the center of gravity G, is* $GO=\frac{GA^2\times A+GB^2\times B+GC^2\times C}{SG\times\overline{A+B+C}}$.

For $A\times SA^2+B\times SB^2+C\times SC^2=A\times\overline{SG^2+GA^2-2SG\times Gf}+B\times\overline{SG^2+GB^2+2SG\times Gg}+C\times\overline{SG^2+CG^2+2SG\times Gh}$, by Eucl.

II. 12 and 13. But (by Cor. 2. Prop. XLIV.) $-A\times Gf+\overline{B\times Gg}$
FIG. 79. $+C\times Gh=0$; therefore $A\times SA^2+B\times SB^2+C\times SC^2=\overline{A+B+C}$ $\times SG^2+A\times GA^2+B\times GB^2+C\times GC^2$. Whence (by Cor. 1.) SO or $SG+GO=\frac{SG^2\times\overline{A+B+C}}{SG\times\overline{A+B+C}}+\frac{A\times GA^2+B\times GB^2+C\times GC^2}{SG\times\overline{A+B+C}}$.

Cor. 3. *Hence* $SG\times GO=$ *the given quantity* $\frac{A\times GA^2+B\times GB^2+C\times GC^2}{A+B+C}$; *and therefore GO is reciprocally as SG.*

For each of the bodies A, B, C, and their distances from G. are given.

Cor. 4. *Hence also if SG be given, GO will be given also. And therefore if the plane of the motion remain the same, in respect to the bodies, and the distance SG remain the same; the distance of O from G will remain the same also.*

Cor. 5. *The percussion or quantity of the stroke at O, by the motion of the system, is the same as it would be at G; supposing all the bodies placed in G, and the angular velocity the same. For the sum of the motions of A, B, C, in the system, acting against O, is as* $A\times SA\times\frac{SA}{SO}+B\times SB\times\frac{SB}{SO}+C\times SC\times\frac{SC}{SO}=\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SO}$ $=SG\times\overline{A+B+C}$ (by this Prop.) *but* $SG\times\overline{A+B+C}$, *denotes the motion of* $A+B+C$, *acting against G, or an obstacle placed there.*

Cor. 6. *In a body or system of bodies, oscillating about a center S, if V be the velocity of O, the center of percussion; the shock or quantity of the stroke at any point P, against an obstacle there, is* $\frac{SG}{SP}\times V\times$ *sum of the bodies, or of the whole system, and therefore are reciprocally as SP. For the velocity of A being denoted by SA, its quantity of motion is* $A\times SA$. *But by the property of the leaver ASP, its quantity of motion against P is* $A\times SA\times\frac{SA}{SP}$, or $\frac{A\times SA^2}{SP}$. *In like manner the motions of B, C against P are* $\frac{B\times BC^2}{SP}$, $\frac{C\times SC^2}{SP}$, *therefore the sum of all, or the whole shock against P, is* $\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP}$, *and that is reciprocally as SP, all the rest being given quantities. But by this* Prop. $\frac{A\times SA^2+B\times SB^2+C\times SC^2}{SP}=\frac{SG}{SP}\times SO\times\overline{A+B+C}$; *where SO denotes the velocity of O, the same as V.*

Cor. 7. *If OT be drawn perpendicular to SO, then OT will be the locus of all the centers of percussion.*

For the direction of O is in the line OT; and therefore it is the same thing which point of the line OT strikes an obstacle.

FIG.

PROP. LVIII.

To find the center of oscillation of a system of bodies, or such a point, in which a body being placed, will vibrate in the same time, and with the same angular velocity as the whole body.

Let the axis of motion be at S, perpendicular to which draw the plane in which the center of gravity G moves; draw SGO. and let O be the center of oscillation; draw the horizontal line Sr, and from the bodies A, B, C, draw Aa, Bb, Cc, perpendicular to SO; and also Ae, Bn, Cd, Gg, Or, perpendicular to Sr. 81.

Put $s = A\times SA^2 + B\times SB^2 + C\times SC^2$. Then (by Cor. 2. Prop. LVI.) the angular velocity which A, B, C, generate in the system by their weight, is $\frac{-Se\times A}{s}$, $\frac{Sn\times B}{s}$, $\frac{Sd\times C}{s}$; and the whole angular velocity generated by them all is $\frac{-Se\times A + Sn\times B + Sd\times C}{s}$. Likewise the angular velocity which any particle p, situated in O, generates in the system, by its weight, is $\frac{Sr\times p}{p\times SO^2}$ or $\frac{Sr}{SO^2}$ or $\frac{Sg}{SG\times SO}$, because of the similar triangle SgG, SrO: But their vibrations, and every part of them, are performed alike; therefore their angular velocities must be every where equal; that is $\frac{-Se\times A + Sn\times B + Sd\times C}{s} = \frac{Sg}{SG\times SO}$; whence by reduction $SO = \frac{Sg}{SG}\times\frac{s}{-Se\times A + Sn\times B + Sd\times C}$. But (by Prop. XLIV.) $-Se\times A + Sn\times B + Sd\times C = Sg\times\overline{A+B+C}$. Therefore the distance of the center of oscillation from the axis of motion, $SO = \frac{s}{SG\times\overline{A+B+C}} = \frac{A\times SA^2 + B\times SB^2 + C\times SC^2 \&c.}{SG\times A+B+C \&c} = \frac{A\times SA^2 + B\times SB^2 + C\times SC^2 \&c}{A\times Sa + B\times Sb + C\times Sc \&c}$. Where $A\times Sa$, $B\times Sb$, &c. must be negative, when a, b, &c. lie on the contrary side of S. And since all these quantities are the same at all elevations of the axis SO; therefore the point O is rightly found; and the system has such a point as is required. Likewise it appears by Cor. 1. of the last Prop. that the center of oscillation is the same with the center of percussion.

Cor.

6

FIG. 81. *Cor.* 1. *If p be any particle of a body, d its distance from S, the axis of motion; G, O, the centers of gravity and oscillation. Then the distance of the center of oscillation of the body, from the axis of motion,* $SO = \frac{\text{sum of all the } p \times dd}{SG \times \text{body}}$.

Cor. 2. *If the bodies A, B, C, be large, and therefore the center of oscillation of each, not in the center of gravity. Let d, e, f, be the respective distances of their centers of gravity, and p, q, r, of their centers of oscillation, from S. Then will the distance of the center of oscillation from S, the axis of motion,* $SO = \frac{dpA + eqB + frC}{A \times Sa + B \times Sb + C \times Sc} = \frac{dpA + eqB + frC}{SG \times \overline{A + B + C}}$.

For let a, b, c, be any particles in A, B, C; and x, y, z, their distances from S respectively. Then by this Prop. $SO = \frac{\text{sum } xxa + \text{sum } yyb + \text{sum } zzc}{SG \times \overline{A + B + C}}$. But $\frac{\text{sum } xxa}{dA} = p$, or sum xxa $= dpA$. and sum $yyb = eqB$, and sum $zzc = frC$; and $SG \times \overline{A + B + C} = Sa \times A + Sb \times B + Sc \times C$.

Cor. 3. *To find the center of oscillation of an irregular body. Suspend it at the given point, and hang up a single pendulum of such a length, that making them both vibrate, they may keep time together. Then the length of this pendulum is equal to the distance of the center of suspension from the center of oscillation of the body.*

Cor. 4. *What has been demonstrated in the last Prop. and Cor.* 1, 2, 3, 4 *for the center of percussion; holds equally true for the center of oscillation.*

SCHOL.

I shall just take notice, that if the distance of the axis of suspension from the center of gravity SG, be made equal to $\sqrt{\frac{GA^2 \times A + GB^2 \times B + GC^2 \times C}{A + B + C}}$; the body will oscillate in the least time possible.

In very small bodies, or any bodies oscillating at a great distance from the axis of motion, the center of oscillation or percussion is in or very near the center of gravity. And the reason why the center of oscillation or percussion is not always in the center of gravity; is because the body in vibration is made

made to turn about a center. But if it be so contrived as always to move parallel to itself, without any circular motion, the centers of gravity, of oscillation, and percussion will be the same. FIG. 81.

The distance of the centers of oscillation and percussion, from the axis of motion, as calculated by Cor. 1. is as follows. Where the axis of motion is at the vertex, and in the plane of the figure.

1. In a right line, small parallelogram, and cylinder, $\frac{2}{3}$ the axis of the figure.

2. In a triangle, $\frac{3}{4}$ the axis
3. In a plane of a circle, $\frac{5}{4}$ the radius
4. In the parabola, $\frac{5}{7}$ the axis
5. Pyramid and cone, $\frac{4}{5}$ axis

} nearly.

6. In a sphere, $r =$ radius, $d =$ distance of the axis of motion from its center. Then the distance of the center of oscillation from the axis of motion, is $d + \frac{2rr}{5d}$.

PROP. LIX.

To find the center of gyration of a system of bodies, or such a point O, as that a given force, acting at a certain place, will in the same time generate the same angular velocity in the system, about an axis SR, as if the whole system was placed in O. 82.

Draw the plane PQS perpendicular to the axis of rotation SR; and let SA, SB, SC be the nearest distances of the bodies A, B, C from the axis SR; and let the force f act at P, in direction PQ perpendicular to PS. Then (by Cor. 2. Prop. LVI.) the angular velocity generated in the system by the force f, will be as $\frac{SP \times f}{A \times SA^2 + B \times SB^2 + C \times SC^2}$, and in the system placed in O, it will be $\frac{SP \times f}{\overline{A+B+C} \times SO^2}$; and if these velocities be made equal, we shall have $SO^2 = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{A+B+C}$. Whence the distance of the center of gyration O from the axis of motion at S, that is $SO = \sqrt{\frac{A \times AS^2 + B \times SB^2 + C \times SC^2}{A+B+C}}$.

Cor. 1. *Let $b =$ quantity of matter in any body ABRCS, p any particle, $d = ap$, its distance from the axis of rotation SR: then the square of the distance of the center of gyration, from the axis of motion, that is $SO^2 = \frac{\text{sum of all the } ddp}{b}$.* 83.

FIG. 82. *Cor.* 2. *If any part of the system be supposed to be placed in the center of gyration of that particular part; the center of gyration of the whole system will continue the same as before.*

For by this Prop. the same degree of force which moved this part of the system before, along with the rest, will move it now without any alteration. And therefore if each part of the system be collected into its proper center of gyration, the center of gyration of the whole will continue the same.

Cor. 3. *If a circle be described from G, the center of gravity of the system; and the axis of rotation be made to pass through any point S in its periphery; the distance of the center of gyration from that point will always be the same.*

For (by Cor. 6. Prop. XLIV.) the quantity $A\times SA^2+B\times SB^2+C\times SC^2$ will be given.

Cor. 4. *The distance of the center of gyration from the axis of motion, is a mean proportional between the distances of the centers of gravity and percussion from that axis.*

It follows from this and the last Prop.

Cor. 5. *The momentum or quantity of motion of the whole system, acting against an obstacle at O the center of gyration, is the same as if all the bodies were placed in O, the angular velocity remaining the same. For the momenta or quantities of motion are as the forces.*

SCHOL.

It is the same thing on whatever side of the axis of rotation SR, the point O or center of gyration be taken, provided it be at its proper distance.

By a computation from Cor. 1. the distance of the center of gyration from the axis of rotation, in the following bodies will be,

1. In a *right line* or small *cylinder* (revolving about the end) $SO =$ length $\times\sqrt{\frac{1}{3}}$.

2. The *plane* of a *circle*, or *cylinder* (revolving about the axis) $SO =$ radius $\times\sqrt{\frac{1}{2}}$.

3. The *periphery* of a *circle* (about the diameter) $SO =$ radius $\times\sqrt{\frac{1}{2}}$.

4. The *plane* of a *circle* (about the diameter) $SO = \frac{1}{2}$ radius.

5. The

5. The *surface* of a *sphere* (about the diameter) $SO =$ radius $\times \sqrt{\frac{2}{3}}$. FIG. 82.

6. A *globe* (revolving about the diameter) $SO =$ radius $\times \sqrt{\frac{2}{5}}$.

7. In a *cone* (about the axis) $SO =$ radius $\times \sqrt{\frac{3}{10}}$.

If the periphery of a circle revolve about an axis in the center, perpendicular to its plane, it is the same thing as if all the matter was collected into any one point in that periphery. And the plane of a circle of double the mater of this periphery, and the same diameter, will in an equal time acquire the same agular velocity.

If the matter of any gyrating body were actually to be placed in its center of gyration, it ought either to be disposed of in the circumference of a circle, whose radius is *SO*, or else into two points *O*, diametrically opposite, equal and equi distant from *S*. For by this means the center of motion *S*, will be in the center of gravity. And the body will revolve without any lateral force towards any side.

PROP. LX.

If EF be any body at rest in free space, G its center of gravity, the points S, O, one the center of suspension, the other of percussion; and if a moving body B strike directly against the point O, the motion generated in the body EF by the stroke shall be such; that in the time that the body makes one revolution about its center of gravity G, the center of gravity will move forward a space, equal to the circumference of a circle, whose radius is SG. 84.

Let the body vibrate about the point *S*, and in a very small time, from the position *SGO*, come into the position *Sgd*. Now the arches *Gg*, *Od*, will be as the velocities of the points *G*, *O*, vibrating about *S*; therefore when it comes into the position *SGO*, if it were disengaged from the point *S*; the center of gravity *G* would still move forward with the same velocity *Gg*; and the body, instead of revolving about *S*, would (by Cor. 1 Prop. XLIX.) revolve about *G* with the same angular motion as before. Therefore if *rGo* be drawn parallel to *Sgd*, *Gg* will represent the velocity of *G*, and *Oo* the velocity of *O* about *G*. And because *Oo*, *Gg* are very small similar arches, therefore their circumferences will be described in equal times; that is, in the time that *O*, or the body itself, makes one revolution

FIG. 84. about G; the point G will advance forward a ſpace, equal to the circumference of a circle, whoſe radius is SG.

Now this is the motion acquired by revolving about S. But (by Prop. LVII.) if a body ſo revolving, ſtrikes an immoveable object at O, both the progreſſive and circular motion will be deſtroyed; and the body will be at reſt. It is evident on the contrary, that if a moving body ſtrike the body at reſt in the point O, with the ſame force; the ſame motion will be reſtored again: and is the ſame as above deſcribed.

Cor. 1. *At the beginning of the motion, and alſo after every revolution of the body, when the line SGO comes into its original poſition, ſo as to be perpendicular to the line of direction OB, the point S will be at reſt for a moment.*

For in this poſition, it will be (by this Prop.) as velocity of O about G : velocity of G :: OG : GS. And by compoſition, vel. O about G + vel. G that is abſolute vel. O : abſolute vel. G :: $OG + GS$ or OS : GS. Therefore ſince the abſolute velocities of O and G are directly as their diſtances from S; it follows that the point S is at reſt.

Cor. 2. *Let a body* $A = \frac{SG}{SO} \times$ body EF. *And if V be the velocity which the body A would receive by the direct ſtroke of B: then I ſay, the abſolute velocity of the body EF (or of its center of gravity G), which it receives by B impinging at O, will be* $\frac{SG}{SO}V$.

For let p be any particle of the body EF, and Sp its diſtance from S. Then (by Cor. 3. Prop. LVI.) if a body $= \frac{\text{ſum of all the } Sp^2 \times p}{SO^2}$ be placed in O, it will receive the ſame angular velocity, by the ſtroke, about S at reſt; as the body EF when ſtruck in O. But (by Cor. 1. Prop. LVIII.) ſum $Sp^2 \times p = SO \times SG \times$ body EF; whence the body $\frac{SO \times SG \times \text{body } EF}{SO^2}$ or A, placed in O, receives the ſame angular velocity about S, as the point O of the body EF. But vel. of O or A : vel. G :: SO : SG. For at the beginning of the motion, S is at reſt, by Cor. 1.

Cor. 3. *The velocity loſt in B by the ſtroke, will be* $\frac{\text{body } EF}{\text{body } B} \times \frac{SG}{SO}V$.

For

For the ſum of the motions of all the bodies, after the ſtroke, is the ſame as the motion of *B* before it, by Prop X. FIG. 84.

SCHOL.

The point *S* is by ſome called the *ſpontaneous center of rotation*; becauſe the body (or ſyſtem of bodies) at the beginning of the motion, moves as it were of its own accord, or without any compulſion, about the center *S* at reſt.

PROP. LXI.

Let DE be any body, C its center of gravity; and if from the center C, the circle BFS be deſcribed; and if about BFS as an axis, a cord ASBFS, be wound, and the end fixed at A. And if O be the center of oſcillation, in reſpect to the center of ſuſpenſion S. Then if the body deſcend by a rotation round the axis BFS, by unwinding the cord ASBF, &c. then I ſay the ſpace deſcended by the whirling body DE, is to the ſpace deſcended in the ſame time, by a body falling freely, as SC to SO. 85.

Through the point of contact *S* and the center of gravity *C*, draw the horizontal line *SCO*. Then (by Prop. LVIII.) the angular velocity of the body about the point of ſuſpenſion *S*, at the beginning of the motion, will be the ſame as if the whole body was placed in *O*. But if a body was placed in *O*, its velocity generated at the beginning, will be the ſame as of a body falling freely. Therefore drawing *Sco* infinitely near *SCO*, and the ſmall arches *Oo*, *Cc*: then the velocity of *O*, is to the velocity of the center of gravity *C*, as *Oo* to *Cc*, or as *SO* to *SC*; that is, the velocity generated by a body falling freely, is to the velocity of the deſcending body *DE*, as *SO* to *SC*. Now ſince the points *S* and *O* are always in the horizontal line *SCO*, and the radius *SC* is given, and likewiſe (by Cor. 4. Prop. LVII.) the diſtance *CO*. Therefore the velocities of *O* and *C* in any times will always be as *SO* to *SC*: that is, the velocity of a body deſcending freely, is to the velocity of the whirling body *DE*, always in the ratio of *SO* to *SC*. And therefore (by Prop. VI.) the ſpaces deſcribed are in the ſame ratio.

Cor. 1. *The weight of the body DE, is to the tenſion of the cord AS; as SO to CO.*

For

FIG. For let the body be ſupported at O, then ſince B is the center of gravity, therefore (by Cor. Prop. L. and Cor. 5. Prop. XIX.) 85. the weight DE is to the preſſure at S, as SO to CO. Now if the point O be let go, the force acting at O will generate a motion about S, whilſt the preſſure at S, and conſequently the tenſion of the cord, is neither increaſed nor decreaſed, but remains the ſame as before.

Cor. 2. *If a circular body as BFS runs down an inclined plane, whilſt the thread ASB unfolds; or if a round body roll down an inclined plane, and by its friction be hindered from ſliding: the ſpace it deſcribes in any time, is to the ſpace deſcribed by a body ſliding down freely without friction, as SC to SO.*

For the forces that generate their motions are both decreaſed, in the ſame ratio, that is as the abſolute gravity to the relative gravity upon the plane; therefore the ſpaces deſcribed will remain in the ſame ratio of SC to SO. And in the rolling body, the friction ſupplies the place of the cord, the ſame as if it had teeth.

Cor. 3. *This motion of the body DE by rotation, is a motion uniformly accelerated. And the tenſion of the cord is always the ſame, through the whole deſcent.*

SCHOL.

86. Let W = weight of a body, S = ſpace deſcribed by a body falling freely. Then the ſpaces deſcribed by rotation or whirling, in the following bodies, as SBF, in the ſame time are,

1. In the circumference of a circle, SBF, or ſurface of a cylinder, ſpace $= \frac{1}{2} S$; tenſion of the ſtring $= \frac{1}{2} W$.

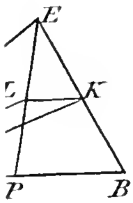
2. In the circumference of a circle, SBF, without weight, and the weight be in the center C; ſpace $= S$, tenſion of the ſtring $= 0$.

3. In the plane of a circle SBF, or a cylinder; ſpace $= \frac{2}{3} S$, and the tenſion of the ſtring $AS = \frac{1}{3} W$.

4. In the ſurface of a ſphere SBF; ſpace $= \frac{3}{5} S$, and the tenſion of the ſtring $AS = \frac{2}{5} W$.

5. In a ſphere SBF; ſpace $= \frac{5}{7} S$, and tenſion of the thread $= \frac{2}{7} W$.

SECT.



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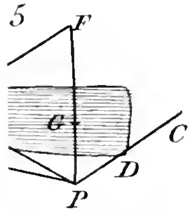
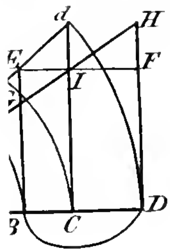
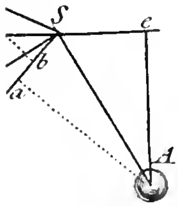
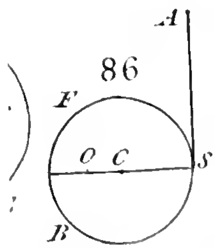




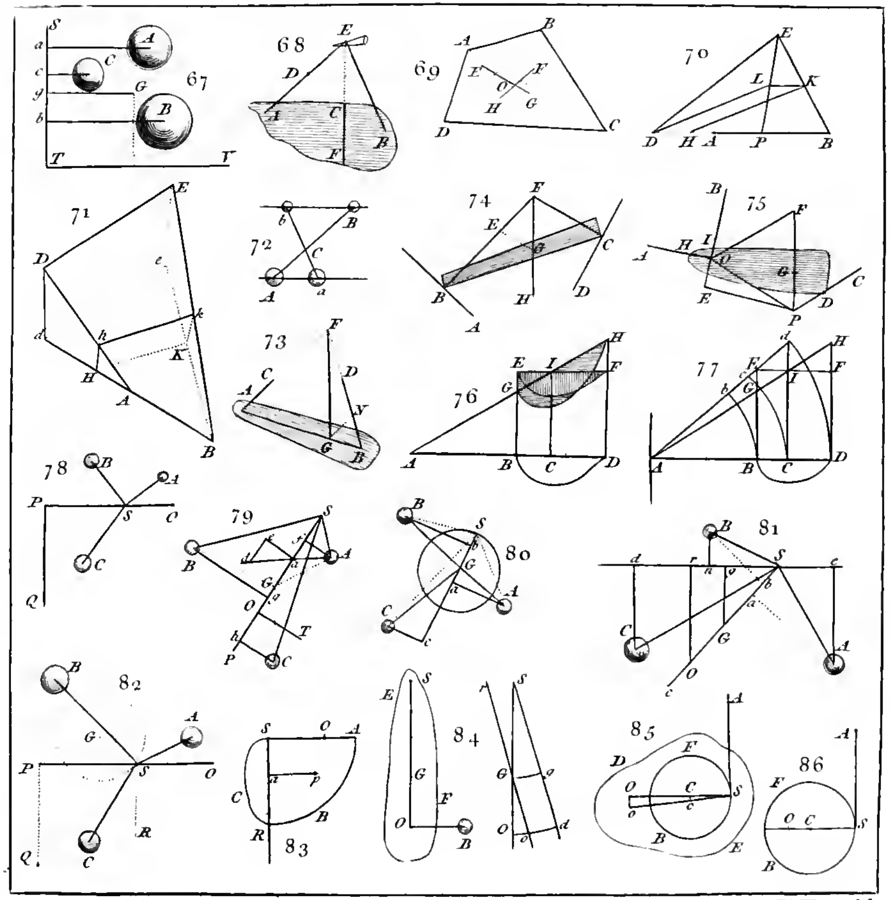
81



86



Pl.IV. *pa.* 86.



Pl. IV. *pa.* 86.

SECT. VIII.

The quantity and direction of the pressure of beams of timber, by their weights; and the forces necessary to sustain them.

FIG. 87.

PROP. LXII.

If a beam of timber be supported at C and B, lying upon the wall ACE, with one end. And if G be the center of gravity of the whole weight sustained: and the line FGH be drawn perpendicular to the horizon, and CF and BH to CB, and BF drawn; I say,

The weight of the whole body	*FH*
Pressure at the top C	*BH*
Thrust or pressure at the base B	*FB,*
are respectively as	*and in these several directions.*

If the beam support any weight, the beam and weight must be considered as one body, whose center of gravity is *G*. Then the end *C* is supported by the plane *BCE*; and (by cor 3. Prop. LIII.) the other end *B* may be supposed to be sustained by a plane perpendicular to *BF*; therefore (by Cor. 1. Prop. LIII.) the weight and forces at *C* and *B*, are respectively as *FH*, *BH*, and *BF*.

Cor. 1. *Produce FB towards Q, then BQ is the direction of the pressure at B. And the pressures at B in directions BQ, FD, DB, are as FB, FD, DB.*

Cor. 2. *Draw Dr perpendicular to BC, and draw CD. Then the weight, pressure at the top, direct pressure at bottom, and horizontal pressure at bottom, are respectively as CB, BD, DC, and Dr.*

FIG. 87. For since the angles EGI, IDF are right; a circle described upon the diameter BF, will pass through G, D. Therefore $<$ $BCD = BFD$ standing on the same arch BD. And because the $<$ GBH and $<$ s at D are right, $BHF = CBD$; therefore the triangles FHB and CBD are similar, and the figure $BHDF$ similar to the figure, $DBrC$, whence $FH : BH : BF : BD$ :: are as $CD : BD : DC$: and Dr.

Cor. 3. *All this holds true for any force instead of gravity, acting in direction GD.*

PROP. LXIII.

88. *If BC be any beam, bearing any weight, G the center of gravity of the whole. And if it lean against the perpendicular wall CA, and be supported in that position; draw BA, CF parallel, and FGD perpendicular to the horizon; and draw FB, then*

$$\left.\begin{array}{l}\textit{The whole weight}\\ \textit{Pressure at the top } C\\ \textit{Thrust or pressure at the bottom } B\\ \qquad\textit{are respectively as}\end{array}\right\}\begin{array}{l}FD\\ BD\\ FB,\\ \textit{and in the same directions.}\end{array}$$

For the end C is sustained by the plane AC; and if the end B be supposed to be sustained by a plane perpendicular to FB. Then (by Cor. 1. Prop. LIII.) the weight, and pressure at top and bottom, are as DF, DB, FB. If you suppose the end B is not sustained by a plane perpendicular to FB, the body will not be supported at all; by Cor. 2. Prop. LIII.

Cor. 1. *If FB be produced to Q, then BQ is the direction of the pressure at B. And the perpendicular pressure at B (FD) is equal to the weight; and the horizontal pressure at B (BD), is equal to the pressure against C.*

PROP. LXIV.

89. *If a heavy beam, or one bearing a weight, be sustained at C, and moveable about a point C; whilst the other end B lies upon the wall BE. And if HGF be drawn through the center of gravity G,*

 per-

perpendicular to the horizon, and BF, CH perpendicular to BC, and CF be drawn; then FIG. 89.

The whole weight	*HF*
Pressure at B	*HC*
Force acting at C,	*CF,*
are respectively as	*and in these directions.*

For the end *B* is sustained by the plane *CB*, and (by Cor. 2. Prop. LIII.) the end *C* may be supposed to be sustained by a plane perpendicular to *FC*, or by a cord in direction *CF*. Then since *HC* is parallel to *BF*, the weight, force at *C*, pressure at *B*, are respectively as *HF*, *CF*, *HC*; by Cor. 1. Prop. LII. or Cor. 1. Prop. LIII.

Cor. But if instead of lying upon the inclined plane at B, the end B laid upon the horizontal plane AB, then the weight, the pressure at B and C, are respectively as BC, GC, and BG; and in this case there is no lateral pressure.

For *BF* will be perpendicular to *BA*, and parallel to *HF*, and consequently *CF* is also parallel to *HF*, therefore (by Cor. 5. Prop. XIX.) the forces at *C*, *G*, *B*, are as *BG*, *BC*, and *CG*.

PROP. LXV.

If a heavy beam BC, whose center of gravity is G, be supported upon two posts BA, CD; and be moveable about the points, A, B, C, D. And if AB, DC, produced, meet in any point H, of the line GF, drawn perpendicular to the horizon. And if from any point F, in the line GF, FE be drawn parallel to AB; I say, 90.

The whole weight	*HF*
Pressure at C	*HE*
Thrust or pressure at B,	*EF,*
are respectively as	*and in these directions.*

For the points *A*, *B*, *C*, *D*, being in a plane perpendicular to the horizon, the body may be supposed to be supported by two planes at *B*, *C*, perpendicular to *AB*, *DC*; or by two ropes *BH*, *CH*. And in either case, the weight in direction *HG*, the pressure at *B*, *C*, in directions *HB*, *HC*, are as *HF*, *EF*, and *HE*.

FIG. 90. *Cor.* *Hence, whether a body be sustained by two ropes BH, CH, or by two posts AB, CD, or by two planes perpendicular to BA, CD: The body then can only be at rest, when the plumb line HGt passes through G, the center of gravity of the whole weight sustained. Or, which is the same thing, when AB, DC intersect in the plumb line HGF passing through the center of gravity.*

SCHOL.

By the construction of these four last propositions, there is formed the *triangle of pressure*, representing the several forces. In which, *the line of gravity* (or plumb line passing through the center of gravity) always represents the absolute weight, and the other sides the corresponding pressures.

PROP. LXVI.

91. *If several beams AB, BC, CD, &c. be joined together at B, C, D, &c. and moveable about the points A, B, C, &c. be placed in a vertical plane, the points A, F, being fixt, and through B C, D drawing ri, sm, tp perpendicular to the horizon. And if several weights be laid on the angles B, C, D, &c. so that the weight on any angle C may be as* $\frac{S.BCD}{S.mCB \times S.mCD}$. *Then all the beams will be kept in equilibrio by these weights.*

Produce DC to r. Then (by Cor. 2. Prop. VIII.) $S. < ABC : S. < ABr ::$ weight B : force in direction $BC = \frac{B \times S.ABr}{S.ABC}$; and $S.BCD : S.DCs ::$ weight : force in direction $CB = \frac{C \times S.DCs}{S.BCD}$; which, to preserve the equilibrium, must be equal to the force in direction BC, that is $\frac{B \times S.ABr}{S.ABC} = \frac{C \times S.DCs}{S.BCD}$; whence $B : C :: \frac{S.ABC}{S.ABr} : \frac{S.BCD}{S.DCs}$. And by the same way of reasoning, $C : D :: \frac{S.BCD}{S.DCs} : \frac{S.CDE}{S.EDt}$. Therefore, ex equo, weight B : weight $D ::$
$$\frac{S.ABC}{S.ABr \times S.BCs} : \frac{S.CDE}{S.DCs \times S.EDt} :: \frac{S.ABC}{S.ABi \times S.CBi} : \frac{S.CDE}{S.CDp \times S.EDp}.$$

Cor. 1. *Produce CD, so that Dw may be equal to Cr, and draw wx parallel to Dp, cutting DE in x. Then the weight C, the forces in directions CB and CD, are as rB, CB and Cr respectively. And weight C is to the weight D, as Br to wx.* FIG. 91.

Cor. 2. *The force or thrust at C, in direction CB, or at B, in direction BC, is as the secant of the elevation of the line BC above the horizon.*

For, force in direction CB : force in direction CD :: CB : Cr :: $S. CrB$ or rCm or sCD : $S. rBC$ :: cos. elevation of CD : cos. elevation of CB :: sec. elevation of CB : sec. elevation CD; because the secants are reciprocally as the cosines.

Cor. 3. *Draw Cp, Dm parallel to DE, CB, then the weights on C and D to preserve the equilibrium, will be as Cm to Dp. And therefore if all the weights are given, and the position of two lines CD, DE; then the positions of all the rest CB, BA, &c. will be successively found.*

For let the force in direction CD or DC be CD, then Cp is the force in direction DE, and Dm, in direction CB. And Dp or the weight D, is the force compounded of DC, Cp; and Cm or the weight C is the force compounded of CD, Dm, by Cor. 2, Prop. VII.

Cor. 4. *If the weights lie not on the angles B, C, D, &c. let the places of their centers of gravity be at g, h, k, l. And let g, h, k, l, also express their weights. And take the weight* $B = \frac{Ag}{AB}g + \frac{bC}{BC}h$, $C = \frac{Bh}{BC}h + \frac{kD}{CD}k$, $D = \frac{Ck}{CD}k + \frac{lE}{DE}l$, *&c. then B, C, D, &c. will be the weights lying upon the respective angles.*

This is evident by Cor. 5. Prop. XIX.

Cor. 5. *If the weights were to act upwards, in the directions mC, pD, &c. or, which is the same thing, if the figure A, B, C, D, E, F, was turned upside down, and the weights remain the same, and the points A, F be fixt as before. All the angles at B, C, D, &c. and consequently the whole figure will remain the same as before; and that whether the lines AB, BC, CD, &c. be flexible or inflexible cords or timbers.*

This will easily appear by the demonstration of the Prop. For the ratio of the forces at any angle C, will be the same, whether they act towards the point C, or from it; by Prop. VIII. that is, it will be the same thing whether the weight at any

FIG. 91. angle C, acts in direction Cm or Cs. And as the forces were supposed before to thrust against C, the same forces now do pull from it.

SCHOL.

92. If $DABF$ be a semi-circle, whose diameter is DF. Draw AG perpendicular to DF. Then the force or weight at any place A, to preserve the equilibrium, will be reciprocally as AG^3, or directly as the cube of the secant of the arch BA.

Likewise it follows from Cor. 5. that if any cords of equal lengths be stretched to the same degree of curvature, the stretching forces will be as the weights of the cords.

SECT. VIII.

The strength of beams of timber in all positions, and their stress by any weights acting upon them, or by any forces applied to them.

PROP. LXVII.

FIG.

The lateral strength of any piece of timber in any place, whose section is a rectangle, is directly as the breadth and square of the depth. 93.

Let BD be any beam, placed horizontally, and fixt at the end BC. And let AFG be the perpendicular section. Divide the depth AF into an infinite number of equal parts at a, b, c, d, &c. whose number is AF or n; through which, suppose lines drawn parallel to FG. And let any force be applied at P in direction DP, to break the beam at AF; then since the strength of the timber is nothing but the force by which the parts of the timber at a, b, c, &c. cohere together: the breaking the timber is nothing but overcoming this force, and separating the parts at a, b, c. Suppose $1 =$ force of cohesion of any of the parts Aa, ab, bc, &c. and imagine QAa, QAb, QAc, &c. so many bended leavers whose fulcrum is at A. And let us see what will be the sum of all the forces applied at Q to break the timber at A. Now (by Cor. 1. Prop. XIX) the power applied at Q to equal or overcome the resistances at A, a, b, c, &c. will be $\frac{0}{AQ}$, $\frac{Aa}{AQ}$, $\frac{Ab}{AQ}$, $\frac{Ac}{AQ}$, $\frac{Ad}{AQ}$, &c. to $\frac{AF}{AQ}$; that is, as $\frac{0}{AQ}$, $\frac{1}{AQ}$, $\frac{2}{AQ}$, $\frac{3}{AQ}$, . . . $\frac{n}{AQ}$. Therefore the effect of all the forces applied to Q, or the whole strength of the beam at A, will be $\frac{0+1+2+3 \cdot \cdot \cdot +n}{AQ}$ or $\frac{nn}{2AQ}$, that is, because AQ is given, as nn or AF^2. Now if the breadth

FG

FIG. 93. FG be increafed in any proportion; it is evident the ftrength of every part Aa, ab, &c. will be increafed in the fame proportion, and therefore the abfolute lateral ftrength will be as $AF^2 \times FG$.

Cor. 1. *In fquare timber, the lateral ftrength is as the cube of the breadth or depth.*

Cor. 2. *And in general, the lateral ftrength of any piece of timber, whofe fections are fimilar figures, are as the cubes of the fimilar fides of the fections.*

Cor. 3. *And in any pieces of timber, whofe fections are fuch figures, that the correfpondent ordinates, parallel to the horizon, are proportional, the ftrengths are as the breadths and fquares of the depths, or as the fections multiplied by the depths.*

289. *Cor.* 4. *The ftrength of cylindrical pieces, or of any fimilar pieces of timber, being forced or twifted round the axis, will alfo be as the cubes of the diameters.*

For let $AD=r$, circumference of the fection $DEFG=c$, $Ap=x$, then the circumference $pqr = \frac{cx}{r}$, and if the cohefion of a particle at p be $=1$. Then the force applied at Q to overcome it, will be $\frac{x}{AQ}$; and the force applied at Q to overcome the cohefion of all the parts, in the circumference pqr, will be $\frac{x}{AQ} \times \frac{cx}{r}$, or $\frac{c}{r \times AQ} \times xx$; and the total force at Q to overcome the cohefion of all the particles in the whole fection $DEFG$ is $= \frac{c}{r \times AQ} \times$ fum of all the $xx = \frac{c}{r \times AQ} \times 1^2 + 2^2 + 3^2 + 4^2 \ldots$ to $r^2 =$ * $\frac{c}{r \times AQ} \times \frac{r^3}{3}$. Therefore, becaufe AQ is given, and the ratio $\frac{c}{r}$; and this force is the ftrength of the beam; therefore the ftrength is as r^3 or AD^3.

SCHOL.

93. What is here faid of timber, is true of any homogeneous bodies whatever fort of matter they are of. But the abfolute

* *Ward's* Math. Guide, Part V. Lem. 3.

ftrength

ſtrength of any beam, lever, rope, &c. when drawn in direction of its length, will be as the ſection of it. For every part does in this caſe bear an equal ſtretch, and the ſum of all the parts is equal to the whole, and that is as the ſection. FIG. 93.

PROP. LXVIII.

The lateral ſtrength of a tube, or hollow cane AB, to that of a ſolid one CD, is as the ſection of the tube (excluding the hollow), to the ſection of the ſolid cane, and the whole diameter of the tube to the diameter of the ſolid cane, nearly. 94.

For (by Cor. 2. of the laſt Prop.) the ſtrength of the ſolid cylinder BF is AF^3, and the ſtrength of the inner ſolid cylinder, whoſe fulcrum is at G, is EG^3, and whoſe fulcrum is at F, is greater than EG^3, and leſs than EF^3, and is nearly $\overline{EG+\frac{1}{2}GI}|^3 = \overline{AF-\frac{3}{2}AE}|^3$, that is $AF^3-\frac{9}{2}AF^2\times AE$ nearly. Therefore the ſtrength of the tube $AFIGE$, is the difference of the ſtrength of theſe cylinders, that is, $AF^3-AF^3+\frac{9}{2}AF^2\times AE$ or $\frac{9}{2}AF^2\times AE$. Likewiſe the ſtrength of the ſolid cylinder DCH, is CH^3. Therefore the ſtrength of the tube FB : ſtrength of the cylinder HD :: $\frac{9}{2}AF^2\times AE : CH^3$. But the ſection of the tube is as AF^2-EG^2 or $AF^2-\overline{AF-2AE}|^2=4AF\times AE$ nearly. Whence, ſtrength of the tube FB : ſtrength of the cylinder HD :: $\frac{9}{2}AF\times AE : CH\times CH^2$ :: $4AF\times AE\times 1\frac{1}{8}AF : CH^2\times CH$ :: ſection of the tube $\times 1\frac{1}{8}$ its diameter : ſection of the cylinder $\times$ its diameter :: ſection tube $\times$ diameter : ſection cylinder $\times$ diameter, nearly.

Otherwiſe,

Let the area of the ring $AEGF$ be diſpoſed into another ring, whoſe diameter is leſs than AF; then ſince every particle of it acts at a leſs diſtance from the fulcrum F, its ſtrength will be leſs in proportion; that is, its ſtrength will be as the diameter of the ring. And when the ring is ſo far diminiſhed as to become an entire circle, the proportion of their ſtrength will not differ far from the proportion of their diameters. Let the diameter of that circle be R; then, ſtrength of the ring or tube : ſtrength of an equal circle :: $AF : R$. And the ſtrength of R : to that of CH :: $R^3 : CH^3$. Therefore, ex equo, ſtrength of the tube BF : ſtrength of the cylinder HD : : $AF\times R^3$: $R\times CH^3$

FIG. 94. $R \times CH^2 :: AF \times R^2 : CH \times CH^2 :: AF \times$ area ring $: CH \times$ area of the circle CH.

Cor. Hence the strength of different tubes are as their sections, and diameters, nearly.

PROP. LXIX.

95. *If any force be applied laterally to a leaver or beam, the stress upon any place is directly as the force and its distance from that place.*

For suppose PAF to be a bended leaver. It is evident the greater the power at P, the greater force is applied at F to separate the parts of the wood. Also the greater the distance AP, the greater power has any given force applied at P, to overcome the cohesion of the wood at F. And therefore the whole stress depends on both.

96. *Cor.* 1. *If two equal weights lie upon the middle of two beams, or upon any other similar places, the stress in these planes, will be as the lengths of the beams.*

For if C be the middle point, then A bears half the weight; therefore the stress at C is as $AC \times \frac{1}{2}$ weight. And because half the weight, or the force acting at A is given; therefore the stress is as AC or half AB, and therefore as AB. And if C be in any other similar situation in both beams, the same thing will follow.

Cor. 2. *If two beams bear two weights proportional to their lengths, and in a like situation; the stress upon each will be as the square of its length.*

Cor. 3. *And if two beams bear two weights reciprocally as their lengths, in a similar situation, the stress where the weights lie, is equal in both.*

PROP.

FIG.

PROP. LXX.

Let AB be any beam of a given length, supported at A and B, and any given weight either suspended at any point C, or equally diffused through the whole length of the beam AB; I say, in either case, the stress of the beam in C, is as the rectangle AC×CB. 97.

CASE I.

Let the given weight be represented by the given length of the leaver AB. Then (by Cor. 5. Prop. XIX.) the weight at A, and the re-action equal thereto, will be CB. And by the last Prop. the stress at C will be as the force acting at $A \times$ distance AC, that is $AC \times CB$.

CASE II.

Let AB be divided into an infinite number n of equal parts, each $=1$. Then as AB represents the whole weight, 1 will be the weight supported upon 1 part of the beam, let it rest at p; then $\frac{Ap}{AB} =$ its pressure on B. Therefore (by the last Prop.) the stress at p is $\frac{Ap \times pB}{AB}$; and the stress at C is $\frac{Ap \times BC}{AB}$, arising from the weight at p. Consequently the stress at C arising from the sum of all the weights between A and C, will be $\frac{0+1+2+3 \ldots AC}{AB} \times BC$; that is, (because AC is the number of them) $\frac{AC^2 \times BC}{2AB}$. And by a like reasoning, the stress at C, arising from the whole weight between B and C, will be $\frac{CB^2 \times AC}{2AB}$. Consequently the whole stress at C is

$$\frac{AC^2 \times BC + CB^2 \times AC}{2AB} = \frac{AC+CB}{2AB} \times AC \times CB = \frac{AC \times CB}{2}.$$

Cor. 1. *The greatest stress of a beam is in the middle, the weight being either suspended there; or equally disposed over the whole length of the beam.*

FIG. 97. *Cor. 2. The ſtreſs of the beam at any point p, by a weight applied to any other point C, is as $Ap \times CB$.*

For $AC \times CB$ is the ſtreſs at C, and (by Prop. laſt) $Ap \times CB$ will be the ſtreſs at p.

Cor. 3. The ſtreſs of the beam at any point C, by a weight ſuſpended there, is double the ſtreſs of the beam at the ſame point C, by the ſame weight preſſing uniformly on all parts of the beam.

For by Caſe I. the ſtreſs at C is $AC \times CB$; and by Caſe II. the ſtreſs at C is $\frac{AC \times CB}{2}$.

Cor. 4. The ſtreſs of a beam at any point C, by a weight ſuſpended there, is double to the ſtreſs at C, when the ſame weight is uniformly diſperſed on all the parts of AC.

Cor. 5. If a weight preſs equally on all the parts of pC, the ſtreſs at C by that preſſure, is to the ſtreſs at C, when ſuſpended at C :: as $Ap + AC$ to $2AC$.

For (by Cor. 2.) the ſtreſs at C, by the weight 1 lying at p, is $Ap \times CB$, and at C, is $AC \times CB$. Therefore the whole ſtreſs at C, by the whole weight on all the points of pC, is the ſum of all the $Ap \times CB = Ap + \overline{Ap+1} + \overline{Ap+2} \ldots AC : \times CB = \frac{Ap + AC}{2} \times pC \times CB$. But the ſtreſs by the whole weight at C, is $AC \times CB \times pC$, and the former is to the latter, as $\frac{Ap + AC}{2}$ to AC.

Cor. 6. If a weight preſs equally on all the parts of Ap, the ſtreſs at any point C by that weight, is to the ſtreſs at C if ſuſpended there :: as Ap to 2AC.

For the ſtreſs at C by all the weight on Ap, is $0 + 1 + 2 \ldots Ap \times CB = \frac{Ap^2}{2} \times CB$. And the ſtreſs by the weight Ap at C is $AC \times CB \times Ap$.

Cor. 7. The ſtreſs at p by a weight at C, is equal to the ſtreſs at C, by the ſame weight at p.

PROP.

FIG. 98.

PROP. LXXI.

If CD be a prominent beam, fixt horizontally at the end C, as in a wall; and if a weight proportional to the length of the beam, be dispersed uniformly on all the parts of the beam. The stress at any point F, will be as DF^2, the square of the distance from the extremity.

For let *FD* be divided into an infinite number of equal parts at *p*, *q*, *r*, *s*, *&c.* and let each be $=1$, and sustain the weight 1. Then (by Prop. LXIX.) the stress at *F*, by the weights at *F*, *p*, *q*, *r*, *&c.* will be 1×0, $1\times Fp$, $1\times Fq$, *&c.* or as 0, 1, 2, 3, *&c.* respectively: therefore the whole stress at *C* will be $0+1+2+3 \ldots FD = \frac{FD^2}{2}$.

Cor. 1. *Hence the stress at F, by any weight suspended at D, will be double the stress at the same point F, when the same weight presses uniformly on all the parts between F and D.*

For (by Prop. LXIX.) the stress at *F* by the weight *DF*, is $DF\times DF$, or FD^2.

Cor. 2. *The stress at the end BC, by the weight P, is the same as the stress upon the middle of a beam of twice the length DC, with twice the weight P laid on its middle, this beam being supported at both ends.*

For the stress now at *C*, is the same as if *DC* was continued to the same length beyond *C*, and a weight equal to *P* suspended at the end; and then the fulcrum *C* will be acted on with twice the weight *P*. And this is the same as if the beam was turned upside down, and twice the weight *P* laid on the middle *C*.

PROP. LXXII.

If there be two beams standing a slope, and bearing two weights upon them, either in the middle, or in any given situation, or equally diffused over the whole length of the beams; the stress upon them will be directly as the weights, and the lengths, and the cosines of elevation. 99. 100.

FIG. 99. 100. For (by Cor. 1. Prop. XXXI.) the weight is to the pressure upon the plane, as radius to the cos. elevation. Therefore the pressure is as cos. elevation × weight : and this is the force acting against the beam. Therefore (by Prop. LXVIII.) the stress will be as its length and this force; that is, as the length, the weight, and cos. elevation.

Cor. 1. *If the weights and length of the beams be the same, the stress will be as the cosine of elevation, and therefore greatest when it lies horizontal.*

Cor. 2. *If the beams lie horizontal, or at any equal inclinations, and the weight be as the length, then the stress is as the square of the length.*

99. Cor. 3. *If the weights are equal, on the horizontal beam AB, and the inclined one AC, and BC be perpendicular to AB : then the stress will be equal upon both.*

For the length × cos. elevation is the same in both, or $AC \times$ cos. $A = AB \times$ radius.

Cor. 4. *But if the weights on the same beams be as their lengths, then the stress will also be as their lengths, AB and AC.*

Cor. 5. *And universally, the stress upon any point of a sloping beam, is as the rectangle of the segments, and the weight, and cos. inclination directly, and the length of the beam reciprocally.*

97. For in the horizontal beam AB, if the weight W lie upon C, the pressure at A will be $\frac{CB}{AB} \times W$. And (by Prop. LXIX.) this pressure or force × distance AC, will be as the stress at C; that is, $\frac{AC \times CB}{AB} \times W$ is as the stress at C. And if the beam lie a slope, the stress (by Cor. 1. Prop XXXI.) will then vary in proportion to the cos. elevation.

PROP. LXXIII.

If any beam of timber be to support any weight, or pressure, or force, acting laterally upon it, the breadth multiplied by the square of the

the depth, or in similar sections, the cube of the diameter, in every place, ought to be proportional to the length multiplied by the weight or force acting on it, or as the stress in that place. And the same is true of several different pieces of timber compared together. FIG.

For every several piece of timber, as well as every part of the same timber or beam, ought to have its strength proportioned to the weight, force or pressure it is to sustain. And therefore the *strength* ought to be universally as the *stress* upon it. But (by Prop. LXVII.) the strength is as the breadth × square of the depth. And (by Prop. LXIX.) the stress is as the weight or force × by the distance it acts at. And therefore these must be in an invariable ratio.

102.

Cor. 1. *If AEB be a prominent beam fixt at the end AE, and sustaining a weight at the other end B. And if the sections in all places be similar figures, and CD be the diameter in any place C, then CB will be every where as CD^3. And if ACB be a right line, EDB will be a cubic parabola. Therefore $\frac{2}{5}$ of such a beam may be cut away without any diminution of the strength.*

But if the beam be bounded by two parallel planes, perpendicular to the horizon, then CB will be as CD^2, and then EDB will be the common parabola. Whence a third part of a beam may be thus cut away.

102.

Cor. 2. *But if a weight press uniformly on every part of AB, and the sections in all points as C, be similar; then BC^2 will be every where as CD^3, and EDB a semi-cubical parabola.* 101.

But if the beam be bounded by parallel planes, perpendicular to the horizon, then BC will be as CD, and EDB a right line. Here half a beam may be cut away without losing any strength.

103.

Cor. 3. *If AB be a beam supported at both ends, and if it bear a weight in any variable point C, or uniformly on all the parts of it. And if all the sections be similar figures, and CD be the diameter in that place C, then will CD^3 be every where as $AC \times CB$.*

But if it be bounded by two parallel planes, perpendicular to the horizon, then will CD^2 be every where as $AC \times CB$, and therefore the curve ADB is an ellipsis, supposing AB a right line.

104.

Cor. 4. *But if a weight be placed at any given point P, and all sections are similar figures, and if CD be any diameter, then will BC be as CD^3, and AQ and BQ are two cubic parabolas.*

But

110. *But if the beam be bounded by two parallel planes, perpendicular*
104. *to the horizon, then BC is as CD^2, and AQ and BQ are two common parabolas.*

Cor. 5. All circular plates, whether great or little, being of the ſame matter and thickneſs, and ſupported all round on the edges, will bear equal weights. The ſame is true of ſquare plates, or any ſimilar ones.

290. For let *AD*, *ad*, be two ſquares, and let them firſt, be only ſupported at the ends *AB*, *CD*, and *ab*, *cd*. Then by this Prop. (ſince the thickneſs is given) $AB : AC \times$ weight on $AD :: ab : ac \times$ weight on *ad*, and weight on *AD* : weight on *ad* :: $\frac{AB}{AC} : \frac{ab}{ac}$, but $\frac{AB}{AC} = \frac{ab}{ac}$, therefore weight on $AD =$ weight on *ad*, when the plates are only ſupported by *AB*, *CD*, and *ab*, *cd*. And for the ſame reaſon the weights will be equal, when only ſupported by the ſides *AC*, *BD*, and *ac*, *bd*. And conſequently the weights will ſtill be equal, when the plates are ſupported by all four ſides, in which caſe, twice the weight will be ſupported; and the ſame holds equally true of all ſimilar figures. For,

291. Let *AD*, *ad*, be two hollow circles, draw the inſcribed ſquares *ABCD*, *abcd*, theſe ſquares are ſupported upon the four ſides *AB*, *BD*, *DC*, *CA*, and *ab*, *bd*, *dc*, *ca*, by the continuity of the plates; therefore the weights will be equal, as was proved before. And that the continuity of the plates will equally ſupport the weights in both circles is plain, becauſe the ſtrength of both ſegments *AB*, *ab*, are equal, the length being as the breadth.

Cor. 6. Hence, the weight a ſquare plate will bear :
To the weight which a bar of the ſame matter and thickneſs will bear : :
As twice the length of the bar :
To its breadth.

And a circular plate is but very little weaker.

SCHOL.

All theſe things appear from the foregoing propoſitions; but it is here ſuppoſed that the timber is homogeneous and of the ſame goodneſs, otherwiſe a proper allowance muſt be made for the defect. And what is here ſaid of pieces of timber, holds equally true of any other ſolid bodies, ſuch as pieces of metal, ſtone, &c. And if pieces of timber or metal be cut into the

figures mentioned in the foregoing corollaries, all the parts will FIG.
be disposed to break together. And if a spring is to be made, 291.
its shape ought to be as in Cor. 1. and then every part will bear a stress proportional to its strength.

PROP. LXXIV.

If a weight A be supported upon the end of a crooked piece of timber ABD, and from the ends, a line AB be drawn perpendicular to the horizon, and from the angle B, the line BC perpendicular to AD; the stress at B will be as the perpendicular BC. 105.

For as the weight *A* acts not in direction *AB*, but in direction *AD*, therefore it is the same as if it were applied at the point *C*; but a force applied at *C*, has a greater power to break the timber at *B*, in proportion as the leaver *BC* is longer. This force therefore, or the stress at *B*, is as *BC*.

Cor. 1. *Hence, if any two forces acting from or against one another, at the ends A, F, of any crooked beam ABDEF, and keep one another in equilibrio; and the line AF, or the direction of the forces being drawn, the stress at any point, is as the perpendicular upon AF. So the stress at b is bc; at B, BC, at D, DI; at E, EK; and at G, H, nothing.* 106.

Cor. 2. *Hence also, that the strength in any part b, may be proportional to the stress there, the breadth multiplied by the square of the depth, must be as the perpendicular bc, reckoning that the depth, which is in the plane passing through AF.*

PROP. LXXV.

Having the length AB and weight W, of a cylinder or prism, that can just support the weight P at the end, to find the length of another beam FG, similar to the former, and of the same matter, that will just break with its own weight, or only support itself. 107.
108.

Since

FIG. Since the weights of similar solids of the same matter, are as the cubes of the lengths, it will be, $AB^3 : W :: FG^3 :$
107. $\frac{FG^3}{AB^3}W =$ the weight of the beam GH. Then by Cor. 2.
108. Prop. LXVII. the strength of the beam AB is AC^3; and of FG, is FH^3. And by Prop. LXIX. the stress at A is $\overline{\tfrac{1}{2}W+P}\times AB$. And the stress at F $\frac{FG^3}{2AB^3}W\times FG$. And since the beams are both supposed to break with these weights; therefore the strength must be as the stress; that is, $\overline{\tfrac{1}{2}W+P}\times AB : \frac{FG^4}{2AB^3}W$ ($:: AC^3 : FH^3$) $:: AB^3 : FG^3$. Whence $\frac{FG^4\times W}{2} = FG^3\times AB\times\overline{\tfrac{1}{2}W+P}$. Or $FG\times W = AB\times\overline{W+2P}$. Whence $W : W+2P :: AB : FG$, the length required.

Cor. 1. *If* $eW=P$. *Then* $FG = AB\times\overline{1+2e}$.

Cor. 2. *Hence there is one and only one beam, that will just break by its own weight, or just sustain itself.*

Cor. 3. *The same Prop. will likewise hold good, in regard to two beams supported at both ends and breaking in the middle, by Cor.* 3. *Prop.* LXX.

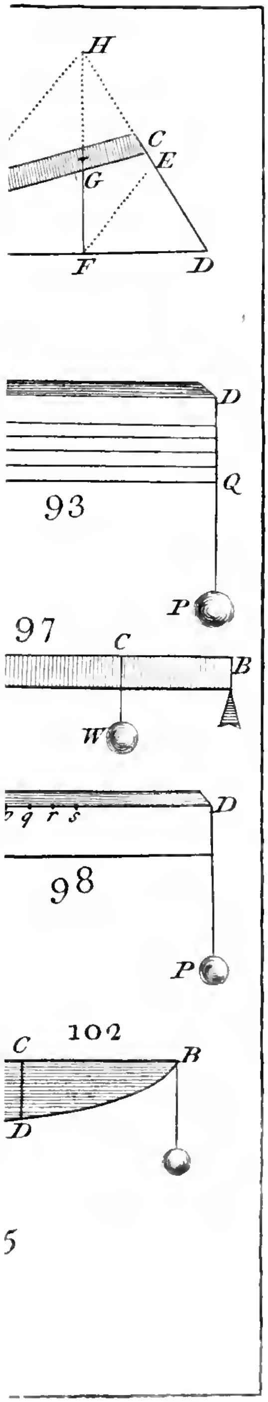
Cor. 4. *If the beam FG break by its own weight, a beam of twice the length of FG, and supporting at both ends, will also break by its own weight; or if one sustain itself, the other will.*

For the stress is the same in both of them, by Cor. 3. Prop. LXX. and Cor. 1. Prop. LXXI. each of them being equal to the stress of a beam, twice the length of FG, and suspended in the middle.

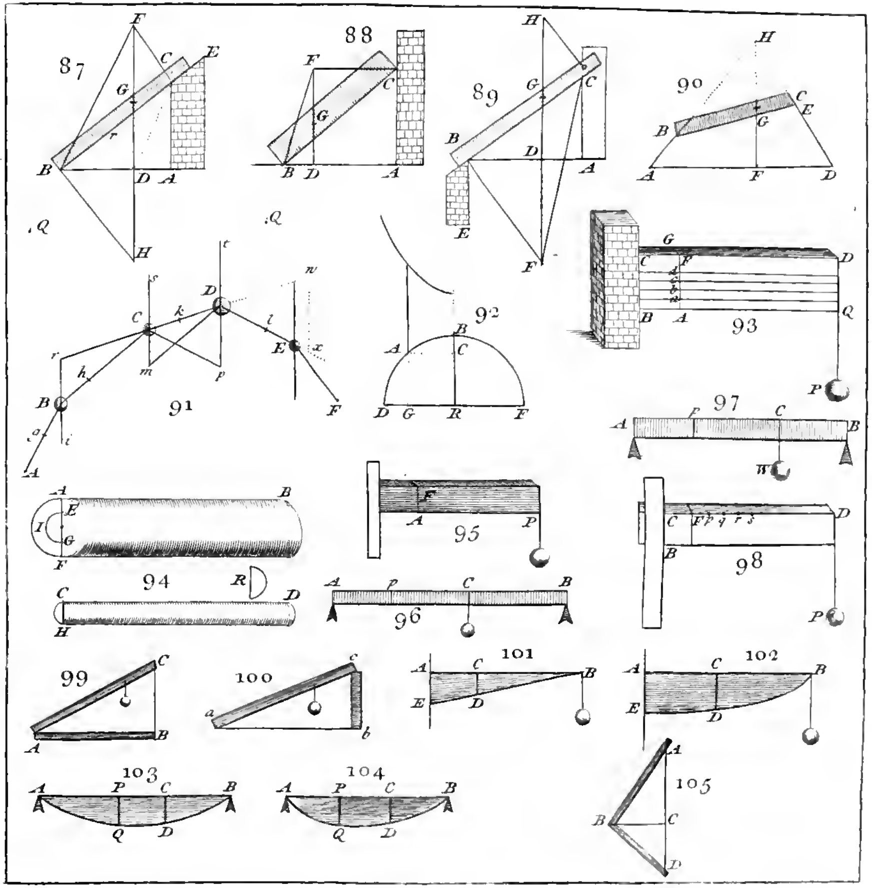
PROP. LXXVI.

109. *If any weight be laid on the beam AB, as at C, or any force applied*
110. *to it at C; the beam will be bent through a space CD proportional to the weight or force applied at C. And the resistance of the beam will be as the space it is bent through nearly.*

In



Pl.V. pa 104.



Pl.V pa 104.

In order to find the law of reſiſtance of beams of timber or ſuch like bodies, againſt any weights laid upon them, or ſtraining them, I took a piece of wood plained ſquare, and ſupporting it at both ends *A*, *B*, I laid ſucceſſively on the middle of it at *C*, 1, 2, 3, 4, 5, 6, 7, and 8 pounds; and I found the middle point *C* to deſcend through the ſpaces 1, 2, 3, 4, 5, 6, 7, and 8, reſpectively. And repeating the ſame experiment with the weights 3, 6, 9 lb. they all deſcended through ſpaces, either accurately or very nearly as the numbers 1, 2, 3. I tried the ſame thing with ſprings of metal, and found the ſpace through which they were bent, proportional to the weight ſuſpended. I alſo tried ſeveral experiments of this kind with wires, hairs, and other elaſtic flexible bodies, by hanging weights at them: and I found that the increaſe of their lengths, by ſtretching, was, in each of them, proportional to the weights hung at them: except when they were going to break, and then the increaſe was ſomething greater. It may be obſerved, that none of theſe bodies regained their firſt figure, when the weights were taken off, except well tempered ſprings; ſo that there are no natural bodies perfectly elaſtic. And even ſprings are obſerved by experience to grow weaker by often bending; and by remaining ſome time unbent, will recover part of their ſtrength: and are ſomething ſtronger in cold than in hot weather. But at any time a ſpring, and all ſuch bodies obſerve this law, that they have the leaſt reſiſtance when leaſt bent, and in all caſes are bent through ſpaces nearly proportional to the weights or forces applied. And therefore I think this law is ſufficiently eſtabliſhed, that the reſiſtance, any of theſe bodies makes, is proportional to the ſpace through which it is bent, or that it exerts a force proportional to the diſtance it is ſtretched to. FIG. 109. 110.

The knowledge of this property of ſpringy bodies is of great uſe in mechanics, for by this means a ſpring may be contrived to pull at all times with equal ſtrength, as in the fuſee of a watch; or it may be made to draw in any proportion of ſtrength required.

The action of a ſpring may be compared to the lifting up a chain of weights, lying upon a plane, or to the lifting a cylinder of timber out of the water endways.

110.

PROP. LXXVII.

111. *To find the lateral strength of any beam of timber, whose transverse section is any figure whatsoever.*

Let ERG be the section of the beam in the place where it breaks. Draw the ordinates IN, in, infinitely near each other, and parallel to the base RG.

Put $ER = d$ $\qquad EI = x$

$RG = b$ $\qquad IR = v$

$IN = y$

The absolute strength of one fibre of the wood $= 1$.

When the beam breaks, it is done by the separation of the parts of the wood at E. Therefore QRE must be esteemed a bended leaver, whose fulcrum is at R. When the beam breaks, the fibres at E are stretch to their full strength, but those nearer R are less stretched, and exert less force or resistance in proportion to their distance from R (by the last Prop.); and therefore the resistance of a fibre at $I = \frac{v}{d}$; and the resistance of all the fibres in the parallelogram In, $= \frac{v}{d} \times In$; and the power of all the fibres in the parallelogram, in regard to the brachium IR, is $= \frac{vv}{d} \times In$. And the sum of all the powers in the whole section $=$ sum of all the $\frac{vv}{d} \times In$.

Let g, p, be the distance of the center of gravity and percussion from RG, as the axis of motion.

Then (by Cor. 1. Prop. LVII.) the sum of all the $vv \times In$ $= gp \times$ sum of all the $In = gp \times$ section ERG. And the sum of all the $\frac{vv}{d} \times In = \frac{gp}{d} \times$ section ERG. Therefore the strength of the beam at E, is $= \frac{gp}{d} \times$ section ERG.

112. *Cor.* 1. *If there be taken* $RO = \frac{gp}{d}$; *then all the fibres of the wood being supposed to be collected in O, and acting there with their full strength; their total strength at O, shall be equal to the strength of the beam, at the section ERG.*

For suppose O to be such a point; then the strength of the beam, or $\frac{gp}{d} \times$ section $ERG = RO \times$ section ERG; and $RO = \frac{gp}{d}$. FIG. 112.

Cor. 2. *If the section be a parallelogram.* $g = \frac{1}{2}d$, *and* $p = \frac{2}{3}d$, *therefore* $RO = \frac{1}{3}d$.

In a circle whose diameter is ER, $g = \frac{1}{2}d$, $p = \frac{5}{8}d$, *and* $RO = \frac{5}{16}d$, $= \frac{1}{3}d$ *nearly, as in the parallelogram.*

In the periphery of a circle (the beam being a hollow cane) $g = \frac{1}{2}d$, *and* $p = \frac{3}{4}d$, *whence* $RO = \frac{3}{8}d = \frac{1}{3}d$ *nearly; as in the parallelogram.*

In a triangle whose base is at E *parallel to the horizon, and vertex at* R; $g = \frac{2}{3}d$, *and* $p = \frac{3}{4}d$, *and* $RO = \frac{1}{2}d$. *And its strength* $= \frac{1}{2}d \times$ *area of the triangle.*

Cor. 2. *The strength of a cylinder when twisted, wrested, or wrung round its axis, is equal to the lateral strength of a triangular beam whose height = radius, base = circumference of the cylinder; and vertex of the triangle at* D. 289.

For let $AD = r$, circ. $DEFG = c$, $Ap = v$, circ. $pqr = \frac{cv}{r}$. The resistance of a fibre at D is 1, and at p is $= \frac{v}{r}$, and the resistance of all the fibres in pqr is $= \frac{cvv}{rr}$; and the power of all the fibres in pqr, in regard to the brachium Ap is $= \frac{cvv}{rr} \times v$; and the total power of all the fibres in the whole section $DEFG$ is $= \frac{c}{rr} \times$ sum of all the $v^3 = \frac{c}{rr} \times 1^3 + 2^3 + 3^3 + 4^3 \ldots$ to $r^3 =$ * $\frac{c}{rr} \times \frac{r^4}{4}$; therefore the strength is $= \frac{crr}{4}$.

And in the triangle, the area being $\frac{cr}{2}$, and $d = r$, the strength $= \frac{1}{2}d \times \frac{cr}{2} = \frac{crr}{4}$, the same as for the twisted cylinder.

SCHOL.

This Cor. 2. does not agree so well with timber as metal, for the texture of wood is not the same in length as breadth. For

* *Ward's* Math. Guide, Part V. Lem. 4.

 all

FIG. all wood is composed of long slender tubular fibres, joined together by a glutinous matter, which is easily separated; and therefore wood is much more easily split than broken.

PROP. LXXVIII.

113. *Given the weight that will break a beam laterally, to find how much will break it when drawn in direction of its length.*

Let DR be the beam.
Put l = its length DE,
W = weight applied at D, that can break it at E,
d = depth ER,
g = distance of the center of gravity of the section ERG from R,
p = distance of the center of percussion of ERG from RG,
Then $\frac{dl}{gp}W$ = weight that will break it when drawn in direction of its length.
But if the beam be supported at both ends, and the weight breaks it in the middle; g and p must be measured from the upper side, and take l for half its length, W for half the weight that breaks it.

For by Cor. 1. of the last Prop. if $RO = \frac{gp}{d}$, then all the fibres of the beam acting at O, will be equal to the strength of the beam; and since W, applied at D, can break it in either case, therefore by the nature of the leaver, it will be, $l \times W = RO \times$ absolute strength $= \frac{gp}{d} \times$ absolute strength, therefore the absolute strength $= \frac{ldW}{gp}$, or the weight that can break it, when drawn in length.

Cor. Hence, if there be taken $RL = \frac{gp}{d}$, then the weight which being applied at L, will just break the beam horizontally, the same will just pull it asunder, when applied lengthways.

For then $l = \frac{gp}{d}$, and weight W = resistance at O = strength of the whole beam.

Therefore

Therefore if a piece of oak, an inch square and a foot long, supported at both ends, bears 315 lb. before it breaks; it will bear, when drawn in length, 2835 lb. or 1 ton, 5 hundreds, 2 stone 7 pound; that is above a ton and a quarter. FIG. 113.

SCHOL.

Here we all along suppose that the fulcrum at *R* remains fixt; but if it should vary by the denting in of the parts at *R*, it will cause a little variation in the strength, and make the beam something weaker laterally. And that it will yield a little this way, is evident from experiments; for the hardest bodies, such as glass in small threads, may be extended in length, and consequently may be contracted by a contrary force; and balls of glass or wood, let fall upon a hard body, will rebound; which they cannot do without the denting in of the parts.

PROP. LXXIX.

If a weight be laid upon the streight beam AB, supported at both ends, its bending or curvature will be nearly as the weight and length directly; and as the breadth and cube of the depth reciprocally. 114.

It is found (by Prop. LXXVI.) that if several weights be laid successively upon a horizontal beam *AB*, the space *CDB*, thro' which the point *D* descends, will be as the weight it bears. Now the parts at *e*, *f*, which are contiguous at the beginning of the descent, are gradually separated; till at last the beam breaks. At which time, when it can bear no more, the infinitely small distance *ef* is a given quantity. If *CD* be supposed very small, then *CD* is as the curvature at *D*, and this curvature is as the infinitely small angle *eDf*, that is, as $\frac{ef}{Df}$, and when it breaks, as $\frac{1}{Df}$.

Let *L* be the length of any beam, *b* its breadth, *Df* its depth, then (by Prop. LXXIII.) its strength, or the weight it will bear, is as $\frac{b \times Df^2}{L}$. Therefore put $\frac{b \times Df^2}{L}$ for the greatest weight, and

FIG. 114. $\frac{1}{Df}$ for the curvature when breaking; W for any other weight, and C the correspondent curvature, And it will be as $\frac{b \times Df^2}{L} : \frac{1}{Df} :: W : C = \frac{LW}{b \times Df^3}$.

Cor. 1. *The quantity of deflexion CD of any beam is as the weight and cube of the length directly, and the breadth and cube of the depth reciprocally.*

For when CD is very small, ADB is very near a circle, or nearer a parabola; suppose it a circle, and let its radius be R, then $2R \times CD = AD^2$ or $\frac{1}{4} LL$; therefore $\frac{CD}{LL} = \frac{1}{8R}$ whence $\frac{CD}{LL}$ is as the curvature C, that is as $\frac{LW}{b \times Df^3}$, or CD is as $\frac{WL^3}{b \times Df^3}$. And if ACB, the original position of the beam, is not a right line, yet CD will still be of the same quantity.

Cor. 2. *In similar homogenous streight bodies the curvature is as the weight directly, and cube of the depth reciprocally; but the deflexion CD is as the weight directly, and depth reciprocally.*

Cor. 3. *In similar bodies bending from a streight line by their own weight, the curvature is given, and the deflexion is as the square of the depth.*

Cor. 4. *In the utmost strength of beams, or their breaking position, the curvature is reciprocally as the depth, and the deflexion as the square of the length directly and the depth reciprocally.*

For then $b \times Df^2$ is as LW.

Cor. 5. *What is said of streight beams is equally true of any beams, in regard to the increase or variation of curvature, and to the deflection from their original position.*

SCHOL.

What is said of beams of timber in this section, is equally applicable to any solid bodies, acted on in a like manner as by weights. There are some bodies in which a very little bending may have a great effect, as in the glasses of large telescopes. For (by Cor. 3. of the last Prop.) the deflexion from their true figure, arising from their own weight, is as the square of the

the diameter, when the glasses are similar. And though this be insensible in small glasses, it may produce some sensible error in large ones; and the same may happen to them in grinding by too much pressure. FIG.

From the foregoing propositions it follows, that if a certain beam of timber be able to support a given weight, another beam of the same timber, similar to the former, may be taken so great as to be able but just to bear its own weight; and any bigger beam cannot support itself, but must break by its own weight; and any less beam will bear something more. For the strength being as the cube of the depth, and the stress being as the matter and length, is as the 4th power of the depth: it is plain the stress increases in a greater ratio than the strength. Whence it follows, that a beam may be taken so large, that the stress may far exceed the strength. And that of all similar beams there is but one that will support itself, and nothing more. Likewise if any beam can make 10 times its own weight, no other similar beam will do the same. And the like holds in all machines, and in all animal bodies. And hence there is a certain limit, in regard to magnitude, not only in all machines and artificial structures, but also in natural ones, which neither art nor nature can go beyond, supposing them made of the same matter, and in the same proportion of parts.

Hence it is impossible that mechanic engines can be increased to any bigness. For when they arrive at a particular size, their several parts will break and fall asunder by their weight; neither can any buildings of vast bigness be made to stand, but must fall to pieces by their great weight, and go to ruin. Vast columns and pyramids will break by their weight and tumble down.

It is likewise impossible for nature to produce animals of any vast size at pleasure, or any such thing as giants, or men of prodigious stature; except some sort of matter can be found to make the bones of, which is so much harder and stronger than any hitherto known; or else that the proportion of the parts be so much altered, and the bones and muscles made thicker in proportion, which will make the animal distorted and of a monstrous figure, and not capable of performing any proper action. And being made similar, and of common matter, they will not be able to stand or move, but being burdened by their own weight must fall down.

Thus it is impossible that there can be any animal so big as to carry a castle upon his back, or any man so strong as to remove a mountain, or pull up a large oak by the roots: nature will not

FIG. admit of these things, whence its impoſſible there can be animals of any ſort beyond a determinate bigneſs.

Fiſh may indeed be produced to a larger ſize than land animals, becauſe their weight is ſupported by the water; but yet they cannot be increaſed to immenſity, becauſe the internal parts will preſs upon one another by their weight and deſtroy their fabric.

On the contrary, when the ſize of animals is diminiſhed, their ſtrength is not dimiſhed in the ſame proportion as the weight. And therefore a ſmall animal will carry far more than its own weight, whilſt a great one cannot carry ſo much as its weight. And hence it is that ſmall animals are more active, will run faſter, jump farther, or perform any motion quicker, for their weight, than large animals; for the leſs the animal the greater the proportion of the ſtrength to the ſtreſs. And nature ſeems to know no bounds as to the ſmallneſs of animals, at leaſt in regard to their weight.

Neither can any two unequal and ſimilar machines reſiſt any violence alike, or in the ſame proportion; but the greater will be more hurt than the leſs. And the ſame is true of animals, for large animals by falling break their bones, whilſt leſſer ones falling higher receive no damage. Thus a cat may fall 2 or 3 yards high and be no worſe, and an ant from the top of a tower.

It is likewiſe impoſſible, in the nature of things, that there can be any trees of immenſe bigneſs; if there were any ſuch, their limbs, boughs, and branches, muſt break and fall down by their great weight. Thus it is impoſſible there can be an oak a quarter of a mile high; ſuch a tree cannot grow or ſtand, but its limbs will drop off by their weight. And hence likewiſe, leſſer plants can better ſuſtain themſelves than large ones can do.

Neither could a tree of an ordinary ſize be able to ſtand if it was compoſed of the ſame tender matter that ſome plants conſiſt of; nor ſuch a plant if it were much bigger than common. And that plants made of ſuch tender matter may better ſupport themſelves; nature has made the trunks and branches of many of them hollow, by which means they are both lighter and ſtronger.

The propoſitions before laid down concerning the ſtrength and ſtreſs of timber, &c. are alſo of excellent uſe in ſeveral concerns of life, and particularly in architecture; and upon theſe principles a great many problems may be reſolved relating to the due proportion of ſtrength in ſeveral bodies, according

to

to their particular positions and weights they are to bear, some FIG.
of which I shall briefly enumerate.

If a piece of timber is to be holed with a mortise hole, the beam will be stronger when it is taken out of the middle, than if it be taken out of either side. And in a beam supported at both ends, it is stronger when the hole is taken out of the upper side than the under one, provided a piece of wood is driven hard in to fill up the hole.

If a piece is to be spliced upon the end of a beam to be supported at both ends, it will be stronger when spliced on the under side of a beam, than on the upper side. But if the beam is supported only at one end, to bear a weight on the other, it is stronger when spliced on the upper side.

When a small leaver, *&c.* is nailed to a body to remove it or suspend it by, the strain is greater upon the nail nearest the hand or point where the power is applied.

If a beam is supported at both ends, and the two ends reach over the props, and be fixt down immoveable, it will bear twice as much weight as when the ends only lie loose or free upon the supporters.

If a slender cylinder is to be supported by two pieces, the distance of the pins ought to be $\frac{5858}{10000}$ parts of the length of the cylinder, that is $\frac{3}{5}$ its length, the pins equi-distant from its ends, and then the cylinder will endure the least bending or strain by its weights.

By the foregoing principles it also follows, that a beam fixt 115.
at one end, and bearing a weight at the other, if it be cut in the form of a wedge and placed with its parallel sides, parallel to the horizon, it will be equally strong every where, and no sooner break in one place than another.

If a beam has all its sides cut into the form of a concave 116.
parabola, whose vertex is at the end, and base a square, a circle, or any regular polygon, such a beam fixt horizontal at one end, is equally strong throughout for supporting its own weight.

By the same principles, if a wall faces the wind, and if the section of it be a right angled triangle, or the foreside be perpendicular to the horizon, and the backside terminated by a sloping plane intersecting the other plane in the top of the wall, such a wall will be equally strong in all its parts to resist the wind if the parts of the wall cohere strongly together; but if it be built of loose materials, it is better to be convex on the backside in form of a parabola.

FIG. If a wall is to support a bank of earth, or any fluid body, it ought to be built concave in form of a semi-cubical parabola, whose vertex is at top of the wall; this is when the parts of the wall stick well together; but if the parts be loose, then a right line or sloping plane ought to be its figure. Such walls will be equally strong throughout.

All spires of churches in the form of cones or pyramids, are equally strong in all parts to resist the wind; but when the parts cohere not together, parabolic conoids are equally strong throughout.

Likewise if there be a pillar erected in form of the logarithmic curve, the assymptote being the axis, it cannot be crushed to pieces in one part sooner than in another by its own weight. And if such a pillar be turned upside down, and suspended at the thick end in the air, it will be no sooner pulled asunder in one part than another by its own weight. And the case is the same if the small end be cut off, and instead of it, a cylinder be added, whose height is half the subtangent.

117. Lastly, let AE be a beam in form of a triangular prism, and if $AD =$ [illegible] AB, and $AI = \frac{1}{3}AC$, and the part $ADIH$ be cut away parallel to the base, the remaining beam $DICEF$ will bear a greater weight P, than the whole $ABCEG$, or the part will be stronger than the whole, which is a paradox in mechanics.

And upon the same principles an infinite number of questions of like kind may be resolved, which are curious enough, and of great use in the common affairs of life.

All I shall here add, is the strength of several sorts of timber, and other bodies, as I have collected from experiments.

In the first edition of this book I had inserted the strength of some sorts of wood, such as I had made experiments upon; in all which, I gave the least weight which the worst of them was just able to bear; lest any body computing the strength of a beam, should overcharge it with too much weight. And since that time, I have made a great many more experiments, not only upon many different sorts of wood, but several other bodies, the result of which I shall here set down. A piece of good oak, an inch square, and a yard long, supported at both ends, will bear in the middle, for a very little time, about 330 pounds *averd.* but will break with more than that weight. This is at a medium; for there are some pieces that will carry something more, and others not so much: but such a piece of wood should not in practice be trusted for any length of time with above a third or a fourth part of that weight. For since this is the extreme weight which the best wood will bear, that of a worse sort must break

break with it. For I have found by experience, that there is a great deal of difference in ſtrength, in different pieces of the very ſame tree, ſome pieces I have found would not bear half the weight that others would do. The wood of the boughs and branches, is far weaker than that of the body; the wood of the great limbs, is ſtronger than that of the ſmall ones, and the wood in the heart of a ſound tree is ſtrongeſt of all. I have alſo found by experience, that a piece of timber, which has born a great weight for a ſmall time, has broke with a far leſs weight, when left upon it for a far longer time. Wood is likewiſe weaker when it is green, and ſtrongeſt when thoroughly dried, and ſhould be two or three years old at leaſt. If wood happens to be ſappy it will be weaker upon that account, and will likewiſe decay ſooner. Knots in wood weaken it very much, and this often cauſes it to break where a knot is. Alſo when wood is croſs-grained, as it often happens in ſawing; this will weaken it more or leſs, according as it runs more or leſs croſs the grain. And I have found by experience, that tough wood croſs the grain, ſuch as elm or aſh, is 7, 8, or 10 times weaker than ſtreight; and wood that eaſily ſplits, ſuch as fir, is 16, 18, or 20 times weaker. And for common uſe it is hardly poſſible to find wood but it muſt be ſubject to ſome of theſe things. Beſides, when timber lies long in a building, it is apt to decay or be worm eaten, which muſt needs very much impair its ſtrength. From all which it appears, that a large allowance ought to be made for the ſtrength of wood, when applied to any uſe, eſpecially where it is deſigned to continue for a long time.

The proportion of the ſtrength of ſeveral ſorts of wood, and other bodies that I have tried, will appear in the following table.

Box, yew, plumbtree, oak — —	11
Elm, aſh — — — —	$8\frac{1}{2}$
Walnut, thorn — — —	$7\frac{1}{2}$
Red fir, hollin, elder, plane — —	7
Crabtree, appletree — —	7
Beech, cherrytree, hazle — —	$6\frac{2}{3}$
Alder, aſp, birch, white fir — —	6
Willow or ſaugh — —	6
Iron — — — —	107
Braſs — — —	50
Bone — — — —	22
Lead — — —	$6\frac{1}{5}$
Fine free-ſtone — — —	1

FIG. In this table I have put ſeveral ſorts of wood into one claſs together, which I found to be pretty near of the ſame ſtrength, as I found ſometimes one ſort to exceed in ſtrength and ſometimes another, there being a great difference even in the ſame ſort of wood; and I do not doubt but other people that ſhall make experiments, will find them as different and various as I have done, and perhaps quite different from mine, juſt according to the goodneſs or badneſs of the wood they uſe. But I have contented myſelf to ſet down what I found from my own experience, as the reſult of a great many trials, without any regard to what other people have done or may do. What I ſhall further add, is this,

A cylindric rod of good clean fir, of an inch circumference, drawn in length, will bear at extremity, 400 lb. and a ſpear of fir 2 inches diameter, will bear about 7 ton, but not more.

A rod of good iron of an inch circumference, will bear near 3 ton weight.

A good hempen rope of an inch circumference, will bear 1000 lb. at extremity.

All this ſuppoſes theſe bodies to be ſound and good throughout, but none of theſe ſhould be put to bear more than a third or a fourth part of the weight, eſpecially for any length of time.

From what has been ſaid, if a ſpear of fir or a rope, or a ſpear of iron of d inches diameter, was to lift $\frac{1}{4}$ the extreme weight, then,

The fir would bear $8\frac{4}{5}\,dd$ hundred weight.
The rope $22\,dd$ hundred weight.
The iron $6\frac{3}{4}\,dd$ ton weight.

SECT.

SECT. IX.

The properties of fluids, the principles of hydrostatics, hydraulics, and pneumatics.

PROP. LXXX.

FIG.

Motion or pressure in a fluid is not propagated in right lines, but equally all around in all manner of directions. 118.

If a force act at *a* in direction *ab*, that motion can be directed no further than these particles lie in a right line as to *c*. But the particle *c* will urge the particles *d*, *f* obliquely, by which that motion is conveyed to *e*, *g*. And these particles *e*, *g*, will urge the particles *n*, *p*, and *r*, *s* obliquely, which lie nearest them. Therefore the pressure as soon as it is propagated to particles that lie out of right lines, begins to deflect towards one side and the other, and that pressure being farther continued, will deflect into other oblique directions, and so on. Therefore the pressure and motion is propagated obliquely ad infinitum, and will therefore be propagated in all directions.

Cor. If any part of a pressure propagated through a fluid, be stopt by an obstacle, the remaining part will deflect into the spaces behind the obstacle. Thus if a wave proceeds from C, and a part goes through the hole A, it expands itself, and forms a new wave beyond the hole, which moves forward in a semi-circle whose center is the hole. 119.

For any part of a fluid pressing against the next is equally re-acted on by the next, and that by the next to it, and so on, from whence follows a lateral pressure (equal to the direct pressure) into the places behind the obstacle.

PROP.

FIG.

PROP. LXXXI.

A fluid can only be at rest when its surface is placed in a horizontal situation.

120. For let *ABCD* be a vessel of water or any fluid: and let *AB* be parallel to the horizon. Suppose the surface of the liquor to be in the position *FE*. Then because the parts of the fluid are easily moveable among themselves; therefore (by ax. 7.) the higher parts at *E* will, by their gravity, continually descend to the lower places at *F*. Also the greater pressure under *E* and the lesser under *F*, will cause the parts at *E* to descend, and those at *F* to ascend. And thus the higher parts of the fluid at *E* descending, and spreading themselves over the lower parts at *F*, which are at the same time ascending: the surface of the fluid will at last be reduced to a horizontal position *AB*. But being settled in this position, since there is no part higher than another, there is no tendency in any one part to descend, more than in another; and therefore the fluid will rest in an horizontal position.

121. Cor. 1. *If the fluid does not gravitate in parallel lines, but towards a fixed point or center C; then the fluid can only be at rest when its surface takes the form of a spherical surface AB, whose center is C.*

For if any parts of the surface of the fluid *A* or *B*, were further from *C* than the rest, they would continually flow down to the places nearer *C*, towards which their weights are directed: till at last they would be all equi-distant from it.

Cor. 2. *Any fluid being disturbed, will of itself return to the same level, or horizontal position.*

122. Cor. 3. *Hence also if a different fluid ABEF rest upon the fluid ABCD; both the surface FE, and the surface AB that divides them, will lie in a level or horizontal situation, when at rest.*

For if any part of the surface *AB* be higher than the rest, it will descend to the same level; and since *FE* is also level, and therefore the heights *AF*, *BE* in every place equal; the pressure of it on all the parts of the horizontal surface *AB*, will be equal. And therefore it cannot descend in one place more than another, but will continue level.

Cor.

FIG.

Cor. 4. *Hence water communicating with two places, or any way conveyed from one place to another; will rise to the same level in both places. Except so far as it is hindered by the friction of the channel it moves thro', or perhaps some very small degree of tenacity or cohesion.* 122.

PROP. LXXXII.

In any fluid remaining at rest; every part of it, at the same depth, is in an equal state of compression.

For let the plane *EF* be parallel to the surface *AB*. Then since the height of the fluid at all the points of *EF*, is equal; therefore the weights standing upon any equal parts of *EF* are equal: and therefore the pressure in all the points of *EF* is equal also. 123.

Cor. 1. *A fluid being at rest, the pressure at any depth is as the depth.*

For this pressure depends on the weight of the superincumbent fluid, and therefore is as its height.

Cor. 2. *In any given place, a fluid presses equally in all directions.*

For (by Prop. LXXX.) as the pressure in any place acts in all directions, it must be the same in all directions. For if it were less in one direction than another, the fluid would move that way, till the pressure becomes equal. And then the fluid would be at rest, and be equally compressed in all directions.

Cor. 3. *The pressure is equal in every part of a plane drawn parallel to the horizon.*

Cor. 4. *When a fluid is at rest, each drop or particle of it, is equally pressed on all sides, by the weight of the fluid above it.*

PROP.

FIG.

PROP. LXXXIII.

123\. *If a fluid be at rest in any vessel, whose base is parallel to the horizon; the pressure of the fluid upon the base, is as the base and perpendicular altitude of the fluid; whatever be the figure of the containing vessel.*

CASE I.

Let *ABCD* be a cylinder or prism, then (by Cor. 1. Prop. LXXXII) the pressure upon a given part of the base, (as a square inch) is as the depth. And the pressure upon the whole base is as the number of parts, or inches, contained in it; and therefore is as the base and altitude of the fluid.

CASE II.

124\. Let the heights and bases of the vessels *ABC*, *DEF* be equal to those of the cylinder *ABCD*; then since any part of the bases *AB* or *DE* is equally prest, as an equal part of the base *CD*; therefore the whole pressure upon the bases *AB* or *DE* is equal to the whole pressure upon the base *CD*. And therefore is as the base and perpendicular height.

Cor. 1. *If two vessels ABC, DEF of equal base and height, tho' never so different in their capacities; be filled with any the same fluid; their bases will sustain an equal quantity of pressure; the same as a cylinder of the same base and height.*

Cor. 2. *The quantity of pressure at any given depth, upon a given surface; is always the same, whether the surface pressed be parallel to the horizon, or perpendicular, or oblique; or whether the fluid continued upwards from the compressed surface, rises perpendicularly into a rectilinear direction, or creeps obliquely thro' crooked cavities and canals; and whether these passages are regular or irregular, wide or narrow.* And hence

125\. *Cor.* 3. *If ABDCF be any vessel containing a fluid; and BL, ED, HFOK, and GC be perpendicular to the horizon, and GHAB the surface of the liquor; and FL, COD parallel to AB. Then the pressure at L and F is as BL or HF; at D, O and C, as ED; at K as HK. And therefore the pressure at L and F is the same. And the pressures at D, O, C are equal.*

Cor.

Cor. 4. *The pressure is every where directed perpendicularly against the inner surface of the vessel. Therefore at K it is directed downward, at L sideways, and at F upwards.* By Prop. IX. FIG. 125.

Cor. 5. *If two vessels AB, CD, communicate with one another by the tube BC, and if any liquor be poured into one AB, it will rise to the same height in the other CD, and will stand at equal heights in both; that is, AD will be a horizontal line.* 126.

For if the fluid stand at unequal heights, the pressure in the higher will be greater than in the lower, and cause it to move towards the lower.

Cor. 6. *If two different fluids sustain one another at rest in two vessels AB, CD, that communicate; their height above their place of meeting, will be reciprocally as their densities or specific gravities.* 127. 128.

Let the fluids join at *C*, and take the perpendicular height of *eC*, equal to that of *AB*. Then if the densities of the fluids were equal they would sustain one another at the equal heights *AB*, *Ce*. Therefore that the pressure of the other fluid may be the same at *C*, its height must be so much greater as the density is less; that is, *CD* : *Ce* or *AB* :: density of *AB* : density of *CD*.

SCHOL.

The truth of the foregoing propositions may be easily proved experimentally. Take several tubes open at both ends, some streight some crooked, with their low ends turned in all directions, and of several sizes, regular and irregular. Put these into a vessel of water to any depth, and the water will rise up to the height of the external surface of the water in them all. But this is to be understood of such tubes as are sufficiently wide; for in capillary tubes immersed in a vessel of water, it rises something above the level, and that to heights reciprocally as the diameters. Likewise, if water can rise and be suspended at the height *B* in the capillary tube *AB*, it will be suspended at the same height *B*, whilst the part of the tube at *B* remains the same, whatever be the figure or wideness of the under part *CD*. And the ascent and suspension of water is the very same in vacuo. The same holds for any other fluids, but different fluids rise to different heights. But quicksilver, instead of ascending in a tube, sinks in it, and has its surface depressed below the common surface, to depths which are reciprocally as the diameters of the tubes. 129.

FIG. tubes. But the forces by which fluids are suspended in capillary
129. tubes come under no hydrostatic laws.

PROP. LXXXIV.

If a homogeneous body be immersed in a fluid of the same density with itself, it will remain at rest in any place and in any position; but a body of greater density than the fluid will sink to the bottom, and a body of less density will rise to the top.

Case I.

130. Let the body *EGF* be immersed in the fluid *AD*. Then since the body is of the same density as the fluid, therefore the body will press the fluid under it, just as much as the same quantity of the fluid put in its place. And therefore the pressure of the body, together with that of the fluid above it, presses the fluid below as much as a column of the fluid of the same depth. Therefore the pressure of the body at *F* against the fluid is equal to the pressure of the fluid at *F* against the body. And therefore these two pressures will remain in equilibrio, and the body will be at rest.

Case II.

If the body is more dense, the pressure against the fluid underneath is greater than that of an equal quantity of the fluid. Therefore the weight of the body will overcome the pressure of the fluid under it, and it will sink. But if the body be lighter, the pressure of the fluid will overcome the weight of the body, and it will rise to the top.

Cor. 1. *If several fluids of different densities be mixt together in the same vessel, the heaviest will get to the lowest place, and the lightest to the top; and those of a mean density to the middle. And in any bodies whatever the heaviest will be the lowest.*

Cor. 2. *Hence bodies placed in fluids have a twofold gravity, the one true and absolute, the other apparent or relative. Absolute gravity is the force with which bodies tend downward; by this all sorts of fluid bodies gravitate in their proper places, and their weight taken*

taken together compose the weight of the whole; for the whole is heavy as may be experienced in vessels full of liquor. FIG. 130.

Relative gravity is the excess of the gravity of the body above that of the fluid. By this kind of gravity fluids do not gravitate in their proper places; that is, they do not preponderate, but hindering one another's descent, remain in their proper places as if they were not heavy.

Cor. 3. Hence an irregular body, or one that is heterogeneous, descending in a fluid, or if it move in any direction, and a line be drawn connecting the center of gravity and center of magnitude of the body, the body will so dispose itself as to move in that line; and that the center of gravity will go foremost and the center of magnitude behind.

For there being more matter and less surface near the center of gravity, that part will be less resisted than near the center of magnitude; therefore the center of magnitude will be more retarded than the center of gravity, and will be left behind.

Cor. 4. Hence no body can be at rest within a fluid, unless it be of the same specific gravity as the fluid.

SCHOL.

What is here said of bodies of greater density sinking in a fluid, must be understood of such as are solid. For if a body be hollow it may swim in a fluid of less density. But if the hollows or cavities be filled with the fluid, it will then sink. Likewise bodies of greater specific gravity being reduced to extremely small particles, may then be suspended in the fluid. But the forces by which this is done belong not to any laws of hydrostatics.

PROP. LXXXV.

Bodies immersed in a fluid and suspended in it, lose the weight of an equal bulk of the fluid.

For (by the last Prop.) if the body *EF* be of the same density as the fluid it loses all its weight, and neither endeavours to ascend or descend. Therefore, if it be lighter or heavier it

FIG. 130. only endeavours to ascend or descend with the difference of the weights of the body and the fluid; and has therefore lost the weight of as much of the fluid.

Cor. 1. *The fluid acquires the weight which the body loses.*

For the sum of the weights of the solid and fluid is the same, both before and after emersion.

Cor. 2. *All bodies of equal magnitude immersed in a fluid lose equal weights, and unequal bodies lose weights proportional to their bulks.*

Cor. 3. *The weights lost by immerging one and the same body in different fluids are as the densities of the fluids, or as their specific gravities.*

Cor. 4. *Hence also, if two bodies of unequal bulks be in equilibrio in one fluid, they will lose their equilibrium in another fluid of different density.*

SCHOL.

Since a body immersed in a fluid loses so much weight as that of an equal quantity of the fluid, therefore it tends downwards only with the difference of these weights; and this is the relative gravity of the body in the fluid. But if the body is specifically lighter than the fluid, it seems to lose more weight than it has, and hence the body will tend upwards with the difference of these weights. And this is the relative levity of the body in the fluid; such as we see in feathers or smoke in the air, or cork in water.

PROP. LXXXVI.

The weight of a solid body floating upon a fluid is equal to the weight of a quantity of the fluid, as big as the immersed part of the solid, cut off by the plane of the surface of the fluid.

For if the body be at rest, the pressure of the body upon the fluid underneath, is just the same as the pressure of the fluid in the

the room of the immersed part. And therefore the weight of one is equal to the weight of the other. FIG.

Cor. 1. *If the body be homogeneous, the weight or magnitude of the whole floating body is to the weight or magnitude of the part immersed :: as the density or specific gravity of the fluid is to the density or specific gravity of the body.*

For the density of the fluid : density of the body :: weight (of the fluid equal to the immersed part, or the weight) of the whole body : weight of the immersed part.

Cor. 2. *If one and the same body float, or swim upon different liquids, the immersed part in each liquid will be reciprocally as their densities. And therefore a body will sink deeper in a lighter fluid than in a heavier.*

PROP. LXXXVII.

If a floating body AFBE, or system of bodies, be at rest in a fluid, and D be the center of gravity of the whole body, and C the center of gravity of the fluid AFB equal to the immersed part of the body; then, I say, the line CD will be perpendicular to the horizon. 131.

For as *C* is the center of gravity of the fluid *AFB*, it is the center of all the forces or weights of the parts of the water in *AFB*, tending downwards; but because the body is at rest, the same point *C* is also the center of all the pressures of the fluid underneath tending upwards, by which the weight of the fluid *AFB* or of the body *AFBE* (equal to it by Prop. last) is sustained. Therefore the sum of all the forces tending upwards to *C*, is equal and contrary to the sum of all the forces tending downwards from *D* (by Ax. 11.) because that pressure sustains the body. But the weight of the body tending from *D* is perpendicular to the horizon. Therefore *CD* is perpendicular to the horizon.

Cor. If the whole body be as heavy or heavier than water, and be immersed in it, the center of gravity will be the lowest, and descend the foremost.

PROP.

FIG.

PROP. LXXXVIII.

If a fluid, considered without weight, be enclosed in a vessel, and strongly compressed on all sides, every part within it will be in the same compressed state.

For if any particle was less pressed than another, the greater pressure would move the fluid towards the less compressed part, till their compression became every where equal; and then the equal pressures would ballance one another, and remain at rest.

132. *Cor.* 1. *Hence any soft body as GHI, whose parts cannot be condensed, being immersed in a fluid enclosed in a vessel, and strongly compressed on every side, the body will retain its figure, and suffer no change from the compression of the ambient fluid. And all its parts will remain at rest among themselves, and in the same compressed state as the fluid.*

Cor. 2. *The motion of any included body as E, or of any number of bodies, will not be at all changed by the compression of the fluid, but will remain the same as before.*

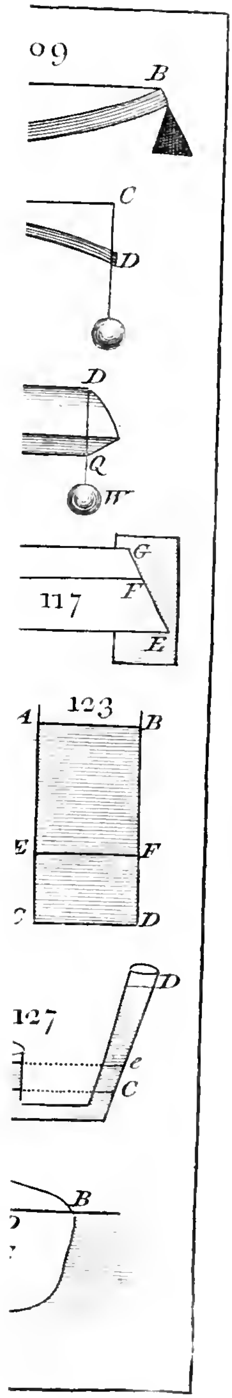
For the compression acting every way alike, can make no alteration in the motion of bodies.

Cor. 3. *In an inflexible vessel, a fluid will not sustain a stronger pressure on one side than another; but will give way to any excess of pressure in a moment of time, and be reduced to an equality of pressure.*

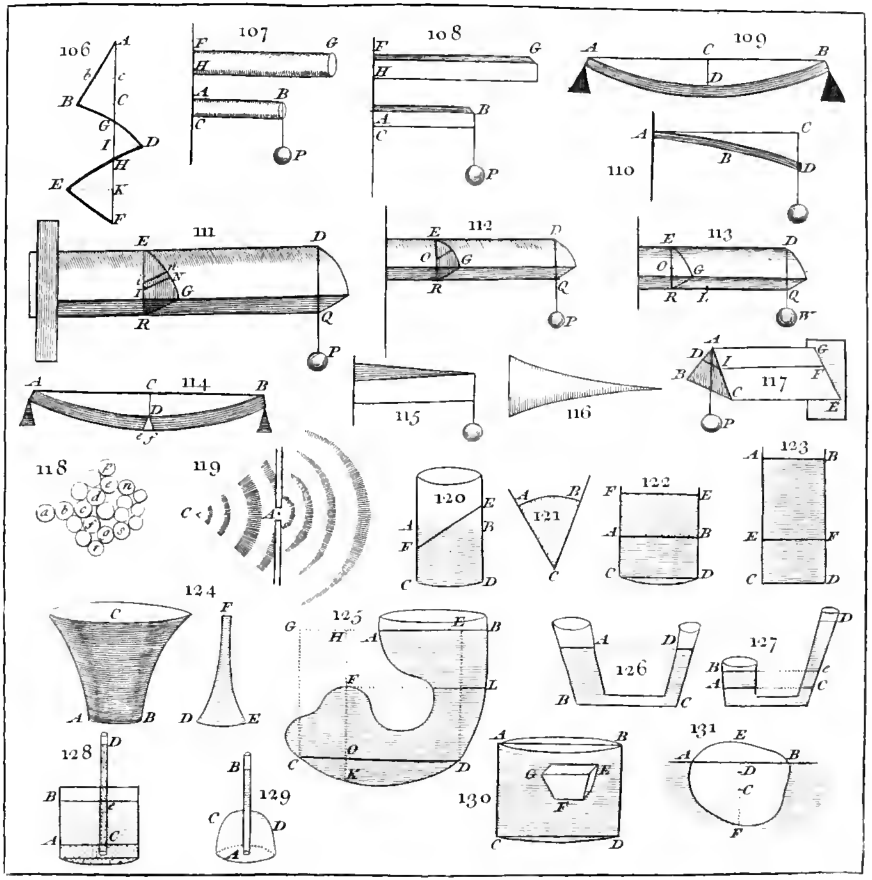
PROP. LXXXIX.

130. *If air, or any elastic fluid of small density, be shut up in a close vessel, every part of it will be in the same compressed state.*

For let *ABCD* be a vessel full of enclosed air, then the air, at equal altitudes within the vessel, will be in the same state of compression; and the compression in the bottom of the vessel can only exceed that at the top, by the weight of a column of air of the height



T.VI. pa. 126.



Pl.VI. pa. 126.

height of the veſſel AC, (by Cor. 1. Prop. LXXXII.) but the weight of ſuch a column of air is inſenſible in reſpect of the external preſſure, or the preſſure of the height of the atmoſphere. And therefore the compreſſion in every part of the veſſel may be looked upon to be the ſame.

FIG.

Cor. 1. *In like manner the compreſſion of the air in any two places near the earth's ſurface is very nearly the ſame.*

For the difference is only the weight of a column of air, whoſe height is the difference of the heights of the two places, which is very inconſiderable.

Cor. 2. *If air be compreſſed in any veſſel by the preſſure of the external air, its elaſtic force is equal to the force and preſſure of the external air.*

Cor. 3. *The air does the ſame thing by its ſpring, as a non-elaſtic fluid does by its weight or preſſure.*

For the ſpring or elaſticity of the air is the force it exerts againſt the force of compreſſion, and therefore muſt be equal to it.

SCHOL.

That the air is a heavy, elaſtic, compreſſible body, is confirmed by many experiments made for that purpoſe. Its properties are theſe.

1. The air has ſome, though a very ſmall degree of weight, which is ſo ſmall, that it hardly becomes ſenſible, but in the weight of the whole atmoſphere, or body of air encloſing the earth.

2. The air is an elaſtic fluid, and capable of being condenſed and rarified. And when it is condenſed or forced into a leſs ſpace, its ſpring, or the force it exerts to unbend itſelf, is proportional to the force that compreſſes it. And the ſpace any given quantity takes up, is reciprocally as the compreſſing force; or its elaſticity is as its denſity.

3. All the air near the earth is in a compreſſed ſtate, by the weight of the atmoſphere or body of the air above, which compreſſes it. And from hence the denſity of the air grows continually leſs, the higher it is above the ſurface of the earth. The weight of the atmoſphere at the ſurface of the earth, is at a medium about $14\frac{1}{2}$ lb. averd. upon every ſquare inch. Bu at different times it differs, by reaſon of winds, hot or cold weather, &c. But the height of the atmoſphere is uncertain, by reaſon it grows

FIG. grows continually more rare towards the top till it vanishes. The weight of the atmosphere is equal to the weight of water 11 yards high.

4. The spring or elasticity of the air is encreased by heat, and decreased by cold, so that if any quantity of air be enclosed in a vessel, it will have a greater spring or pressure when heated, and will lose part of its spring by cold.

PROP. XC.

To find the specific gravity of bodies.

CASE I.

If it be a solid body heavier than water, weigh it exactly, first in air, and then in water, or some fluid whose specific gravity you know; and let

The absolute weight of the body $= A$,
The weight in water, &c. $= B$,
The specific gravity of water, &c. $= C$,
The specific gravity of the body $= D$,

Then will $D = \frac{A}{A-B}C$, *the specific gravity of the body.*

CASE II.

For a solid body lighter than water. Take any piece of metal and tie it to a piece of the light body, so that the compound may sink in water; and putting A, C, D, as in Case I. and

$E =$ *weight of the metal in water.*
$F =$ *weight of the compound in water.*

Then $D = \frac{AC}{A+E-F}$, *the specific gravity of the light body.*

CASE III.

For a fluid. Take a solid body of known specific gravity, which will sink in the fluid. And putting the same letters as in Case I *Then will* $C = \frac{A-B}{A}D$, *the specific gravity of the fluid.*

Or

FIG.

Or thus:

Take a body that will ſink in the fluid and alſo in water, and let

A = abſolute weight of the body,
B = its weight in water,
G = its weight in the fluid,
C = ſpecific gravity of water,
Z = ſpecific gravity of the fluid required,

Then $Z = \frac{A—G}{A—B}C$.

But for mercury, or for powders, duſt, or ſmall fragments of bodies, you muſt uſe a glaſs or metal bucket, obſerving to balance its weight both in air and water. And for bodies that will diſſolve in water, weigh them in oil of turpentine inſtead of water. When the body is weighed in the fluid, it muſt be ſuſpended by a horſe hair, or a fine ſilk thread. Note, if the body and fluid be near the ſame ſpecific gravity, your work will be more exact.

To demonſtrate the rules, it is evident (by Prop. LXXXV.) that a body weighed in water, loſes the weight of as much water. Therefore in Caſe I. the weight of an equal quantity of water is $A—B$. But (by Def. 11.) the ſpecific gravities are as the weights of equal quantities of matter; therefore $A—B : A :: C : D$.

And in Caſe II. $F—E$ = weight of the light body in water, (which is negative when E is greater than F) and the weight of an equal quantity of water is $A—\overline{F—E}$ or $A+E—F$, therefore (as in Caſe I.) it is $A+E—F : A :: C : D$. And the rule is equally true, whether D be lighter or heavier than water.

In Caſe III. ſince $D = \frac{A}{A—B}C$, therefore $C = \frac{A—B}{A}D$. Or in the other rule; $A—B$ = weight of as much water, and $A—G$ = weight of as much of the fluid, and the ſpecific gravities being as the weights of equal quantities of the matter; therefore $A—B : A—G :: C : Z$.

Cor. 1. *Hence if a piece of metal, or any ſort of matter is offered, to know what ſort it is of. Find its ſpecific gravity by the rule above, which ſeek in the following table; and the neareſt to it gives the name of the body, or what kind it is of.*

FIG.

Cor. 2. *And to find the solid content of a small body heavier than water. Weigh it in air and water, and the difference of the weights reduced to grains, being divided by 256; the quotient is the cubic inches it contains.*

For a cubic inch of water weighs 256 grains; or a cubic foot weighs 76lb. troy, or 62 ½ lb. averdupoise, which is but 254 grains to an inch.

Cor. 3. *Hence also the solidity of a body being known, the weight may be found, and the contrary. Thus, put* n = 0.5275 *ounces troy, or* 0.5787 *ounces averdupoise; and* D = *specific gravity of the body by the following table. Then as* 1 : nD : : *solid content in inches : weight in ounces; and one being given finds the other.*

For the weight of a cubic inch of water is .5275 oz. troy, or .5787 oz. averdupoise.

A TABLE of the specific gravities of bodies.

SOLIDS.

Fine Gold	19.648
Standard Gold	18.888
Lead	11.340
Fine Silver	11.092
Standard Silver	10.536
Copper	9.000
Copper Halfpence	8.915
Fine Brass	8.350
Cast Brass	8.100
Steel	7.850
Iron	7.644
Pewter	7.471
Tin	7.320
Cast Iron	7.000
Lead Ore	6.200
Copper Ore	5.167
Lapis Calaminaris	5.000
Load-stone	4.930
Crude Antimony	4.000
Diamond	3.517
White Lead	3.160
Island Crystal	2.720

Marble

FIG.

Marble	2.707
Pebble Stone	2 700
Coral	2.700
Jaſper	2.660
Rock Cryſtal	2 650
Pearl	2.630
Glaſs	2.600
Flint	2.570
Onyx-ſtone	2.510
Common Stone	2.500
Glauber Salt	2 250
Cryſtal	2.210
Oyſter Shells	2.092
Brick	2 000
Earth	1.984
Nitre	1.900
Vitriol	1.880
Alabaſter	1.874
Horn	1 840
Ivory	1.820
Brimſtone	1.800
Chalk	1.793
Borax	1.717
Allum	1.714
Clay	1.712
Dry Bone	1.660
Humane Calculus	1.542
Sand	1.520
Gum Arabic	1.400
Opium	1.350
Lignum Vitæ	1.327
Coal	1.250
Jet	1.238
Coral	1.210
Ebony	1.177
Pitch	1.150
Roſin	1.100
Mahogany	1.063
Amber	1.040
Brazil Wood	1.031
Box Wood	1.030
Common WATER	1.000
Bees Wax	.955
Butter	.940

FIG.

Logwood	.913
Ice	.908
Ash (dry)	.838
Plumbtree (dry)	.826
Elm (dry)	.801
Oak (dry)	.800
Yew	.760
Crabtree	.700
Beech (dry)	.700
Walnut-tree (dry)	.650
Cedar	.613
Fir	.580
Cork	.238
New fallen Snow	.086

FLUIDS.

Quickfilver	14.000
Oil of Vitriol	1.700
Oil of Tartar	1.550
Honey	1.450
Spirit of Nitre	1.315
Aqua Fortis	1.300
Treacle	1.290
Aqua Regia	1.234
Spirit of Urine	1.120
Human Blood	1.154
Sack	1.033
Urine	1.032
Milk	1.031
Sea Water	1.030
Serum of human Blood	1.030
Ale	1.028
Vinegar	1.026
Tar	1.015
WATER	1.
Distilled Waters	.993
Red Wine	.990
Linfeed Oil	.932
Brandy	.927
Oil Olive	.913
Spirit of Turpentine	.874
Spirit of Wine	.866
Oil of Turpentine	.810
Common Air	.0012

In this table you have the mean ſpecific gravities. For there is ſome difference in different pieces of the ſame ſort of body, by reaſon of their different goodneſs, fineneſs, compactneſs, texture, dryneſs, being more or leſs free from mixture, &c. And ſometimes by a greater degree of heat or cold, which affect all bodies a little, from whence there will ariſe a ſenſible difference in different parcels of the ſame ſort of matter, in almoſt all bodies, whether ſolid or fluid. Particularly in wood there is great difference, for green wood is far heavier than dry wood, and ſome green wood will ſink in water, as elm. FIG.

PROP. XCI.

The center of preſſure of any plane ſuſtaining a fluid preſſing againſt it, is the ſame as the center of percuſſion, ſuppoſing the axis of motion to be at the interſection of this plane with the ſurface of the fluid.

The center of preſſure is that point againſt which a force being applied equal and contrary to the whole preſſure, it will juſt ſuſtain it, ſo as the body preſſed on, will incline to neither ſide. 133.

Let AF be the ſurface of the water, O the center of preſſure, draw AO, and parallel to AF, draw cbd. Then the preſſure againſt any ſmall part cd, is as cd and the depth of the fluid, that is as $cd \times Ab$. And the force to turn the plane about O, is $cd \times Ab \times bO$, or $cd \times Ab \times AO - cd \times Ab^2$. And the ſum of them all muſt be equal to o. Therefore $AO = \frac{\text{ſum of all } cd \times Ab^2}{\text{ſum of all } cd \times Ab}$; and therefore (by Prop. LVII.) O is the ſame as the center of percuſſion.

Cor. 1. *The center of preſſure, upon a plane parallel to the horizon, or upon any plane where the preſſure is uniform, is the ſame as the center of gravity of that plane.*

For the preſſure acts upon every part, in the ſame manner as gravity does.

Cor. 2. *The quantity of preſſure upon any plane ſurface, is equal to that of the ſame plane, placed parallel to the horizon, at the depth*

FIG. 133. *depth where its center of gravity is. And the same is true of any number of surfaces taken together.*

For the whole pressure is as the sum of all the $Ab \times cd$; and upon the whole figure placed at the center of gravity, it is $ABC \times$ distance of the center of gravity from A. But (by Cor. 3. Prop. XLIV.) these products are equal. And the same may be proved for several surfaces, or the surface of any solid, taking the center of gravity of all these surfaces.

PROP. XCII.

To find the center of equilibrium of a body, or a system of bodies, immersed in a fluid.

The center of equilibrium is the same with respect to bodies immersed in a fluid, as the center of gravity is to bodies in free space: It is a certain point, upon which if the body or bodies be suspended, they will rest in any position.

67. Let A, B, C be three bodies, or the quantities of matter in them; p, q, r their relative gravities in the fluid; $1 =$ absolute gravity. Then pA, qB, rC are the weights of A, B, C in the fluid. Let G be the center of equilibrium. Then, by the same reasoning as in Prop. XLVII. the sum of the forces of A, B, C is $pA \times Aa + qB \times Bb + rC \times Cc = Gg \times \overline{pA + qB + rC}$, the sum of the forces or weights when situated in G. *Whence* $Gg = \frac{Aa \times pA + Bb \times qB + Cc \times rC}{pA + qB + rC}$, *the distance of the center of equilibrium from ST, in the fluid.* And if any body, as A, is lighter than the fluid; then its relative gravity p will be negative. And if any body is situated on the other side of the plane, its distance from it must be taken negative.

Cor. If the body or bodies be homogeneous, the center of equilibrium is the same as the center of gravity.

SCHOL.

The relative gravity is found thus. Take the specific gravity of the fluid from that of the body, and divide the remainder by the specific gravity of the body. And these specific gravities are had by Prop. XC.

PROP.

FIG. 81.

PROP. XCIII.

If a system of bodies oscillate in a fluid, without resistance; to find the length of an isocronal pendulum vibrating in vacuo.

Because particles of different specific gravities, placed in any given point, will require different times of vibrating in the fluid; therefore we must find the point where a particle of infinite density being placed, will vibrate in the same time as the system: and this will be the center of oscillation. For this particle will lose nothing of its weight in the fluid; its relative gravity being the same as the absolute. Whence the vibrations of this particle will be performed in the same time as in vacuo.

Let A, B, C be three bodies, or their quantities of matter; p, q, r their relative gravities in the fluid; $1 =$ absolute gravity. Then pA, qB, rC are the weights of the bodies in the fluid. Let G be the center of equilibrium; and O the center of oscillation sought. Put $s = A \times SA^2 + B \times SB^2 + C \times SC^2$. Then (by the same reasoning, and construction, as in Prop. LVIII.) the angular velocities which the bodies A, B, C generate in the system are, $\frac{-Se \times pA}{s}$, $\frac{Sn \times qB}{s}$, $\frac{Sd \times rC}{s}$; and the whole angular velocity generated by them all, is $\frac{-Se \times pA + Sn \times qB + Sd \times rC}{s}$. Likewise the angular velocity which the particle P, situated in O, generates in the system, is $\frac{Sr \times P}{P \times SO^2}$ or $\frac{Sr}{SO^2}$. But their vibrations are performed alike: therefore their angular velocities must be equal. That is $\frac{-Se \times pA + Sn \times qB + Sd \times rC}{s} = \frac{Sr}{SO^2} = \frac{Sg}{SG \times SO}$. Whence $SO = \frac{Sg}{SG} \times \frac{s}{-Se \times pA + Sn \times qB + Sd \times rC}$. But (by Prop. XCII.) $-Se \times pA + Sn \times qB + Sd \times rC = Sg \times \overline{pA + qB + rC}$ therefore $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SG \times \overline{pA + qB + rC}}$, *the length of an isocronal pendulum, out of the fluid.*

Cor.

FIG. 81. *Cor.* 1. *Hence if the bodies are homogeneous, then* $p = q = r$; *and* $SO = \frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{p \times SG \times \overline{A + B + C}}$.

Cor. 2. *The system makes an exceeding small vibration in the fluid, in the same time that a simple pendulum, whose length is* $\frac{A \times SA^2 + B \times SB^2 + C \times SC^2}{SG \times \overline{pA + qB + rC}}$, *makes a vibration in vacuo.*

For the velocity of the system being very small; the resistance is inconsiderable.

Cor. 3. *Hence if* SA *be the length of a simple pendulum* (A), *vibrating in a fluid: then* $\frac{SA}{p}$ *is the length of an isocronal pendulum in vacuo.*

For in a single body, $SO = \frac{A \times SA^2}{SA \times pA}$ or $\frac{SA}{p}$.

Cor. 4. *And if p be negative, or the pendulum specifically lighter than the fluid; the pendulum will turn upside down, and vibrate upwards in the fluid. And the length of an isocronal pendulum out of the fluid, will be* $\frac{SA}{p}$, *as before.*

SCHOL.

The center of percussion is the same in a fluid as out of it. For there is nothing concerned in that, but the quantities of matter and velocities; both of which are the same in the fluid, as out of the fluid.

The relative gravities p, q, r are found by the Schol. of the last Prop.

PROP. XCIV.

If a fluid runs thro' any tube, pipe, or canal, and always fills it; its velocity in any place will be reciprocally as the area of the section in that place.

Let

FIG. 134.

Let AB, CD be two ſections at A and C; and let the quantity of the fluid $ABDC$, in a very ſmall time, be tranſlated into the part $abdc$ of the pipe. Draw Pp, Qq perpendicular to AB, CD, or parallel to the motion of the fluid; then Pp, Qq being indefinitely ſmall, will be the velocities of the fluid at P and Q, or the ſpaces deſcribed in that ſmall part of time. Then, becauſe the pipe is always full, the quantity $ABDC = abdc$. Take from both, the part $abDC$, which is common; and there remains $ABba = CDdc$, that is the ſection $APB \times Pp =$ ſection $CQD \times Qq$.

Cor. 1. *The quantity of motion of the fluid in the pipe AD, at any ſection CD; is equal to the motion of a cylinder of that fluid, whoſe baſe is CD, and length the ſame with the pipe from the beginning to that ſection, and velocity that of the fluid at CD. Or the quantity of motion at CD, is as the length of the pipe to that ſection CD.*

For ſince the velocity in A is as $\frac{1}{APB}$, the motion of $ABba$ is $\frac{ABba}{APB}$, that is as Pp; and the motion of the whole, is as the ſum of all the Pp, or the length of the pipe, without any regard to the diameter AB. The cylinder, whoſe baſe is CQD, and height PQ is $= PQ \times CQD$; and its motion with velocity at C, is $= \frac{PQ \times CQD}{CQD} = PQ$ the length of the pipe, as before.

Cor. 2. *If water is driven through the pipe PR by ſome given force acting at P, and the length of the pipe PR be given; the quantity of water diſcharged at R, in a ſecond, or ſome given time, will be as the ſection at R.*

For if the force is given, the motion generated in a given time, will be given; and this motion, being as the quantity of water $\times$ velocity at R; therefore the quantity forced through R, will be reciprocally as the velocity, or directly as the ſection at R, by this Prop.

Cor. 3. *The velocity and quantity of motion, is the ſame very near in crooked tubes as in ſtreight ones; and in pipes divided into ſeveral branches; taking the ſection of all the branches as the ſection of one tube.*

PROP. XCV.

135. *In any pipe whose section is ABCD, the stress or force to split any part of the pipe at B, is equal to half the pressure of the fluid upon the plane BD, drawn perpendicular to the curve in B, and of the same length as that part of the pipe.*

Let Ee be any small part of the surface; draw EO perp. to the curve Ee. Draw En, en perp. to BO, and er perp. to EN. And let OE represent the pressure of a particle of the fluid, then $EO \times Ee =$ pressure upon Ee. The force OE may be divided into the two ON, NE; of which ON tends only to split the tube at A, but that in direction NE is the force to separate the parts at B. Therefore $EN \times Ee$ is the stress at B. But the triangles Eer, EON are similar, and $Ee \times EN = EO \times er$, or $EO \times Nn$. Therefore the part of the pressure on Ee, in direction NE, from whence the stress at B arises, is $= EO \times Nn$, that is $=$ to the pressure upon the plane Nn. Consequently the stress arising from the pressure on BE is $=$ pressure on BN, and from the pressure on BA is $=$ pressure on BO. And the stress at D by the pressure on AD, is equal to the pressure on OD.

Also we suppose the same forces acting in the semi-circle BCD: but these serve only to keep the forces, acting upon BAD, in equilibrio.

Cor. 1. *The stress on any part of a pipe full of water, is as the diameter of the pipe, and the perpendicular height of the water above that place. And consequently the thickness of the metal ought to be in that ratio.*

Cor. 2. *In any concave surface, cask, or vessel, generated by revolving round an axis, and filled with a fluid; the stress as to splitting is equal to half the pressure upon the plane passing through its axis. And the stress on both sides at B and D, equal to the whole pressure on that plane.*

Cor. 3. *Hence the internal pressure on any length of the pipe, is to the stress it suffers as to splitting :: as* 2×3.1416, *to* 1.

Cor. 4. *Hence it follows, that the stress, arising from any pressure, upon any part, to split it longitudinally, transversely, or in any*

any direction, is equal to the pressure upon a plane, drawn perpendicular to the line of direction. Thus the stress arising from the pressure on BE is = pressure on BN. FIG. 135.

Cor. 5. And if the pipe be flexible; it will, by the pressure, be put into a cylindrical form; or such that the section is a circle.

For if *BD* be greater than *AC*, there will be a greater pressure in direction *OA* than in direction *OB*. And the greater pressure will drive out the sides *A* and *C*, till *AC* become equal to *BD*; and *ABCD* be a circle. Besides, a circle is more capacious than any other figure. And if a pipe be not flexible, yet the pressure of the fluid will always endeavour to put it into a circular figure.

Cor. 6. And if an elastic compressed fluid be inclosed in a vessel, flexible, and capable of being distended every way; it will form itself into a sphere, for the same reason.

PROP. XCVI.

If a close flexible tube AB full of air, be immersed wholly or in part in the water CDEF; the force to split it, in any place O, is proportional to AO, its height from A. 136.

For the air compressed at *A* is in equilibrio with the external pressure of the water. At *B* and *O* the air is in the same compressed state as in *A*, but the external pressure at *B*, is less by the weight of the column of water *AB*: and at any place *O*, the external pressure is less than in *A*, by the weight of *AO*; therefore the internal pressure at *O* exceeds the external, by the weight of the column of water *AO*. And the stress at *O* is as that excess.

Cor. 1. The stress is greatest at the top BG, and at A is nothing.

Cor. 2. If the tube be inflexible; the stress will be according to the state of the compressed air within it. If the air within be the same as the external air at B, then the stress at O is as BO. If it be less then the external air, the increase of the stress will be also as BO; acting at the outside. If it be of very great density, the increase of stress at O is as AO.

 For

FIG. 136. For the pressure within is uniform; but without, it is as the depth of the water.

PROP. XCVII.

The quantity of a fluid flowing in any time through a hole in the bottom or side of a vessel, always kept full; is equal to a cylinder whose base is the area of the hole, and its length the space a body will describe in that time, with the velocity acquired by falling through half the height of the liquor above the hole.

137. Let *ADB* be a vessel of water, *B* the hole, and take *BC*=*BD* the height of the water. And let the cylinder of water *BC* fall by its weight through half *DB*, and it will by that fall, acquire such a motion, as to pass through *DB* or *BC* uniformly in the same time, by Cor. 3. Prop. XIV. But (by Prop. LXXXIII. and Cor. 2.) the water in the orifice *B* is pressed with the weight of a column of water, whose base is *B* and height *BD* or *BC*; therefore this pressure is equal to the weight of the cylinder *BC*. But equal forces generate equal motions: therefore the pressure at *B*, will generate the same motion in the spouting water, as was generated by the weight of the cylinder of water *BC*. Therefore in the time of falling through half *DB*, a cylinder of water will spout out, whose length (or the space passed uniformly over) is *BC* or *BD*. And in the same time repeated, another equal cylinder *BC* will flow out, and in a third part of time, a third, &c. Therefore the length of the whole cylinder run out, will be proportional to the time, and consequently the velocity of the water at *B* is uniform. Therefore in any time, the length of a cylinder of water spouting out, will be equal to the length described in that time, with the velocity acquired by falling through half *DB*.

Cor. 1. *Hence in the time of falling through half DB, a quantity of the fluid runs out, equal to a cylinder whose base is the hole; and length, the height of the fluid above the hole.*

Cor. 2. *The velocity in the hole B is uniform, and is equal to that a heavy body acquires by falling through half DB.*

Cor.

FIG. 138.

Cor. 3. *But at a small distance without the hole, the stream is contracted into a less diameter, and its velocity increased; so that if a fluid spout through a hole made in a thin plate of metal, it acquires a velocity nearly equal to that, which a heavy body acquires by falling the whole height of the stagnant fluid above the hole.*

For since the fluid converges from all sides towards the center of the hole *BF*; and all the particles endeavouring to go on in right lines, but meeting one another at the hole, they will compress one another. And this compression being every where directed to the axis of the spouting cylinder; the parts of the fluid will endeavour to converge to a point, by which means the fluid will form itself into a sort of a conical figure, at some distance from the hole as *BEGF*. By this lateral compression, the particles near the sides of the hole are made to describe curve lines as *HE*, *KG*; and by the direct compression, the fluid from the hole is accelerated outwards at *EG*; and thus the stream will be contracted at *E*, in the ratio of about $\sqrt{2}$ to 1, and the velocity increased in the same ratio.

It must be observed, however, that the particles of the fluid don't always move right forward; but near the edge of the hole, often in spiral lines. For no body can instantly change its course in an angle, but must do it gradually, in some curve line.

Cor. 4. *The fluid at the same depth, spouts out nearly with the same velocity, upwards, downwards, sideways, or in any direction. And if it spout vertically, ascends nearly to the upper surface of the fluid.*

Cor. 5. *The velocities of the fluid, spouting out at different depths, are as the square roots of the depths.*

For the velocities of falling bodies are as the square roots of the heights.

Cor. 6. *Hence if* $s = 16\frac{1}{12}$ *feet,* D = *depth of the vessel to the center of the hole,* F = *area of the hole, all in feet,* t = *time in seconds. Then the quantity of water running out in the time* t, *by this Prop. will be* $tF\sqrt{2Ds}$ *feet, or* $6.128tF\sqrt{2Ds}$ *ale gallons.*

SCHOL.

There are several irregularities in spouting fluids, arising from the resistance of the air, the friction of the tubes, the bigness

FIG. 138. neſs and ſhape of the veſſel, or of the hole, *&c.* A fluid ſpouts fartheſt through a thin plate; if it ſpout through a tube inſtead of a plate, it will not ſpout ſo far; partly from the friction, and partly becauſe the ſtream does not converge ſo much, or grow ſmaller. A jet de'au ſpouts higher, if its direction be a little inclined from the perpendicular; becauſe the water in the uppermoſt part of the jet, falls down upon the lower part, and ſtops its motion. We find by experience, a fluid never ſpouts to the full height of the water above the hole: but in ſmall heights falls ſhort of it, by ſpaces, which are as the ſquares of the heights of the fluid. And all bodies projected upwards, fall ſhort of thoſe projected in vacuo, by ſpaces which are in the ſame ratio; from the reſiſtance of the air.

By experiments, if the height of a reſervoir be 5 feet, a jet will fall an inch ſhort; and the defect will be as the ſquare of the height of the reſervoir. But ſmall jets fail more than in that proportion, from the greater reſiſtance of the air.

PROP. XCVIII.

139. *If a notch or ſlit, fghi, in form of a parallelogram, be cut out of the ſide of a veſſel full of water, ADE; the quantity of water flowing out of it, will be* $\frac{2}{3}$ *the quantity flowing out of an equal orifice, placed at the whole depth gi, or at the height hi; in the ſame time: the veſſel being ſuppoſed to be always kept full.*

For draw the parabola *gob*, whoſe axis is *gi*, and baſe *bi*, and ordinate *ro*; then ſince the velocity of the fluid at any place *r*, is as $\sqrt{gr}$, (by Cor. 5. of the laſt Prop.) that is (by the nature of the parabola) as the ordinate *ro*; therefore *ro* will repreſent the quantity diſcharged at the depth or ſection *rn*. Alſo *bi* will repreſent the quantity diſcharged at the depth or baſe *hi*. Conſequently the ſum of all the ordinates *ro*, or the area of the parabola, will repreſent the quantity diſcharged at all the places *rn*. And the ſum of all the lines *hi* or *rn*, or the area of the parallelogram *fghi*, will repreſent the quantity diſcharged by all the ſections *rn*, placed as low as the baſe *hi*. But the parabola is to the parallelogram, as $\frac{2}{3}$ to 1.

Cor.

Cor. 1. *Let* $s = 16\frac{1}{12}$ *feet.* $D = gi$, *the depth of the slit.* $F =$ *area of the slit, fhig. Then the quantity flowing out in any time or number of seconds* t, *is* $= \frac{2}{3} tF\sqrt{2Ds}$. FIG. 139.

This follows from Cor. 6. of the last Prop.

Cor. 2. *The quantity of fluid discharged through the hole rnbi, is to the quantity which would be discharged through an equal hole placed as low as hi; as the parabolic segment robi, to the rectangle rnbi.*

This appears from the reasoning in this proposition.

SECT. X.

The resistance of fluids, their forces and actions upon bodies; the motion of ships, and position of their sails.

PROP. XCIX.

A body descending in a fluid, adds a quantity of weight to the fluid, equal to the resistance it meets with in falling.

For the resistance is equal to the gravity lost by the body. And because action and re-action are equal and contrary, the gravity lost by the body, is equal to that gained by the fluid. Therefore the resistance is equal to the gravity gained by the fluid.

Cor. 1. *If a body ascends in a fluid; it diminishes the gravity of the fluid, by a quantity equal to the resistance it meets with.*

Cor. 2. *This increase of weight arising from the resistance, is over and above the additional weight mentioned in Cor.* 1. *Prop.* LXXXV.

Cor. 3. *If a heterogeneous body descend in a fluid; it will endeavour to move with its center of gravity foremost, leaving the center of gravity of as much of the fluid, behind.*

For the side towards the center of gravity contains more matter, and will more easily make its way through the fluid; and be less retarded in it.

PROP. C.

If any body moves through a fluid; the resistance it meets with, is as the square of its velocity.

For the resistance is as the number of particles struck, and the velocity with which one particle is struck. But the number of particles of the fluid which are struck in any time, is as the velocity of the body. Therefore the whole resistance is as the square of the velocity.

Cor. 1. *The resistances of similar bodies moving in any fluids, are as the squares of their diameters, the squares of their velocities, and the densities of the fluids.*

For the number of particles struck with the same velocity, are as the squares of the diameters, and the densities of the fluids.

Cor. 2. *If two bodies A, B, with the same velocity, meet with the resistances p and q; their velocities will be as* $\frac{1}{\sqrt{p}}$ *and* $\frac{1}{\sqrt{q}}$, *when they meet with equal resistances.*

For let b be the common velocity, then $p : bb :: q : \frac{bbq}{p}$, and $b\sqrt{\frac{q}{p}}$ = velocity of A to have the resistance q; and since b = velocity of B to have the same resistance q; therefore vel. A: vel. $B :: b\sqrt{\frac{q}{p}} : b :: \frac{1}{\sqrt{p}} : \frac{1}{\sqrt{q}}$.

PROP. CI.

The center of resistance of any plane moving directly forward in a fluid, is the same as the center of gravity.

The center of resistance is that point, to which if a contrary force be applied, it shall just sustain the resistance.

Now the resistance is equal upon all equal parts of the plane, and therefore the resistance acts upon the plane, after the same

FIG. manner, and with the same force as gravity does; therefore the center of both the resistance and gravity must be the same.

Cor. 1. *In any body moving through a fluid; the line of direction of its motion will pass through the center of resistance, and center of gravity of the body.*

For if it do not, the forces arising from the weight and resistance, will not balance one another, which will cause the body to librate or oscillate in the fluid; till by degrees the situation of these two centers will fall into the line of their motion.

Cor. 2. *And for the same reason; if a globe, moving in a fluid, oscillates or turns round its axis; that side, which in oscilating moves against the fluid, suffers a greater force or resistance; and therefore the body is driven from that part, and made to recede from that side, and deflect to the other side; and perhaps describe a curve line in the fluid.*

PROP. CII.

140. *If a non-tenacious fluid, such as the wind, &c. move against the sail SA or any plane surface, in direction WS; it shall urge it in a direction WA perpendicular to that surface, with a force, which is as the square of the velocity, the square of the sine of the angle of incidence, the magnitude of the sail, and the density of the fluid.*

Draw WA, AC perpendicular to SA, SW; and the force of the fluid upon SA, is as the force of one particle, and the number of them falling on SA.

But (by Cor. 1. Prop. IX.) the force of one particle is as its velocity $\times$ S. incidence WSA.

And the number of them (supposing the density to be given) is as their velocity$\times CA$, or (supposing the sail SA given) as the velocity$\times S$. WSA.

Therefore the force of the fluid upon the sail SA, is as the square of the velocity, and the square of the sine of WSA.

Increase the density of the fluid, and the magnitude of the sail, in any ratio; and its evident the force of the fluid against the sale, will be increased in the same ratio.

Cor.

FIG.

Cor. 1. *And if a thin body move in a fluid at rest; the same law holds in respect of the resistance it meets with, in the direction perpendicular to its surface.* 140.

Cor. 2. *If a fluid with a given velocity move in direction WS, against the sail SA; its force to move the sail in any given direction SB, is as the square of the S.WSA × by the S.ASB. And if WSB be a right angle, as S.WSA × $\frac{1}{2}$ sine of twice WSA.* 141.

For let *SD*, *DB* be perpendicular to *SA*, *SB*. Then *SD* is the whole force acting at *S*, and *SB* the force in direction *SB*, and *SB* is as *S.SDB* or *ASB*. And if *ASB* be the comp. of *WSA*, then *S.WSA* × cos. *WSA* is as *S.* twice *WSA*, by trigonometry.

Cor. 3. *The force of a fluid in direction WS, to move the sail or body SA in the same direction WS; is (ceteris paribus) as the cube of the sine of incidence WSA.*

For then *WSB* will be one continued streight-line.

Cor. 4. *But the force of a given stream of a fluid, against any sail SA, to move it perpendicular to its surface, is simply as the S. angle of incidence: but to move it in the same direction with itself, as the square of the S. incidence: all things else remaining the same.*

This follows from Cor. 1. Prop. IX. and Cor. 2. of this.

SCHOL. I.

If the angle *WSB* be given; the fluid will have the greatest force possible against the sail, to move it in direction *SB*; when its position is such, that the sine of the diff. of the angles $\overline{WSA - ASB}$, may be $\frac{1}{3}$ the sine of the sum *WSB*.

SCHOL. II.

If the fluid be tenacious it will urge the body in the same direction with itself, and with a force which is as the sine of incidence: or universally, as the sine of incidence, the square of the velocity, the magnitude of the sail, and density of the fluid. 140.

For by reason of the tenacity of the fluid, the sail is acted on by both the forces *WA*, *AS*, which are equivalent to *WS*.

FIG.

PROP. CIII.

142. *If a very thin and light body SA, plain on both ſides, be placed in a very denſe fluid, which moves in direction WS; and the body can make little or no way thro' the fluid, but only in the direction of its length SA. And if the body be obliged to move parallel to itſelf in a given direction SD. I ſay the body will be ſo moved in the fluid, that its abſolute velocity will be* $= \frac{S.WSA}{S.ASD} \times$ *velocity of the fluid.*

Draw *DT* parallel to *AS*, and produce *WS* to *T*. Then whilſt a part of the fluid moves from *S* to *T*, the body will be moved into the line *TD*; and ſince *SD* is the direction of its motion, the point *S* will be found in *D*. And therefore the velocities of the fluid, and of the body, will be as *ST* to *SD*; that is, as *S.TDS* or *DSA*, to *S.STD* or *WSA*.

Cor. 1. *If WS the direction of the fluid, is perpendicular to SD the direction of the body; then the velocity of the body SA will be* $=$ *tan. WSA* $\times$ *velocity of the fluid.*

For $\frac{S.WSA}{S.ASD} = \frac{S.WSA}{cof.\ WSA} =$ tan. *WSA*, radius being $= 1$.

Cor. 2. *And hence if the body SA continually turn round an axis parallel to WS; then the velocity of SA in direction perpendicular to WS, will be as the tan. WSA*$\times$*velocity of the fluid.*

For *SA* in this caſe will always have the ſame poſition to the direction of the fluid, as before.

143. *Cor.* 3. *If a very thin body SA be obliged to move parallel to itſelf, through a very denſe fluid at reſt; and if it be drawn with a given velocity in direction always parallel to SW. Its abſolute velocity in the fluid, will be reciprocally as the coſ. WSA, and in direction SA.*

Draw *AC* perpendicular to *SW*. Then by reaſon of the denſity and reſiſtance of the fluid, the body will not be able to move laterally, but only in direction *SA*. But the velocities of the point *S* in directions *SW*, *SA* are as *SC* to *SA*, or as coſ. *CSA* to rad. Therefore vel. in direction *SA* $= \frac{rad.}{cof.\ CSA} \times$ vel. in direction *SW*.

PROP.

FIG.

PROP. CIV.

If a plane surface SA, moving parallel to itself, with velocity and direction SD, be acted upon by a fluid moving with velocity and direction WS. And if WF be drawn parallel and equal to SD; and FS drawn. I say the fluid acts upon the plane in the angle FSA, with the relative velocity FS. 144.

For complete the parallelogram *WSDF*; and let the body be at rest, and the fluid move with the contrary motion *DS* or *FW*; and then their relative motions will be the same as before: and the fluid will have the two motions *FW*, *FD* in respect of the body *SA* at rest. Therefore (by Cor. 2. Prop. VII.) the motion compounded out of these is *FS*; which is the absolute motion of the fluid, supposing the body at rest; or the relative motion of it, in respect of the moving body; and therefore acts on it in the angle *FSA*.

Cor. If F falls in the line SA, then the fluid acts not at all upon the body. And if it fall on the contrary side of it; then the fluid acts on the contrary side of the body SA.

SCHOL.

The fluid will move with the surface in direction *SD*, with the greatest force; when it has such a position, that the sine of the diff. of the angles, $\overline{FSA - ASD}$, may be $\frac{1}{3}$ the sine of the angle *FSD*.

And when the angle *WSA* is given, the fluid will have the greatest force upon the sail *SA*, to move it in direction *SD*, when the *S*. angle *ASD* is equal to $\frac{WS}{WF} \times \frac{1}{3}$ the *S*. of *WSA*.

PROP. CV.

Let SA be the sail of a ship, SD the position of her keel; SK, DK perpendicular to SA, SD. And if DE, DS be as the resistances the ship has ahead and aside, with equal velocities; and if DC is 145.

a mean

FIG. 145. *a mean proportional between DE and DK; then SC will be the way of the ship nearly.*

For let SK perpendicular to SA represent the force of the wind upon the sail. The force SK is resolved into the forces SD, DK; SD is the direct force, and DK the force producing her lee way. By Prop. C. her resistance ahead with velocity SD : resist. ahead with vel. $DE :: SD^2\ DE^2$,

and resist. ahead with vel. DE : res. aside with vel. $DE :: DE : SD$, and res. aside with vel. DE : res. aside with vel $DC :: DE^2 : DC^2$.

Therefore ex equo.

Res. ahead with vel. SD : res. aside with vel. $DC :: SD^2 \times DE^3 : DE^2 \times SD \times DC^2 :: SD \times DE : DC^2$.

But the resistances are as the forces producing them, therefore $SD : DK :: SD \times DE : DC^2 = DE \times DK$.

Cor. 1. *Let r = ship's resistance ahead, R = ship's resistance aside, with the same velocity. Then $R : r ::$ radius $\times$ cotan. ASD : tan. square of DSC, the leeway.*

For let rad. $= 1$. tan. $DSK = t$. Then $1 : t :: SD : DK = t \times SD$; and $SD : DC$ or $\sqrt{DE \times DK}$ or $\sqrt{t \times SD \times DE} :: 1$: tan. $DSC = \sqrt{\frac{t \times DE}{SD}}$.

Cor. 2. *Hence the tangent of the lee-way, in the same ship, is as the square root of the cotangent of the angle ASD, which the sail makes with the keel. Therefore if the lee-way be known for any position of the sail, it will be known for all.*

SCHOL.

The lee-way of a ship is generally something more than is here assigned; because her hull and rigging will make her drive a little to the leeward, directly from the wind.

PROP. CVI.

146. *If the wind with a given velocity, in direction WS, fall on the sail SA of a ship, making little or no lee-way; it will urge the ship in direction of the keel SD, with a force, which is as $S.WSA^2 \times S.ASD$.*

Draw

FIG. 144.

Draw SC perpendicular to SA, and CD to SD. And (by Prop. CII.) the force acting upon the sail in direction SC, is as the square of the sine of $IWSA$. But the forces in directions SC and SD are as SC to SD, or as radius 1 to the sine of SCD or ASD. Therefore the force in direction $SD = S.ASD \times$ force in direction SC $= S.ASD \times S.^2 WSA$.

Cor. 1. *The force acting in direction DC perpendicular to the keel, is as* $\overline{S.WSA}^2 \times$ *cos.* ASD.

Cor. 2. *The force in direction SD will be universally as* $\overline{S\,WSA}^2 \times S.ASD$, *and the square of the velocity of the wind, and magnitude of the sail.* 146.

Cor. 3. *The velocity of the ship in direction SD, is as* $S.WSA \times \sqrt{S.ASD} \times$ *velocity of the wind.*

For the square of the velocity of the ship in any direction, is as the resistance in the water, or (its equal) the force of the wind upon the sail in that direction; that is (by Cor 2.) as $\overline{S.WSA}^2 \times S.\ ASD$, and the square of the velocity of the wind. The density, and sail being given.

Cor. 4. *Let the angle WSA be given. And if SDC be a semicircle described on any given line SC; then the force in any direction SD of the keel, is as the cord SD; and the velocity as* $\sqrt{SD}$.

Cor. 5. *The velocity of the ship to windward, is as* $S.WSA \times \sqrt{S.ASD} \times$ *cos.* WSD.

For draw SP perpendicular to WS, and DG to SP; and the velocities in directions SD, GD are as SD to GD, or as radius 1 to $S.DSG$: therefore the velocity in $GD = S.DSG \times S.WSA \times \sqrt{S.ASD}$.

Cor. 6. *The force of the sail SA to turn the ship about, is as* $\overline{S.WSA}^2 \times$ *cos.* ASD.

This appears by Cor. 1. supposing the sail placed in the head of the ship.

PROP.

FIG.

PROP. CVII.

If a ſtream of any fluid as water, flows directly againſt any plane ſurface; its force againſt that plane is equal to the weight of a column of the fluid, whoſe baſe is the ſection of the ſtream; and its length twice the height deſcended by a falling body, to acquire the velocity of the fluid.

Let $s = 16\frac{1}{12}$ feet, the height deſcended by a falling body in 1 ſecond.

v=velocity of the fluid, or the ſpace it deſcribes in one ſecond.

B=baſe of the cylinder or column of water.

Then $2s$=velocity generated by gravity in falling through s. Therefore (by Cor. 1. Prop. XIV.)

$4ss : s :: vv : \frac{vv}{4s} =$ height fallen to gain the velocity v.

And $\frac{vv}{2s} =$ twice that height. Alſo $\frac{vv}{2s}B =$ a cylinder of twice that height.

Now the motion which the cylinder's weight will generate in 1 ſecond, is $2s \times \frac{vv}{2s}B$; or vvB; the motion being as the body × by the velocity. And the force of the fluid againſt the plane, is equal to the reſiſtance of the plane. And the motion deſtroyed in 1 ſecond by the reſiſtance of the plane, is $v \times Bv$ or vvB; which was alſo the motion generated by the weight of the cylinder $\frac{vv}{2s}B$, in the ſame time. But equal forces in the ſame time generate or deſtroy equal motions. Therefore the weight of the cylinder $\frac{vv}{2s}B =$ force of the fluid againſt the plane.

Cor. 1. *The force of a ſtream of water againſt any plane, is equal to the weight of a column of water, whoſe baſe is the ſection of the ſtream, and height $\frac{vv}{2s}$; or the height of the water, if it flow through a hole at the bottom of a reſervoir.*

It follows from this Prop. and Prop. XCVII. Cor. 2.

Cor. 2. *Moreover if any part of the water lie upon the plane; the force will be augmented by the weight of ſo much water.*

Cor.

FIG.

Cor. 3. *The forces of different streams of water against any plane, are as their sections and the squares of the velocities.*

Cor. 4. *If the plane be also in motion; the relative velocity of the water against the plane, must be taken instead of the absolute velocity.*

SCHOL.

A cubic foot of water contains 6.128 ale gallons, and weighs $92\frac{1}{2}$ lb. aver.

The density of water to that of air at a mean is as 850 to 1. Sir Isaac Newton found the resistance of water to that of air (by the oscillations of a pendulum) to be as 446 to 1. or between 446 and 478 to 1; or at a mean as 460 to 1. See Schol. Prop. XXXI. Book II. Principia.

LEMMA.

If the quadrant EDA revolve about the radius CA, and describe an hemisphere; and from all the points of its surface, as D, d, perpendiculars DB, db, be let fall upon the base EC. I say the sum of all the perpendiculars BD, in the surface EDA, is to the sum of as many radii CD; as 1 to 2. 147.

For take Dd infinitely small, and complete the square $CAHN$, and draw CH, also draw $DFGR$, $dfgr$ parallel to EC. By the similar triangles CDF, Dnd; $DF \times Dd = CD \times nd$; also $BD = CF = FG$.

The surface of the spherical annulus $DdfF$ is $3.1416 \times 2DF \times Dd$ or $3.1416 \times 2CD \times nd$, that is (because $3.1416 \times 2CD$ is given) as nd or Ff. And the sum of all the BD's in the annulus is as $BD \times$ by its surface, that is as $BD \times Ff$, or $FC \times Ff$. Therefore the sum of all the BD's in the annulus is expressed by the area $FfgG$. And for the same reason, the sum of as many radii, by $FfrR$. Therefore the sum of all the BD's in the hemisphere : is to the sum of as many radii :: as the sum of all the $FfgG$: to the sum of all the $FfrR$ :: that is as the triangle CAH : to the square $CAHN$, or as 1 to 2.

PROP. CVIII.

If a cylinder moves uniformly forward, in direction of its axis, in a fluid of the same density; it meets with a resistance equal to the 148.

the

FIG. *the force which can generate its motion, in the time it describes twice its length.*

148. Let *AB* be the cylinder moving from *A* towards *G*, and take *FBCG* equal to *ASBF*. And let us first suppose that the cylinder *AB*, whilst it moves forward, pushes against the several parts of the fluid, and drives them successively before it, in direction of its axis, from the several places through which it passes. So that in equal times it moves equal quantities of the fluid, and communicates to them the same velocity that it moves with. It is evident that the cylinder, after it has moved uniformly forward, the length of its axis has removed the cylinder of the fluid *FBCG* equal to itself *ASBF*, and has communicated a motion to it equal to its own. And since action and re-action are equal, the force that uniformly generated this motion, is equal to the uniform resistance the cylinder suffered in the mean time. And therefore the resistance is equal to the force by which its own motion can be generated, in the time it describes its length.

All this is true, upon supposition that every particle of the fluid is driven directly forward, with the same velocity the cylinder has. But since, in reality, the motion generated in the
147. fluid is not directly forward, but (by Prop. LXXX.) diverges
148. on all sides, and in all manner of directions *CD*, *Cd*, &c. Therefore if the quadrant *AE* be divided into an infinite number of equal parts, *Dd*, and to all the points *D*, *d*, the radii *CD*, *Cd*, &c. be drawn, representing the motions of the particles in all directions; and from any one *D*, the perp. *DB* be drawn on *EC*. Then the motion *CD* ($=CA$) is resolved into the two motions *CB*, *BD*; of which *CB* does not affect the cylinder; and the direct motion of the particle *D* is only *BD*, which is less than *CD*. Therefore the force to generate this motion, and consequently the resistance of a particle at *D* (equal to this force) must be less than before in proportion of *CD* to *BD*. Therefore the former resistance, when all the particles are driven directly forward; to the resistance when they diverge on all sides; is as the sum of all the radii *CD*, drawn to every point of the surface of a sphere, to the sum of all the corresponding sines *BD*; that is (by the Lem.) as 2 to 1. Therefore the resistance the cylinder meets with now, is but half the former resistance. Consequently, since the force to generate any motion is reciprocally as the time; the resistance will be equal to the force that can generate its motion, in the time that it describes twice its length.

8 *Cor.*

Cor. 1. *If a cylinder moves in direction of its axis, in a fluid of the same density, and with the velocity acquired by falling in vacuo, from a height equal to its length: it meets with a resistance equal to its weight.* FIG. 148.

For the force that generates its motion, in the time of its moving twice its length (or of falling through once its length), is its gravity.

Cor. 2. *If a cylinder moves uniformly forward in any fluid; its resistance is to the force by which its whole motion may be generated, in the time of moving twice its length; as the density of the fluid, to the density of the cylinder.*

For if the density of the fluid be increased in any ratio; the resistance will be increased in the same ratio.

Cor. 3. *The resistance of a cylinder moving in any fluid, is equal to the weight of a cylinder of that fluid, of the same base, and its length equal to the height a body falls in vacuo, to acquire its velocity.* By Cor. 1.

Cor. 4. *Let* $s = 16\frac{1}{12}$ *feet,* B = *base of the cylinder,* v = *its velocity, or the space described in* 1 *second. Then its resistance is* = *weight of the cylinder* $\frac{vv}{4s}B$, *of the fluid.*

SCHOL.

If the cylinder move in a fluid inclosed in a vessel; instead of the absolute velocity, the relative velocity in the fluid must be taken, in order to find the resistance. And besides, if the vessel be narrow, the resistance will be increased more or less, because the fluid, being confined by the vessel, cannot then diverge in all directions. And if it be so confined, that it cannot diverge at all, but is obliged to move directly forward; the resistance then will be double; which is the greatest it can possibly have, or the utmost limit of its resistance. Also by comparing the last Cor. with Cor. 1. Prop. CVII. it appears that the force of a cylinder of water against a plane, is double the resistance an equal cylinder would meet with, moving in water with the same velocity. And this will not appear strange, when we consider, that in the first case the whole motion of the water is destroyed by the resistance of the plane; but in the latter case, the water diverges every way from the moving cy-

 linder,

FIG. 118. linder, and does not partake of its direct motion. But if the water was not suffered to diverge, but was driven directly forward with the motion of the cylinder; the resistance would then be doubled; and these two cases would become the same.

LEMMA.

147. *If the quadrant ADE revolve about the radius CA, and generate an hemisphere; and on every point B of the base, perpendiculars BD be drawn. I say the sum of all the BD^2 on the base, is to the sum of as many CD^2, as 1 to 2.*

Let $CD=r$, $CB=x$, $BD=y$, $c=3.1416$; then $2cx=$ circumference of BC. Then,

The sum of all the rr : to the sum of all the yy, in the annulus Bb, is as $2cx \times rr$: to $2cx \times yy$ :: $rrx : yyx$.

And sum of all the rr : sum of all the yy, in the hemisphere, is as sum $rrx \times Bb$: sum $yyx \times Bb$, on the base,

Or as sum of $rrx \times Bb$: sum of $\overline{rr-xx} \times x \times Bb$;

Or as the sum of $rrx \times Bb$: sum $rrx \times Bb -$ sum $x^3 \times Bb$, in the base.

† But the sum of all the $x \times Bb = \overline{1+2+3+4, \&c. \text{ to } r \times 1} = \frac{1}{2}rr$.

† And sum of all the $rrx \times Bb = \frac{1}{2}r^4$. putting $Bb=1$.

† Also the sum of all the $x^3 \times Bb = \overline{1^3+2^3+3^3+4^3, \&c. \text{ to } r^3 \times 1} = \frac{1}{4}r^4$.

Therefore the sum of all the rr : sum of all the yy, in the hemisphere; is as $\frac{1}{2}r^4 : \frac{1}{2}r^4 - \frac{1}{4}r^4$, or as $\frac{1}{2}$ to $\frac{1}{4}$, that is as 2 to 1.

PROP. CIX.

If a globe move uniformly forward in a compressed infinite fluid; its resistance is to the force by which its whole motion may be destroyed or generated, in the time of describing $\frac{8}{3}$ parts of its diameter; as the density of the fluid, to the density of the globe, very nearly.

149. Let the globe move in the direction CA. Draw the tangent DH, and BDG parallel to CA, and GH perp. to DH; and let GD

† See *Ward's* Math. Guide, Part V.

GD be the force of a particle of the fluid against the base *B*, in direction *GD*: then *GH* will be the force acting against *D*, in direction *DC*. And this force is to the force in direction *GD* as *DC* to *DB*. Whence the force against *B*, is to the force against *D*, in direction *GD*; in a ratio compounded of *GD* to *GH*, and *DC* to *DB*; that is as DC^2 to DB^2. Therefore the force of all the particles of the fluid against the base, is to their force against the convex surface; as the sum of all the DC^2, to the sum of all the DB^2 on the base; that is (by the Lem.) as 2 to 1. Therefore the resistance of the surface of the sphere, is but half the resistance of the base, or of a cylinder of the same diameter. FIG. 149.

Now the globe is to the circumscribing cylinder as 2 to 3; and half of that force (which can destroy all the motion of this cylinder, whilst it describes 2 diameters) will destroy all its motion, whilst it describes 4 diameters. And therefore the same force that destroys the cylinder's motion, in the time of moving 4 diameters, will destroy the globe's motion whilst it moves $\frac{2}{3}$ of this length, or $\frac{8}{3}$ of its own diameter. But (by Cor. 2. Prop. CVIII.) half the resistance of the cylinder, that is the resistance of the globe, is to this force; as the density of the fluid, to the density of the cylinder or globe.

Cor. 1. *The resistance of a sphere is but half the resistance of a cylinder, of the same diameter.*

Cor. 2. *The resistance of a globe moving in any fluid, is equal to the weight of a cylinder of that fluid, of the same diameter; and its length equal to half the height, through which a body falls in vacuo, to acquire the velocity of the globe.* By Cor. 3. Prop. CVIII.

Therefore if $s = 16\frac{1}{12}$ *feet,* v = *velocity of a globe, or the space it moves in* 1 *second,* D = *its diameter: then its resistance is equal to the weight of a cylinder of the fluid, of the same diameter* D, *and its length* $\frac{vv}{8s}$. *And if* $v = 4\sqrt{\frac{Ds}{3}}$, *its resistance is equal to the weight of an equal globe of the fluid.*

Cor. 3. *The greatest velocity a globe can obtain, by descending in a fluid; is that which it would acquire by falling in vacuo, through a space that is to* $\frac{4}{3}$ *the diameter; as the difference between the density of the globe and the density of the fluid, is to the density of the fluid.*

For let *G*, *F* be the densities of the globe and the fluid; *D* the diameter of the globe. Then since a globe is equal to a

FIG. 149. cylinder whose height is $\frac{2}{3}$ D. Therefore the weight of the globe $=$ weight of a cylinder of the fluid, whose length is $\frac{2}{3}$ $D \times \frac{G}{F}$. And (by Prop. LXXXV.) the weight of the globe in the fluid is $=$ weight of a cylinder of the fluid, whose length is $\frac{2}{3} D \times \frac{G-F}{F}$. But (by Cor. 2.) the resistance of the globe moving with the velocity acquired by falling in vacuo, through the height $\frac{4}{3} D \times \frac{G-F}{F}$ is $=$ weight of a cylinder of the fluid whose length is $\frac{2}{3} D \times \frac{G-F}{F}$. Therefore the weight of the globe in the fluid is equal to the resistance: and consequently it cannot accelerate the globe.

And hence if $v = 4\sqrt{\frac{G-F}{3F} Ds}$, *the resistance is equal to the weight of the globe in the fluid.*

Cor. 4. *Two equal and homogeneous globes moving in a resisting medium; will, in times that are reciprocally as the first velocities, describe equal spaces; and lose a given part of their motions.*

For the motion lost, in describing two very small equal spaces, is as the resistance and time; that is (because the space is given) as the square of the velocity directly and the velocity inversely; that is directly as the velocity. And so in describing any spaces, the motion lost will always be as the first motion; and the time reciprocally as the first velocity.

Cor. 5. *Two homogeneous globes, moving with equal velocities in a fluid; lose equal velocities in describing spaces proportional to their diameters.*

For the velocity lost in each, by describing two small spaces proportional to the diameters; will be as the resistance and time directly, and the body inversely; that is (because the resistance is as the square of the diameter, and the time as the diameter), as the cube of the diameter directly, and the cube of the diameter inversely: therefore the velocity lost is equal in both. And the like for any succeeding correspondent parts.

SCHOL.

The resistance of fluids is of three kinds. 1. *Tenacity* or *cohesion* or the parts of the liquor, which is the force by which the particles of the fluid stick together, and causes them not to separate easily; and this is the same for all velocities. 2. *Friction*

tion or *attrition*, where the parts of the fluid do not slide freely by one another; and this is as the velocity. 3. The *density* or *quantity of matter* to be removed; and this is as the square of the velocity. The two former kinds are very small in all fluids, except viscid and glutinous ones; and upon this account the foregoing theory regards only the last kind. And therefore the resistance there described is the very least the body can possibly meet with. But since all fluids have some small degree of friction and tenacity, they will increase the resistance a little. Also when the velocity is very great; the compression of the fluid ought to be so too, to cause the fluid to return with equal ease behind the moving body; and when this does not happen, the resistance is increased upon that account. For a fluid yielding to a projectile, does not recede ad infinitum; but with a circular motion comes round to the places which the body leaves. Likewise when bodies move in a stagnant fluid near the surface; the fluid cannot dilate itself upwards, to give way to the moving body; and this will considerably increase the resistance. Also if a body moves in a fluid inclosed in a vessel; the relative velocity of the body in the fluid must be esteemed its true velocity. But the resistance it meets with will be increased, because the fluid has not liberty to diverge every way. And the straiter the vessel, the more is the resistance increased; and it may by this means be increased till it be near double; beyond which it cannot go. For all that a body can do is to drive the fluid wholly before it, without any diverging. So that the least resistance a globe can have is the same as is laid down in Cor. 2. of the last Prop and the greatest can never exceed the double of it; so that it will always be between these limits. If the fluid in which the body moves be elastic and spring from the body; the resistance will be greater than if it was non-elastic. But these irregularities are not considered in the foregoing theory.

There are some bodies that may be reckoned in a middle state between solids and fluids. And in some of these the tenacity and friction is so great, as in many cases far to exceed the resistance arising from their density only. For example, it appears by experiments, that if a hard body be suspended at several heights, and be let fall upon any soft substance, such as tallow, soft clay, wax, snow, &c. it will make pits or impressions, which are as the heights fallen, that is as the squares of the velocities. Likewise nails give way to a hammer in a ratio which is as the square of the velocity. Comparing this with Schol. Prop. XIV. It appears that in these cases, the resistance

FIG. 149. refiftance is the fame for all velocities: which argues a very great degree of tenacity. Again, bodies projected into earth mixt with ftones; the impreffions are found to be between the fimple and duplicate ratio of the velocities. Therefore in this cafe, the refiftance is in a lefs ratio than the fimple ratio of the velocity: and therefore thefe fort of bod es have both friction and tenacity. And in different forts of bodies, there is great difference and variety in their nature and conftitution.

Tenacity may be compared to the force of gravity, which is always the fame; with this difference, that tenacity acts always contrary to the motion of the body, and when the body is at reft, it is nothing. *Attrition* may be compared to the motion of a body ftriking always a given number of particles of matter in a given time, with any velocity: and therefore the refiftance of fuch a body will be as the velocity.

SECT.

SECT. XI.

Methods of communicating, directing, and regulating any motion in the practice of mechanics.

PROP. CX.

To communicate motion from one body to another, or from one place to another.

1. The easiest and simplest method of communicating motion from one thing *A* to another *B*, is by a rope or a leaver *AB*, reaching between the two places, or things. 150.

2. Motion is communicated from one wheel or roller *DC* to another *AB*, by a perpetual or endless rope *ABCD*, going once or oftner about them: or if you will, by a chain. That the rope slip not, make knots on it, and channels in the wheels, if necessary. 151.

3. Motion is communicated from one wheel *ABC*, to another *DEF*; by the teeth in the two wheels working together. 152.
Or thus, where the axis of *A* having but one tooth; one revolution of it answers to the motion of only one tooth in *B*. 153.

4. Motion is communicated from one place to another, by one or more beams or leavers, *MB*, *BC*, *CE*, *EF*, *FH*, &c. moveable about the centers *A*, *B*, *C*, *D*, *E*, *F*, *G*; of which *A*, *D*, *G*, &c. are fixed. Here if the point *M* be moved, the point *H* will be moved; for *MB*, *BC*, *CE*, &c. all move one another to the last, *FH*. 154.

5. Motion may also be communicated from *A* to *B*, by a pinion at *A*, and a streight ruler with teeth, which bite one another. 155.

FIG.

PROP. CXI.

By help of one uniform motion given; to produce another, either uniform or accelerated.

152. 1. A uniform motion is produced in the wheel *DEF*; by moving the wheel *ABC* uniformly, which carries it. Also a uni-
151. form motion is produced in wheels moving by cords, as *AB*, *CD*: for one being moved uniformly, moves the other also uniformly.

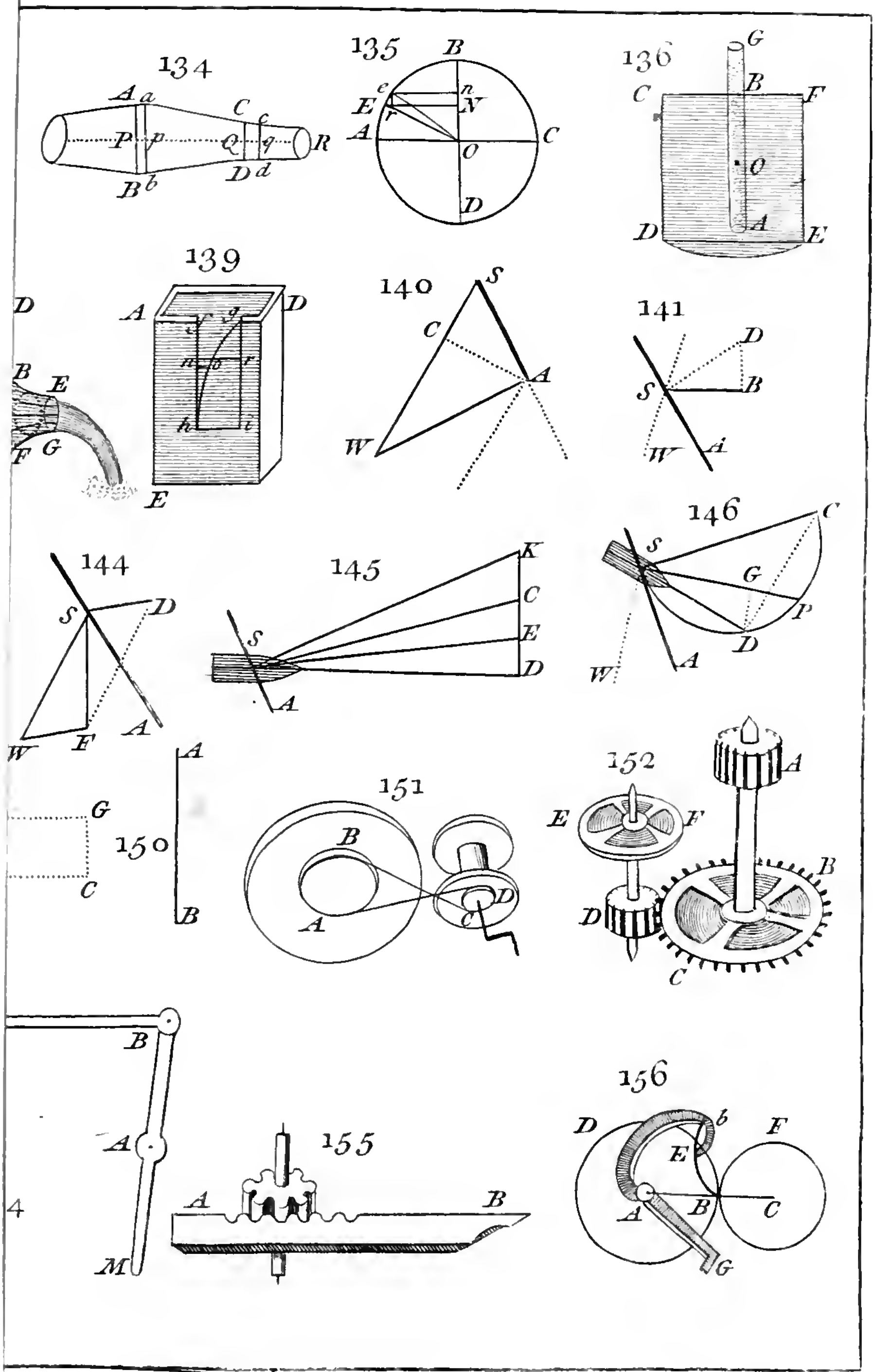
156. 2. The wheel *BF* may be made to move uniformly about the center *C*, by the motion of the wheel *BD*. On the base *BF* with the generating circle *BD*, describe the epicycloidal tooth *BE*. Then the point *B* of the wheel *AB*, moving uniformly about the center *A*, and passing over the tooth *BE*; will move the wheel *BF* uniformly about *C*. Here the acting tooth *AB* ought to be made crooked as *Ab*, that it touch not the end *E*, of the tooth *BE*, if it act on the concave side. Or else the plane of the wheel *BD* must be raised above the plane of *BF*, and a tooth made at *B* to bend down perpendicular to the plane of the wheel, as *AG*, to catch the tooth *BE*.

157. 3 The leaver *AB* may be made to move up and down with either a uniform or accelerated motion, after this manner. Let *AE* be a wheel whose axis is parallel to the leaver, and directly above it. Take any arch *N*4, and divide it into any number of equal parts at 1, 2, 3, &c. through which from the center *O*, draw *Oa*, *Ob*, *Oc*, *Od*; and make 1*a*, 2*b*, 3*c*, 4*d*, &c. respectively equal to 1, 2, 3, 4 equal parts. And through the points *N*, *a*, *b*, &c. draw the curve *Nabcd*. Then the part *NdF* being made of solid wood, and fixed to the wheel; and the wheel being turned uniformly about, in the order *ENA*; the part *NF* will give a uniform motion to the leaver *AB*, about the center of motion *C*. And you may fix as many of these teeth to the wheel as you will.

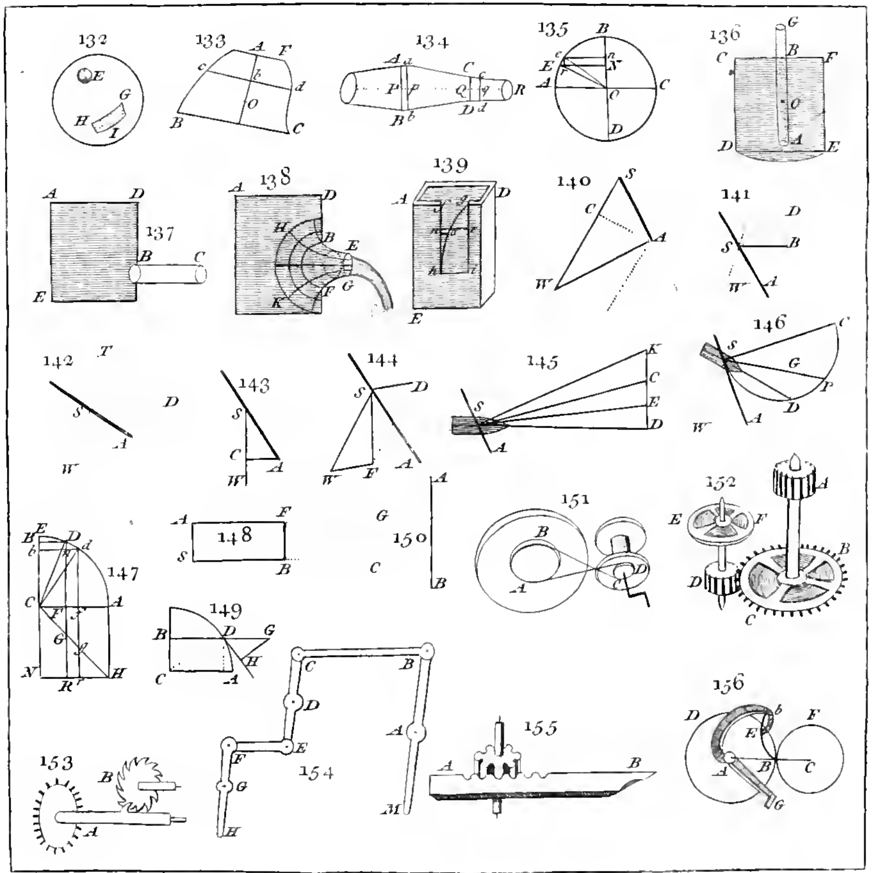
Again, in the tooth *AD*, if *A*1, 12, 23, &c. be taken equal, and 1*a*, 2*b*, 3*c*, 4*d*, &c. be taken equal to 1, 4, 9, 16, &c. equal parts; and the curve *Aabcd* be drawn, and the tooth formed. Then the leaver will be moved with a uniformly accelerated motion.

The accelerated motion is proper for lifting a given weight at the end *B*, as a hammer; or for working a pump, by a chain going over the end *B*.

4. The



Pl.VII. pa. 162.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Pl.VII pa.162.

4. The leaver *AB*, may alſo be moved thus, by help of a FIG.
machine *GFD*, moving uniformly along *GD*. Make *IH*, *IF* 158.
right lines; and make as many ſuch teeth as you will; and theſe will give a uniform motion to the leaver.

Make the curves *EFE* all parabolas, equal and equi-diſtant; whoſe vertices are at *F*; and their baſes meet at *E*, and theſe will make the leaver riſe and fall with an accelerated motion. Such parabolic teeth as theſe may be placed on a wheel, whoſe axis is perpendicular to the horizon.

5. One wheel may move another with an accelerative motion 159.
thus. On the circle or wheel *EF*, take *Ea*, *ab*, *bc*, &c. equal to each other. And on the edge of the wheel *BD* take *B*1, a very ſmall part; and 13, 35, 57, &c. 3, 5, 7, &c. times *B*1, ſuppoſe the plane of the wheel *EF* to be extended as far as the marks 1, 3, 5, 7, &c. then turn the wheel *EF*, till *E* fall on *a*; then mark the point 1 on the plane of the wheel *EF*. Then turn *EF* till *E* comes to *b*; and mark the point 3 on the plane of the wheel *EF*. Likewiſe let *E* come to *c*, *d*, &c. and mark the points 5, 7, &c. on the plane of the wheel *EF*; then *E* 1 3 5 7 *r* is the figure of the tooth of the wheel *EF*, which being uniformly moved, will move *DB* with an accelerative motion.

PROP. CXII.

To change the direction of any motion.

1. The direction of any motion may be changed, by the leaver 160.
of the firſt kind, for the two ends have oppoſite motions. Likewiſe a bended leaver will change the direction to any other direction.

2. The direction of motion may be changed by the help 161.
of pulleys, with a rope going over them. Thus the direction *AB* is changed ſucceſſively into the directions *BC*, *CD*, *DE*, *EF*, *FG*.

3. The direction may be changed by wheels, whoſe axles are 162.
perpendicular to one another. Thus the direction *AB* is changed into the direction *EF*; by the wheel *C*, working in the crown wheel *D*.

4. The direction may be changed, by making the lanthorn *B*, 163.
inclined in any given angle, to be moved by the coggs of the

FIG. wheel *A*. Here the rungs at *F*, where they work, must be paral-
163. lel to the plane of the wheel *A*, or perpendicular to the coggs.
164. The same thing may be done by wheels with teeth, as *C*, *D*.
In both cases the axles of the two wheels must be in one plane.

PROP. CXIII.

To regulate any motion, or to make it uniform.

165. 1. Any motion is made uniform, by the help of a pendulum *AB*, suspended at *A* and vibrating. As the pendulum vibrates, it causes *CDE* to vibrate also, about the axis *DE*. The weight *I* carries the wheel *R*, and *R* moves *LF*. Now whilst the pendulum vibrates towards *M*, a tooth of the wheel *GF* goes off the pallat *I*, and another catches the pallat *H*; and when the pendulum returns towards *N*, it draws the pallat *H* off the tooth, and another catches the pallat *I*; and so on alternately. So that at every vibration of the pendulum, a tooth goes off one or other of the palats.

166. 2. A uniform motion is effected by the pendulum *CP*, vibrating in the arch *NM* about the center of motion *C*. As the pendulum vibrates, it causes the piece *ADE* to vibrate along with it about the axis of motion *DE*. By this motion the leaf *a* catches hold of a tooth of the horizontal wheel *GF*, in its going; and the leaf *b* of another tooth, in returning. A wheel with a weight is applied to the pinion *L*, to keep the pendulum going.

167. 3. A pendulum may also be applied thus for the same purpose. *FG* is a thick wheel, or rather a double wheel, whose axis is parallel to the horizon. *nP* a pendulum vibrating upon the axis *DE*, which is parallel to the planes of the wheel *FG*; *anb* two wings perpendicular to *DE*, and to *nP*; 1, 1, 1 pins in the rim *G*; and 2, 2, 2 pins in the rim *F*. These pins are in the planes of the wheel; but not perpendicular to the circumference, but inclined in an angle of about 45 degrees, and the pins in one end are against the spaces in the other; *ab* is parallel to the axis of the wheel *FG*, but neither in the same horizontal or perpendicular plane; but almost the radius of the wheel below, and something more forward. Whilst the pendulum *P* vibrates in the arch *MN*, about the axis *DE*, the wing *a* catches hold of a tooth in the end *F*; and when it returns, the wing *b* catches hold of a tooth in the end *G*. Thus the pins act-
ing

ing alternately againſt the wings *a*, *b*, keep the pendulum going, by help of the weight *W*. FIG. 167.

4. A ſteady motion is continued by applying the heavy wheel *ABC*, to the machine : or the croſs bar *DE* loaded with two equal weights at *D* and *E*. Or a cylinder of ſome heavy matter may be applied ; being made to revolve about its axis. By theſe the force of the power, which would be loſt, is kept in the wheel, and is equally diſtributed in all parts of the revolution. Such a wheel is of great uſe in ſuch machines as act with unequal force at different times, or in different parts of a revolution. For by its weight it conſtantly goes on at the ſame rate, and makes the motion uniform, and every where equal. By reaſon of its weight a little variation of force will not ſenſibly alter its motion : and its friction, and the reſiſtance of the air will hinder it from accelerating. If the machine ſlackens its motion, it will help it forward ; if it tends to move too faſt, it will keep it back. 168. 169.

Every ſuch regulating wheel ought to be fixed upon that axis, where the motion is ſwifteſt. And ought to be the heavier, the ſlower it is deſigned to move ; and the lighter, the ſwifter the motion is. And in all caſes the center of motion muſt be in the center of gravity of the wheel. And the axis may be placed parallel to the horizon, as well as perpendicular to it.

If the machine be large, and the axis of the heavy wheel be perpendicular to the horizon ; the heavy wheel may be made to roll on the ground, round that axis ; by putting the wheel upon another axis fixed in the former at right angles to it ; and thus the weight is taken off the firſt axis. And two ſuch wheels may be applied on oppoſite ſides.

5. Any ſwift motion may be moderated by a fly *AB*, moveable about the axis *CD*. This is made of thin metal ; at *s* is a ſpring to keep the axis and fly pretty ſtiff together. This bridles the rapidity of the motion of the machine, to which it is applied, by reaſon of its great reſiſtance in the air ; and therefore it hinders the motion from accelerating beyond a certain degree. This ſort of fly is uſed in clocks, and is ſo uſeful in any motion that requires to ſtop, or move a contrary way. 170.

None of theſe regulating wheels or flies add any new power to the machine ; but rather retard the motion by their friction and reſiſtance.

FIG.

PROP. CXIV.

To deſcribe ſeveral ſorts of knots.

As ropes are made uſe of in ſeveral ſorts of machines, and eſpecially aboard of ſhips; it is proper for a mechanic to know how to tye them together. Therefore I ſhall here deſcribe ſeveral ſorts of knots, not ſo much to teach how to tye them, as to ſhew the form they appear in, when they are tyed. For the method of tying them is beſt learned from thoſe that can tye them already.

171. 1. *A thumb knot.* This is the ſimpleſt of all; and is uſed to tye at the end of a rope, to hinder its opening out. Alſo it is uſed by taylors at the end of their thread.

172. 2. *A loop knot.* This is uſed to join pieces of ropes together.

173. 3. *A draw knot*, is the ſame as the laſt; only one (or both) of the ends returns the ſame way back, as *a b c d e*. By pulling at *a* the part *bcd* comes through, and the knot is looſed.

174. 4. *A ring knot.* This ſerves alſo to join pieces of rope together.

175. 5. *Another knot* for tying ropes together. This is made uſe of when any rope is often to be looſed.

176. 6. *A running knot*, to draw any thing cloſe. By pulling at the end *a*, the rope is drawn through the loop *b*, and the part *cd* is drawn cloſe about a beam, *&c.*

177. 7. *Another knot*, to tye any thing to a poſt; here the end may be put through as oft as you will.

178. 8. *A very ſmall knot.* There is a thumb knot made at the end of each piece; and the end of the other is to go through it. Thus the rope *ac* runs through the loop *d*, and *bd* through *c*. And then drawn cloſe by pulling at *a* and *b*: if the ends *e*, *f* be drawn, the knot will be looſed again.

179. 9. *A fiſher's knot*, or *water knot*. This is the ſame as the 4th, only the ends are to be put twice through the ring, which in that was but once; and then drawn cloſe.

180. 10. *A maſhing knot* for nets; and is to be drawn cloſe.

181. 11. *A barber's knot*, or a knot for cawls of wigs. This muſt be drawn cloſe.

194. 12. *A bowline knot.* When this is drawn cloſe, it makes a loop that will not ſlip, as fig. *R*. This ſerves to hitch over any thing.

193. 23. *A wale knot* is made with the three ſtrands of a rope, ſo that it cannot ſlip. When the rope is put through a

hole, this knot keeps it from ſlipping through, 'tis repreſented at *S*, fig. 193. If the three ſtrands are wrought round once or twice more, after the ſame manner, it is called *crowning*. By this means the knot is made bigger and ſtronger. A thumb knot art. 1, may be applied to the ſame uſe as this. FIG. 193.

Concerning the ſtrength of ropes, ſee the latter end of Sect. VIII.

SECT. XII.

The powers and properties of compound engines; of forces acting within the machine; of friction.

PROP. CXV.

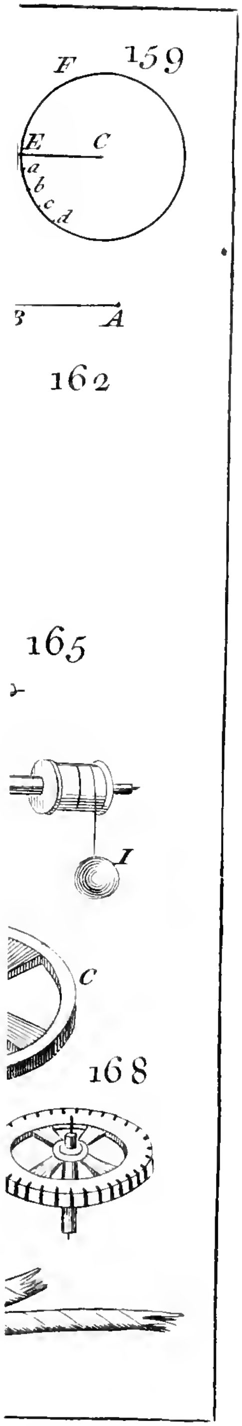
In any compounded machine, if the power and weight keep the machine, and all its parts, in equilibrio. Then the power it to the weight, in the compound ratio of the power to the weight in every ſimple machine, of which the whole is compoſed.

For let the compound machine be divided into all its ſimple mechanic powers; and in the firſt let the power be to the weight as *A* to *B* Then conſidering the weight *B* in the firſt, as the power in the ſecond, to which it is equal (by Ax. 3.) let that power in the ſecond machine be to the weight as *B* to *C*. Then ex equo, the firſt power *A* is to the ſecond weight *C*; in the compound ratio of *A* to *B* and *B* to *C*. In like manner, if the weight in the ſecond be taken for the power in the third, and this power be to the weight as *C* to *D*; then the firſt power *A* is to the laſt weight *D*, in the compound ratio of *A* to *B*, *B* to *C*, and *C* to *D*; and ſo on thro' the whole.

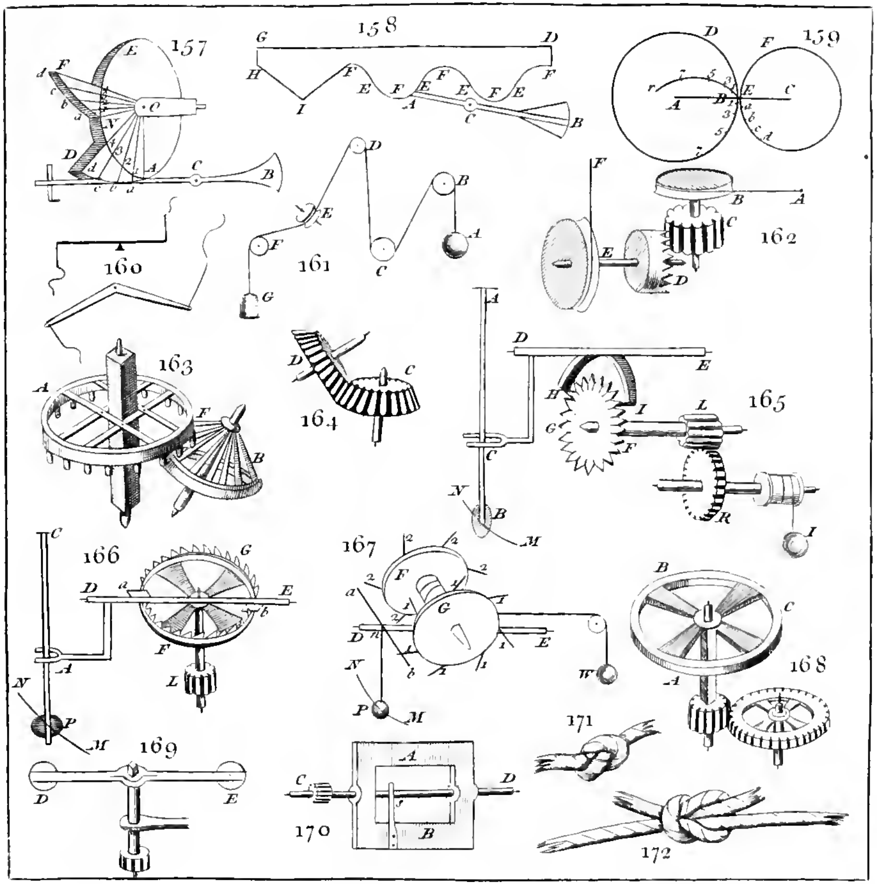
Cor. In any machine compoſed of wheels; the power is to the weight; in the compound ratio of the diameter of the axel where the weight is applied, to the diameter of that where power is applied; and the number of teeth in the pinion of each axis, beginning at the power, to the number of teeth in each wheel they work in, till you come at the weight.

Or, inſtead of the teeth, you may take their diameters.

PROP.



Pl.VIII. pa.168.



Pl.VIII. pa. 168.

FIG.

PROP. CXVI.

If the power and weight be in equilibrio on any machine; if they be put in motion, the velocity of the weight will be to that of the power; as the power is to the weight.

For ſince they are in equilibrio; one of them cannot of itſelf move the other; therefore if they be put into motion, the momentum or quantity of motion of the weight, will be equal to that of the power: and therefore their velocities will be reciprocally as their quantities.

Cor. 1. *Hence it follows, that if any weight is moved by help of a machine; what is gained in power is loſt in time.*

For in whatever proportion the power is leſs than the weight; in the ſame proportion will the weight be ſlower than the power.

Cor. 2. *Hence the motion of the weight is not at all increaſed by any engine, or mechanical inſtrument; only the velocity of the weight is ſo much diminiſhed thereby; that the quantity of motion of the weight, may not exceed the quantity of motion of the power. And therefore it is a vain fancy for any one to think that he can move a great weight with a little power, and with the ſame velocity as with a greater power.*

For the advantage gained by the power is loſt by the velocity. If any power is able to raiſe a pound with a given velocity; it is impoſſible by the help of any machine whatever, that the ſame power can raiſe two pounds with the ſame velocity. Yet it may, by help of a machine, be made to raiſe 2 pounds with half that velocity; or even 1000 pounds with the thouſand part of the velocity. But ſtill there is no greater quantity of motion produced, when 1000 pounds weight is moved, than when 1 pound: the 1000 pounds being proportionally ſlower. The power and uſe of machines conſiſts only in this, that by their means the velocity of the weight may be diminiſhed at pleaſure, ſo that a given weight may be moved with a given power; or that with a given force any given reſiſtance may be overcome. Mechanic inſtruments being only the means whereby one body communicates motion to another: and not deſigned to produce a motion that had no exiſtence before.

FIG.

Cor. 3. Hence also it is plain, that a given power or quantity of force, applied to move a heavy body by help of a machine, can produce no greater quantity of motion in that body, than if that force was immediately applied to the body itself. Nor not quite so much, by reason of the friction and resistance of the engine. And if the power be given, you may chuse whether you will move a greater weight with a less velocity, or a less weight with a greater velocity. But to do both, is utterly repugnant to the eternal laws of nature.

PROP. CXVII.

182. *If any machine CD, is to be moved by the help of leavers, wheels, &c. And if the power that moves it acts intirely within the machine, and exerts its force against some external object B. Then the force applied within to move the machine, will be just the same as if the machine was at rest, and the object B was to be moved: supposing B to be as easily moved as the machine.*

For suppose first, the leaver *AFB* to be fixt, and to make a part of the machine; and let the external force acting at *B*, which is capable to move the machine, be 1. Now suppose the leaver *AFB*, moveable about *F*; and a force applied at *A*, so great, as to act at *B*, with the force 1. Then the action and re-action at *B* being the same as before, it is plain the machine will be moved as before. But the force now acting at *A*, is $\frac{BF}{AF} \times 1$; just the same as if the point *F* were fixt, and *B* was to be moved. And if more leavers, or any number of wheels be added, the thing will still be the same.

Otherwise.

Let the absolute force to move the machine be 1, and the force acting at *A* be f; and let us first consider it as acting out of the machine. Then *B* being fixt, is the fulcrum; therefore the force acting at *F*, is $\frac{AB}{FB} \times f$. Now if the acting force be considered in the machine, it will not be urged forward with all this force, for the re action will be equal to f, the power at

A.

A. Therefore the abfolute force the machine is moved with, FIG. 182.
is $\frac{AB}{FB}\times f-f$ or $\frac{AB-FB}{FB}\times f$, that is $\frac{AF}{FB}\times f$, but this is $=$ 1, therefore $f=\frac{FB}{AF}\times 1$.

Cor. 1. *Hence if the abfolute external force, to move any body or machine, be given; and the machine is to be moved by an internal power: that power may be found, by fuppofing the machine at reft, and the external object B was to be removed, and to require the fame abfolute force to move it.*

For it is the fame thing, as to the power, whether the machine, or the external object be moved, whilft the other is at reft.

Cor. 2. *If the power acting within the machine be not communicated to fome external object, it will have no force at all to move the machine. And any force that both begins and ends within it, does nothing at all to move it.*

For the power acting only againft fome part of the machine, will be deftroyed by the contrary and equal re-action. And the body being acted on by thefe equal and contrary forces, will not be moved at all. Thus if a man, fitting in the head of a boat, pull the ftern towards him by a rope; the boat will not be moved at all out of its place, by that force.

PROP. CXVIII.

To determine the friction, and other irregularities in mechanical engines.

The propofitions hitherto laid down, fuppofe all bodies perfectly fmooth, that they flide over one another without any friction, and move freely without any refiftance. But fince there is no fuch thing as perfect fmoothnefs in bodies; therefore in rubbing againft one another they meet with more or lefs friction, according to their roughnefs; and in moving in any medium, will be refifted according to the denfity of the medium. Even ropes going over pullies cannot be bended without fome force.

Among machines, fome have a great deal more friction than others, and fome very little. Thus a pendulum has little or no friction, but what arifes from the refiftance of the air. But

5 1 G. a carriage has a great deal of friction. For upon plain ground a loaded cart requires the strength of several horses to draw it along; and all or most of this force is owing to its friction. All compounded machines have a great deal of friction, and so much the more, as they consist of more parts that rub against one another. And there is great variety in several sorts of bodies, as to the quantity of friction they have, and even in the same bodies under different circumstances: upon which account it will be impossible to give any standing rules, by which its quantity can be exactly determined. All we can do is to lay down such particular rules, as have been deduced from experiments made upon particular bodies; which rules will require some variation under different circumstances; according to the judgment and experience of the artist.

1. Wood and all metals when oiled or greased have nearly the same friction. And the smoother they are, the less friction they have. Yet metals may be so far polished as to increase friction, by the cohesion of their parts.

Wood slides easier upon the ground in wet weather than in dry; and easier than iron in dry weather. But iron slides easier than wood in wet weather. Lead makes a great deal of resistance. Iron or steel running in brass, makes the least friction of any. In wood acting against wood, grease makes the motion twice as easy, or rathar $\frac{2}{3}$ easier. Wheel naves greased or tarr'd, go 4 times easier than when wet.

Metals oiled make the friction less than when polished, and twice as little as when unpolished.

In general, the softer or rougher the bodies, the greater is their friction.

2. As to particular cases: a cubic piece of soft wood of eight pound weight, moving upon a smooth plane of soft wood, at the rate of three feet per second; its friction is about $\frac{1}{3}$ the weight of it. But if it be rough, the friction is little less than half the weight.

Upon the same supposition, other soft wood upon soft wood very smooth; the friction is about $\frac{1}{4}$ the weight.

Soft wood upon hard, or hard upon soft, $\frac{1}{5}$ or $\frac{1}{6}$ the weight.

Hard wood upon hard wood, $\frac{1}{7}$ or $\frac{1}{8}$ the weight.

Polished steel moving on steel or pewter, $\frac{1}{4}$ the weight: moving on copper or lead, $\frac{1}{5}$ the weight: on brass $\frac{1}{6}$ the weight. Metals of the same sort have more friction than different sorts.

The friction, *ceteris paribus*, increases with the weight, almost in the same proportion. The friction is also greater with a greater velocity, but not in proportion to it, except in very

few

few cases. A greater surface also causes something more friction, with the same weight and velocity. Yet friction may sometimes be increased, by having too little surface to move on: as upon clay, &c. where the body sinks. FIG.

3. The friction arising from the bending of ropes about machines, differs according to their stiffness, the temper of the weather, degree of flexibility, &c. but, *ceteris paribus*, the force or difficulty of bending a rope, is as the square of the diameter of the rope, and its tension, directly; and the diameter of the cylinder, or pulley, it goes about, reciprocally.

A rope of 1 inch diameter, whose tension, or weight drawing it, is 5lb. going over a pulley 3 inches diameter; requires a force of 1lb. to bend it.

4. The resistance of a plane moving through a fluid, is as the square of the velocity: and (putting v=velocity in feet, in a second) it is equal to the weight of a column of the fluid, whose base is the plane, and height $\frac{vv}{64}$. And in a globe it is but half so much.

5. The friction of a fluid running through a tube is as the velocity and diameter of the tube.

But the friction is greater in respect to the quantity of the fluid, in small tubes, than in large ones; and that reciprocally as their diameters. But the absolute quantity of the friction in tubes, is but very small, except the velocity be very great, and the tube very long.

But if a pipe be divided into several lesser ones, whose number is n; the resistance arising from the friction will be increased as $\surd n$. For the area of the section of any one pipe, will be $\frac{1}{n}$; and the friction, being as the circumference, will be as $\frac{1}{\surd n}$; and therefore the friction in all of them, will be $\frac{n}{\surd n}$, or as $\surd n$.

6. As to the mechanic powers. The single leaver makes no resistance by friction. But if by the motion of the leaver in lifting, the fulcrum or place of support be changed further from the weight; the power will be decreased thereby.

7. In any wheel of a machine, running upon an axis; the friction on the axis is as the weight upon it, the diameter of the axis, and the angular velocity. This sort of friction is but small.

FIG. 8. In the pulley, if p, q be two weights, and q the greater; and if $W = \frac{4pq}{p+q}$, then W is the weight upon the axis of the single pulley. And it is not increased by the acceleration of the weight q, but remains always the same.

The friction of the pulley is very considerable, when the sheaves rub against the blocks; and by the wearing of the holes and axles.

The friction on the axis of the pulley is as the weight W, its angular velocity, the diameter of the axis directly, and the diameter of the pulley inversely. A power of 100lb. with the addition of 50lb. will but draw up 500lb. with a tackle of 5.

And 15lb. over a single pulley will draw up only 14lb.

9. In the screw there is a great deal of friction. Those with sharp threads have more friction than those with square threads. And endless screws have more than either. Screws with a square thread raise a weight with more ease than those with a sharp thread.

In the common screw, the friction is so great, that it will sustain the weight in any position given, when the power is taken off. And therefore the friction is at least equal to the power. From whence it will follow, that in the screw.

The power must be to the weight or resistance; at least as twice the perpendicular height of a thread, to the circumference described by one revolution of the power; if it be able to raise the weight, or only sustain it. This friction of the screw is of great use, as it serves to keep the weight in any given position.

10. In the wedge, the friction is at least equal to the power, as it retains any position it is driven into. Therefore in the wedge,

The power must be to the weight; at least as the base to the height; to overcome any resistance.

11. To find the friction of any engine, begin at the power, and consider the velocity and the weight at the first rubbing part; and estimate its quantity of friction, by some of the foregoing articles. Then proceed to the next rubbing part, and do the same for it. And so on through the whole.

And note, something more is to be allowed for increase of friction, by every new addition to the power.

Cor. Hence will appear the difficulty, or rather impossibility of a perpetual motion; or such a motion as is to continue the same for ever,

ever, or at least as long as the materials will last, that compose the moving machine. FIG.

For such a motion as this ought continually to return undiminished, notwithstanding any resistance it meets with, which is impossible. For although any body once put into motion, and moving freely without any resistance, or any external retarding force acting upon it, would for ever retain that motion. Yet in fact we are certain, that no body or machine can move at all, without some degree of friction and resistance. And therefore it must follow, that from the resistance of the medium, and the friction of the parts of the machine upon one another, its motion will gradually decay, till at last all the motion is destroyed, and the machine is at rest. Nor can this be otherwise, except some new active force, equal to all its resistance, adds a new motion to it. But that cannot be from the body or machine itself; for then the body would move itself, or be the cause of its own motion, which is absurd.

PROP. CXIX.

To contrive a proper machine that shall move a given weight with a given power; or with a given quantity of force, shall overcome any other given resistance.

If the given power is not able to overcome the given resistance, when directly applied, that is, when the power applied is less than the weight or resistance given; then the thing is to be performed by the help of a machine made with *leavers*, *wheels*, *pullies*, *screws*, &c. So adjusted, that when the weight and power are put in motion on the machine; the velocity of the power may be at least so much greater than that of the weight; as the weight and friction of the machine taken together, is greater than the power. For on this principle depends the mechanism or contrivance of mechanical engines, used to draw or raise heavy bodies, or overcome any other force. The whole design of these being to give such a velocity to the power in respect of the weight; as that the momentum of the power may exceed the momentum of the weight. For if machines are so contrived, that the velocities of the agent and resistant are reciprocally as their forces, the agent will just sustain the resistant: but with a greater degree of velocity will overcome it. So that if the

FIG. the excefs of velocity in the power is fo great, as to overcome all that refiftance which commonly arifes from the friction or attrition of contiguous bodies, as they flide by one another, or from the cohefion of bodies that are to be feparated, or from the weights of bodies to be raifed. The excefs of the force remaining, after all thefe refiftances are overcome, will produce an acceleration of motion proportional thereto, as well in the parts of the machine, as in the refifting body. Now how a machine may be contrived to perform this to the beft advantage, will appear from the following rules.

1. Having affigned the proportion of your power and the weight to be raifed: the next thing is to confider how to combine leavers, wheels, pulleys, &c. fo that working together they may be able to give a velocity to the power, which fhall be to that of the weight, fomething greater than in the proportion of the weight to the power. This done, you muft eftimate your quantity of friction, by the laft Prop. and if the velocity of the power be to that of the weight, ftill in a greater proportion, than the weight and friction taken together, is to the power; then your machine will be able to raife the weight. And note, this proportion muft be fo much greater, as you would have your engine work fafter.

2. But the proportion of the velocity of the power and weight, muft not be made too great neither. For it is a fault to give a machine too much power, as well as too little. For if the power can raife the weight, and overcome the refiftance, and the engine perform its proper effect in a convenient time, and works well, it is fufficient for the end propofed. And it is in vain to make more additions to the engine, to increafe the power any further: for that would not only be a needlefs expence, but the engine would lofe time in working.

3. As to the power applied to work the engine, it may be either a living power, as men, horfes, &c. or an artificial power, as a fpring, &c. or a natural power, as wind, water, fire, weights, &c.

When the quantity of the power is known; it matters not as to the effect, what kind of power it is. For the fame quantity of any fort will produce the fame effect. And different forts of powers, may be applied in an equal quantity, a great variety of ways.

The moft eafy power applied to a machine is weight, if it be capable of effecting the thing defigned. If not, then wind, water, &c. if that can conveniently be had, and without much expence.

A fpring

A ſpring is alſo a convenient moving power for ſeveral machines; but it never acts equally as a weight does; but is ſtronger when much bent, than when but a little bent, and that in proportion to the degree of bending, or the diſtance it is forced to. But ſprings grow weaker by often bending, or remaining long bent; yet they recover part of their ſtrength by lying unbent. FIG.

The natural powers, wind and water, may be applied with vaſt advantage to the working of great engines, when managed with ſkill and judgment. The due application of theſe has much abridged the labours of men; for there is ſcarce any labour to be performed, but an ingenious artificer can tell how to apply theſe powers to execute his deſign, and anſwer his purpoſe. For any conſtant motion being given, it may by a due application be made to produce any other motions we deſire. Therefore theſe powers are the moſt eaſy and uſeful, and of the greateſt benefit to mankind. Beſides, they coſt nothing, nor require any repetition or renewing, like a weight or a ſpring, which require to be wound up. When theſe cannot be had, or cannot ſerve our end, we have recourſe to ſome living power, as men, horſes, &c.

4. Men may apply their ſtrength ſeveral ways, in working a machine. A man of ordinary ſtrength turning a roller by the handle, can act for a whole day againſt a reſiſtance equal to 30lb. weight; and if he works 10 hours in a day, he will raiſe a weight of 30 lb. $3\frac{1}{2}$ feet in a ſecond; or if the weight be greater, he will raiſe it ſo much leſs in proportion. But a man may act, for a ſmall time, againſt a reſiſtance of 50 lb. or more.

If two men work at a windleſs, or roller, they can more eaſily draw up 70lb. than one man can 30 lb. provided the elbow of one of the handles be at right angles to that of the other. And with a fly, or heavy wheel applied to it a man may do $\frac{1}{3}$ part more work; and for a little while act with a force, or overcome a continual reſiſtance of 80 lb. and work a whole day when the reſiſtance is but 40lb.

Men uſed to carrying, ſuch as porters, will carry, ſome 150lb. others 200 or 250lb. according to their ſtrength.

A man can draw about 70 or 80lb. horizontally; for he can but apply about half his weight.

If the weight of a man be 140lb. he can act with no greater a force in thruſting horizontally, at the height of his ſhoulders, than 27 lb.

As to horſes. A horſe is, generally ſpeaking, as ſtrong as 5 men. A horſe will carry 240 or 270lb.

FIG. A horſe draws to greateſt advantage, when the line of direction is a little elevated above the horizon, and the power acts againſt his breaſt. And can draw 200 lb. for eight hours in a day at $2\frac{1}{2}$ miles an hour. If he draw 240 lb. he can work but ſix hours, and not go quite ſo faſt. And in both caſes, if he carries ſome weight, he will draw better than if he carried none. And this is the weight a horſe is ſuppoſed to be able to draw over a pulley out of a well. In a cart, a horſe may draw 1000 lb.

The moſt force a horſe can exert is, when he draws ſomething above a horizontal poſition.

The worſt way of applying the ſtrength of a horſe, is to make him carry or draw up hill. And three men in a ſteep hill, carrying each 100 lb. will climb up faſter than a horſe with 300 lb.

Though a horſe may draw in a round walk of 18 feet diameter; yet ſuch a walk ſhould not be leſs than 25 or 30 feet diameter.

5. Every machine ought to be made of as few parts, and thoſe as ſimple as poſſible, to anſwer its purpoſe; not only becauſe the expence of making and repairing will be leſs, but it will alſo be leſs liable to any diſorder. And it is needleſs to do a thing with many, which may be done with fewer parts.

6. If a weight is to be raiſed but a very little way, the leaver is the moſt ſimple, eaſy, and ready machine. Or if the weight be very great, the common ſcrew is moſt proper. But if the weight is to be raiſed a great way, the wheel and axle is a proper power, and blocks and pullies are eaſier ſtill; and the ſame may be done by help of the perpetual ſcrew.

Great wheels to be wrought by men or cattle, are of moſt uſe and convenience, when their axles are perpendicular to the horizon; but if by water, *&c.* then it is beſt to have their axles horizontal.

7. As to the combination of ſimple machines together, to make a compound one. Though the leaver when ſimple, can not raiſe a weight to any great height; and in this caſe is of little ſervice; yet it is of great uſe when compounded with others. Thus the ſpokes of a great wheel are all leavers perpetually acting; and a beam fixed to the axis to draw the wheel about by men or horſes, is a leaver. The leaver alſo may be combined with the ſcrew, but not conveniently with pullies or with the wedge. The wheel and axle is combined with great advantage with pullies. The ſcrew is not well combined with pullies; but the perpetual ſcrew combined with the wheel, is very ſerviceable. The wedge cannot be combined with any other mechanical

chanical power; and it only performs its effect by percussion; FIG.
but this force of percussion may be increased by engines.

Pullies may be combined with pullies, and wheels with wheels. Therefore if any single wheel would be too large, and take up too much room, it may be divided into two or three more wheels and trundles, or wheels and pinions, as in clock-work; so as to have the same power, and perform the same effect.

In wheels with teeth, the number of teeth that play together in two wheels, ought to be prime to each other, that the same teeth may not meet at every revolution. For when different teeth meet, they by degrees wear themselves into a proper figure. Therefore they should be contrived, that the same teeth meet as seldom as possible.

8. The strength of every part of the machine ought to be made proportional to the stress it is to bear. And therefore let every leaver be made so much stronger, as its length and the weight it is to support is greater. And let its strength diminish proportionally from the fulcrum, or point where the greatest stress is, to each end. The axles of wheels and pullies must be so much stronger, as they are to bear greater weight. The teeth of wheels, and the wheels themselves, which act with greater force, must be proportionally stronger. And in any combination of wheels and axles, make their strength diminish gradually from the weight to the power, so that the strength of every part be reciprocally as the velocity it has. The strength of ropes must be according to their tension, and that is as the squares of their diameters, (see the end of Sect. VIII.) And in general whatever parts a machine is composed of, the strength of every particular part of it must be adjusted to the stress upon it, according to Sect. VIII. Therefore in square beams the cubes of the diameters must be made proportional to the stress they bear. And let no part be stronger or bigger than is necessary for the stress upon it: not only for the ease and well-going of the machine, but for the diminishing the friction. For all superfluous matter in any part of it, is nothing but a dead weight upon the machine, and serves for nothing, but to clog its motion. And he is by no means a perfect mechanic, that does not only adjust the strength to the stress, but also contrive all the parts to last equally well, that the whole machine may fail together.

9. To avoid friction as much as possible, the machine ought not to have any unnecessary motions, or useless parts; for a multiplicity of parts, by their weight and motion, increase the friction. The diameters of the wheels and pullies ought to be

FIG. large, and the diameters of the arbors or ſpindles they run on, as ſmall as can be conſiſtent with their ſtrength. All ropes and cords muſt be as pliable as poſſible, and for that end are rubbed with tar or greaſe; the teeth of wheels muſt be made to fit and fill up the opens; and cut in the form of epicycloids. All the axles, where the motion is, and all teeth where they work, and all parts that in working rub upon one another, muſt be made ſmooth: and when the machine goes, muſt be oiled or greaſed. If a joint is to go pretty ſtiff and ſteady, rub a little greaſe upon it.

183. The axis *a* of a wheel, may have its friction diminiſhed, by cauſing it to run on two rollers, *B*, *C*, turning round with it, upon two centers.

184. Likewiſe inſtead of the teeth of wheels, one may place little wheels as *A*, *B*, running upon an axis in its center. And this will take away almoſt all the friction of the teeth. And in lanterns or trundles, the rounds may be made to turn about, inſtead of being fixed.

In all machines with wheels, the axes or ſpindles ought not to ſhake, which they will do, if they be too ſhort. And their ends ought juſt to fill their holes.

When the teeth of a wheel are much worn away, it makes that wheel move irregularly about, increaſes the friction, and requires more force; and may cauſe the teeth of two wheels to run foul upon one another, and to ſtop their motion, and endanger breaking the teeth. To prevent this, proper care ſhould be taken to dreſs the teeth, and keep them to their proper figure.

10 When any motion is to be long continued; contrive the power to move or act always one way, if it can be done. For this is better and eaſier performed than when the motion is interrupted, and the power is forced to move firſt one way, and then another. Becauſe every new change of motion requires a new additional force to effect it. Beſides, a body in motion cannot ſuddenly receive a contrary motion, without great violence. And the moving any part of the machine contrary ways by turns, with ſudden jerks, tends only to ſhake the machine to pieces.

11. In a machine that moves always one way, endeavour to have the motion uniform. Some methods of doing this may be ſeen in Prop. CXIII. and if one uniform motion be required to produce a motion either uniform or accelerated, ſome light may be had from Prop. CXI. Likewiſe how to communicate motion, conſult Prop. CX. And to change the direction, ſee Prop. CXII.

12. But when the nature of the thing requires that a motion is to be suddenly communicated to a body, or suddenly stopt: to prevent any damage or violence to the engine, by a sudden jolt; let the force act against some spring, or beam of wood, which may supply the place of a spring. FIG.

13. In regard to the size of the machine; let it be made as large as it can conveniently. The greater the machine, the exacter it will work, and perform all its motions the better. For there will always be some errors in the making, as well as in the materials; and consequently in the working of the machine. The resistance of the medium in some machines has a sensible effect. But all these mechanical errors bear a less proportion to the motion of the machine, in great machines than in little ones, being nearly reciprocally as their diameters; supposing they are made of the same matter, and with the same accuracy, and are equally well finished. Therefore in a small machine they are more sensible; but in a great one almost vanish. Therefore great machines will answer better than smaller, in all respects, except in strength; for the greater the machine the weaker it is, and less able to resist any violence.

14. For engines that go by water, it is necessary to measure the velocity and force of the water. To get the velocity, drop in pieces of sticks, *&c.* and observe how far they are carried in a second, or any given time.

But if it flow through a hole in a reservoir or standing receptacle of water. The velocity will be found from the depth of the hole below the surface; by Cor. 2. Prop. XCVII. And its force by Cor. 1. Prop. CVII.

Thus let $s = 16\frac{1}{12}$ *feet*, $v =$ velocity of the fluid per second. $B =$ the area of the hole. $H =$ height of the water; all in feet. Then the velocity $v = \sqrt{2sH}$; and its force $=$ the weight of the quantity $\frac{vv}{2s}B$ or HB of water, or $= \frac{62\frac{1}{2}}{112}HB$ hundred weight; because a cubic foot is $= 62\frac{1}{2}$ lb. averd. Also a hogshead is about $8\frac{1}{2}$ feet, or 531 lb. and a ton is 4 hogsheads.

When you have but a small quantity of water, you must contrive it to fall as high as you can, to have the greater velocity, and consequently more force upon the engine.

15. If water is to be conveyed through pipes to a great distance, and the descent be but small; so much larger pipes must be used, because the water will come slow. And these pipes ought not to be made straiter in some places than others; for the

FIG. the quantity of water conveyed through them, depends upon the bigness of the bore at the ſtraiteſt place.

Pipes of conduct coming directly from an engine, ſhould be made of iron, with flanches at the ends to ſcrew them together, with lead between, or elſe of wood; for lead pipes will bulge out at every ſtroke of the engine and burſt. But pipes next a jet muſt be lead. Pipes ſhould not turn off at an angle, but gradually in a curve: pipes of elm will laſt 20 or 30 years in the ground. But they muſt be laid ſo deep, that the froſt may not reach them, or elſe the water muſt be let out; otherwiſe the froſt will ſplit them.

The thickneſs of any pipe muſt be as the diameter of the bore, and alſo as the depth from the ſpring. For a lead pipe of 6 inches bore, and 60 or 70 feet high, the thickneſs muſt be half an inch. And in wooden pipes 2 inches.

Water ſhould not be driven through pipes faſter then 4 feet per ſecond, by reaſon of the friction of the tubes. Nor ſhould it be much wire-drawn, that is, ſqueezed through ſmaller pipes; for that creates a reſiſtance, as the water-way is leſs in narrow pipes.

And in pump work, where water is conveyed through pipes to higher places, the bores of the pipes ſhould not be made too ſtrait upwards; for the ſtraiter they are near the top, the leſs water will be diſcharged. Nor ſhould the pipe that brings the water into the pump be too ſtrait, for the ſame reaſon. The wider theſe are, the eaſier the pump works.

When pipes are wind bound, that is, when air is lodged in them that the water can hardly paſs, it muſt be diſcharged thus. Going from the ſpring till you come to the firſt riſing of the ground, dig it open till the pipe be laid bare; then with a nail driven into it, at the higheſt part, or rather a little beyond, make a hole in the top; and all the air will blow out at the hole, and when the water comes, batter up the hole again. Do the ſame at every eminence, and all the air will be diſcharged. If the water runs faſt through the pipes, the air will be beyond the eminence; but ſtopping the water, the air will aſcend to the higheſt part. If air be driven in at firſt along with the water, the nail hole muſt be left open, or a cock placed there to open occaſionally. Sometimes a ſmall leaden pipe is placed over the other, communicating with it in ſeveral places, in which is a cock at top to open upon occaſion.

192. 16. When any work is to be performed by a water wheel moved by the water running under it, and ſtriking the paddles or

or laddle boards. The channel it moves in ought to be something wider than the hole of the adjutage, and so close to the floats on every side, as to let little or no water pass; and when past the wheel, to open a little that the water may spread. It is of no advantage to have a great number of floats or paddles, for these past the perpendicular are resisted by the back water, and those before it are struck obliquely. The greatest effect that such a wheel can perform, in communicating any motion, is when the paddles of the wheel move with $\frac{1}{3}$ the velocity of the water; in which case, the force upon the paddles is $\frac{4}{9}$ only; supposing the absolute force of the water against the paddles, when the wheel stands still, to be 1. So that the utmost motion which the wheel can generate, is but $\frac{4}{27}$ of that which the force of the water against the paddles at rest, would produce. This is when the wheel is at the best; but oftentimes far less is done. FIG. 192.

Machines to raise water, though well made, seldom lose less than $\frac{1}{5}$ the computed quantity of water to be raised. The best contrived engine is scarce $\frac{1}{5}$ part better than the worst contrived engine, when they are equally well executed.

A man with the best water engine cannot raise above one hogshead of water in a minute, 10 feet high, to work all day.

17. When a weight is to be raised with a given corporeal power, by means of the wheel and axle; so that the weight may receive the greatest motion possible in a given time. The radius of the wheel and axle, and the weight to be raised, ought to be so adjusted, that the radius of the axle (EF) : may be to the radius of the wheel (AB) :: as $\frac{2}{5}$ the power (P) : to the weight to be raised (W) : or, which comes to the same thing, the velocity gained by the power in descending must be $\frac{2}{5}$ the velocity which would be gained by gravity in the same time. 30.

This only holds good, when the power is a heavy body, as well as the weight: but does not take place, when the power is some immaterial active force, such as that of an elastic medium, the strength of a spring, &c. whose weight is inconsiderable.

18. *These principles also are very useful, and necessary to be known, where water works are concerned.*

The pressure of the atmosphere upon a square inch, is 14.7 lb. *averd.* at a medium.

The weight of a column of water, equal to the weight of the atmosphere, is 11 $\frac{1}{4}$ *yards.*

A cubic

FIG. A cubic foot of water weighs 62 ½ lb. *averd.* and contains 6.128 *ale gallons.*

An ale gallon of water contains 282 *inches,* and weighs 10.2 lb. *averd.*

A tun of water ale meaſure, weighs 1.1 *tun averd.* at 63 gallons the hogſhead.

A cylinder of water a yard high, and *d* inches diameter, contains $\frac{1}{10}$ *dd ale gallons*; and weighs 1 $\frac{1}{45}$ *dd pounds averd.*

SECT.

FIG.

SECT. XIII.

The description of compound machines or engines, and the method of computing their powers or forces: with some account of the advantages and disadvantages of their construction.

PROP. CXX.

To describe several sorts of engines, and to compute their forces or effects.

There are two things required to make a good mechanic or engineer. The first is a good invention for contriving all the parts of a machine, to perform its motions and effects in the most simple and easy manner. The next is, to be able to compute the power or force of it; to know whether it can really perform the effect expected from it or not. The foundation of both these has been already laid down in this book. What seems to be necessary farther, is to give some examples in practice, by shewing the construction of several mechanic engines, and computing their powers. As there is great skill and sagacity in contriving fit and proper ways to perform any motion, so this is principally to be attained by practice, and a thorough acquaintance with machines of several kinds. I shall therefore give the mechanical construction of several sorts of machines, made for several different purposes, which will assist the reader's invention, and give him some idea how he may proceed in contriving a machine for any end proposed. Of which I shall only give a short explanation of the principal parts, not troubling the reader with any description of their minuter parts, nor how they are joined together, or strengthened, *&c.* It is sufficient here to shew the disposition and nature of the principal members: the rest belongs to carpenters, joiners, smiths, *&c.* and is easily understood by any one.

To

FIG.

To compute their powers.

1. As to ſimple machines, they are eaſily accounted for, and their forces computed, by the properties of the mechanic powers.

2. For compound machines; ſuppoſe any machine divided into all the ſimple ones that compoſe it. Then begin at the power and call it 1; and by the properties of the mechanic powers, find the force with which the firſt ſimple machine acts upon the ſecond, in numbers. Then call this force 1, and find the force it acts upon the third, in numbers. And putting this force 1. find the force acting on the fourth, in numbers; and ſo on to the laſt. Then multiply all theſe numbers together, the product will give the force of the machine, ſuppoſing the firſt power 1.

3. When pullies are concerned in the machine, all the parts of the ſame running rope, that go and return about ſeveral pullies, freely and without interruption, muſt be all numbered alike for the force. And if any rope act againſt ſeveral others, it muſt be numbered with the ſum of all theſe it acts againſt.

4. In a combination of wheels; take the product of the number of teeth in all the wheels that act upon and drive others, for the power; and the product of the teeth in all the wheels moved by them, for the weight. Or inſtead of the teeth take the diameters.

Or thus,

When a machine is in motion, if you meaſure the velocity of the weight, and that of the power, in numbers; then the firſt number to the ſecond, gives the proportion of the power to the weight.

Otherwiſe thus,

In wheel work, there are always two wheels fixt upon one axis, or elſe one wheel, and a pinion, trundle, or barrel, which ſupplies the place of a wheel. Of theſe two, call that wheel the *leader*, which is acted on by the power, or by ſome other wheel; and the other, on the ſame axis, call the *follower*, which drives ſome other forward. Then having either the number of teeth, or the diameter, of each, *take the product of all the leaders, for the weight; and the product of all the followers, for the power.* Here the leader receives the motion, and the follower gives it.

5. And

5. And if the velocity of the power or weight be required. FIG.
Take the product of all the leaders, for the velocity of the power; and the product of all the followers, for the velocity of the weight.

Other things that are more complex and difficult, must be referred to the general laws of motion.

EXAMPLE I.

Sciffars, pinchers, &c. may be referred to the leaver of the first kind. A *handfpike* and *crow* are leavers of the first kind. *Knives* fixt at one end, to cut wood, bread, *&c.* are leavers of the second kind. The *bones* in animals, alfo *tongs*, are leavers of the third kind. A *hammer* to draw a nail is a bended leaver.

Ex. II.

A *windlefs*, and a *capftain* in a fhip, and a *crane* to draw up goods out of a fhip or boat, may be referred to the wheel and axel.

Ex. III.

All *edge tools* and inftruments with a *fharp point*, to cut, cleave, flit, chop, pierce, bore, *&c.* as *knives, hatchets, fciffars, fwords, bodkins*, &c. may be reduced to the wedge.

Ex. IV.

The *bar AB* bearing a weight *C*, may be referred to the 186.
leaver; where the weight upon *A* : to the weight upon *B* :: is as *BC* : to *AC*.

Ex. V.

Likewife if two horfes draw the weight *W*, in the directions 187.
A 1, *B* 2, by help of the *fwingtree AB*; this may be referred to the leaver. And the ftrength or force at *A*, to that of *B* :: is as *BC* : to *AC*.

Ex. VI.

ACB is a *balance*, where the brachia *AC*, *CB* being equal, 188.
the weights in the two fcales *D*, *E* will be equal. The properties of a good balance are, 1. That the points of fufpenfion of the fcales, and the center of motion of the beam, be in one right line. 2. That the brachia or arms be exactly of equal length from the center of motion. 3. That they be as

FIG. 188. long as possible with conveniency. 4. That there be as little friction as possible in the motion. 5. That the center of gravity of the beam be in, or but very little below, the center of motion. 6. That they be in equilibrio when empty.

If one brachium *AC* be longer than the other *CB*, then the weight in the scale *E* must be greater than that in *D*, to make an equilibrium. And then you will have a deceitful balance, which being empty, or loaded with unequal weights, shall remain in equilibrio. For $AC : CB :: $ weight in E : weight in D; by the property of the leaver. But changing the weights from one scale to the other will discover the deceit; for the balance will be no longer in equilibrio.

Ex. VII.

190. The *steelyard AB* is nothing but a leaver, whose fulcrum is *C*, the center of motion. If the weight *P* placed at *D* reduces the beam *AB* to an equilibrium; and there be taken the equal divisions *D* 1, 1 2, 2 3, 3 4, *&c.* then the weight *P* placed successively at 1, 2, 3, 4, *&c.* will equi-ponderate with weights as *W*, suspended at *B*, which are also as the numbers 1, 2, 3, 4, *&c.* respectively. Moreover if the divisions *D*1, 1 2, 2 3, *&c.* be each $= CB$. Then if *P* be successively placed at 1, 2, 3, *&c.* the weight *W* to balance it, will be respectively equal to P, $2P$, $3P$, *&c.* that is to 1, 2, 3 pounds, *&c.* if *P* is a pound.

For by the property of the leaver $CP \times P + CD \times P = CB \times W$, that is $PD \times P = CB \times W$. And $CB : PD :: P : W$, universally. Whence, if DP or $D1 = CB$, then $W = P$. If DP or $D2 = 2CB$, then $W = 2P$, *&c.* But if *CB* be greater than *D*1, 1 2, *&c.* then will the constant weight *P* be greater than *W*, $2W$, &c.

The properties necessary for a steelyard to have, are these.

1. That the fixt weight *P* being placed at *D*, where the divisions begin, shall make the beam in equilibrio.

2. That the divisions *D* 1, 1 2, 2 3, *&c.* be equal to one another.

3. That *CB* may be of any length, provided the weight *P* be rightly adjusted to it: *viz.* so that $CB : D1 :: P : 1$ pound, if *W* be pounds. Or $CB : D1 :: P : 1$ stone, if *W* be stones.

4. That the beam be strait, and the upper edge in a line with the centers *C*, *B*.

5. That it move easily and freely on its center *C*.

Many

Many steelyards are likewise graduated on the under side, FIG.
which may be used by turning them upside down. Generally 190.
one side is for small weights, and the other for great ones. And each side is adjusted by the foregoing rules; and all the crooks hanging at it (except the moveable one for the weight) must go to the weight of the beam.

Ex. VIII.

Let AB be a *cheese press*; CE, FG are leavers moveable about 189.
the points D, E, F, G, by applying the hand at C. S the stone or weight. H the cheese.

If $CD = 5$, $DE = 2$, $FG = 6$, $GH = 2$, $FR = 1$, $FH = 4$. Then in the leaver CE, D is the fulcrum. Call the power at C, 1; then the force at E or F is $\frac{5}{2}$. And in the leaver FG, whose fulcrum is G; if the power at F be 1, the force at R is $\frac{6}{5}$, therefore the power at C, to the weight S, is as 1 to $\frac{5}{2} \times \frac{6}{5}$ or 3. Also the weight of the stone at R, to the pressure at H, as 2 to 5, or 1 to $\frac{5}{2}$. And the power at C, is to the pressure at H, as 1 to $3 \times \frac{5}{2}$ or $7\frac{1}{2}$.

Ex. IX.

Let EG be a *spinning wheel*. Diameter of the rim $EF = 18$. 191.
Diameter of the twill $ab = 2$. Diameter of the whorle $cd = 3$. $EabF$ the band going about the twill. $EcdF$ the band going about the whorle. Therefore whilst the rim makes 1 revolution, the twill makes 9, and the whorle and feathers 6. Therefore there are 3 revolutions of the twill, for 2 of the feathers n. And consequently the difference of the revolutions which is 1, is the quantity taken up by the twill, whilst the thread trn is twined by these 2 revolutions of the feathers. The greater the difference of the revolutions of the twill and feathers, the more the wheel takes up. And the nearer an equality, the more she twines. If they make equal revolutions in the same time, she will not take up at all. And if the feathers make no revolutions, she will twine none. The greater the proportion of the rim to the whorle and twill, the faster she will do both.

Ex. X.

A machine to raise a weight by the force of the running water IH, carrying the wheel LK, by means of the floats F, F. Let the diameter of the wheel LK be 10; of GB, 2; of DC,

11;

FIG. 192. 11; of AE, 3. Let the power of the water againſt the floats F, be 1. Then the force at B to move the wheel CD, will be 5; again if the power at B be 1, the force at A will be $3\frac{2}{3}$. Therefore the force of the water, to the weight W, is as 1 to $5\times3\frac{2}{3}$, or as 1 to $18\frac{1}{3}$.

When the wheels and axles and weight are ſo adjuſted, that the velocity of the floats at F, is $\frac{1}{3}$ the velocity of the water there; then the weight W will have the greateſt motion of aſcent poſſible. For if any one thing be changed, whether the weight, or the diameter of any wheel or axle, whilſt the reſt remain the ſame, the motion will be leſſened.

Ex. XI.

193. In the machine FB, which raiſes the weight W, by means of the wheel EG, and the perpetual ſcrew BE. Let the circumference deſcribed by the power C be 30 inches, the diſtance of two threads of the perpetual ſcrew E, be 1 inch. Diameter of the wheel $EG=5$ feet, of $DA=2$. Therefore if the power at C be 1, the force acting at E to turn the wheel EG will be 30. And if the power at E be 1, the force at D will be $2\frac{1}{2}$. Therefore the power at C, to the weight W, is as 1 to $30\times2\frac{1}{2}$ or 1 to 75.

Note, it is the ſame thing whether CB be ſtrait or crooked, whilſt the diſtance BC, in a ſtrait line is the ſame; and in meaſuring, you muſt always take the ſtrait line BC.

Ex. XII.

194. In a machine compounded of wheels to raiſe a weight; let $AB=5$, diameter of the barrel $MN=2$, the number of teeth in the wheels and nuts, as follows; $CD=10$, $CE=40$, $FG=12$, $FH=50$, $KI=12$ $IL=64$. Then the power applied to B, is to the weight W, as $1\times10\times12\times12$ to $5\times40\times50\times64$; that is as 1440 to 640000, or as 1 to $444\frac{4}{9}$.

But if the power was at W, to move the weight B; then the ratio will be inverted. For then the power will be to the force at B, as 444 to 1. Or if the velocity of B was required, you will have the velocity of W to that of B, as 1 to 444.

Ex. XIII.

195. A machine to raiſe a weight by help of the *triangle* $ABEF$, the windleſs CC, and two pullies P, Q. Let the diameter HG

HG where the rope goes, be$=2$, radius $CD=5$. Then if the power at D be 1, the force at H is 5. And if the force at H, drawn by one rope, be 1, the force at W drawn by two ropes, will be 2. Therefore the power at D, to the weight W, is as 1 to 2×5 or 10. If the leg AB be wanting, the other two may be set against a wall, or upheld by ropes, and then it is called a pair of *sheers*. FIG. 195.

Ex. XIV.

If the weight A is to be lifted by the 3 pullies C, D, E, of which C is fixt. Call the power at B, 1. Then the force stretching AE is 1; and both together is equal to the force of $DE=2$; and force $DA=2$; whence, force $DC=4$; likewise force $CA=4$ Therefore the whole force acting at A is $1+2+4=7$, and the power at B to the weight A; as 1 to 7. 196.

Ex. XV.

In this machine, $bACD$ is a running rope fixt at D, B a fixt pulley. Let the power at b pulling the rope bA be 1. That on AC 1, and CD 1. Then will AB be 2, and BC 2, BE 4. And the weight W opposing AC, BC and DC, will be $1+2+1=4$. Whence the power at b, to the weight W, is as 1 to 4. 197.

Ex. XVI.

Another machine with pullies. A a fixt pulley; the ends of the several ropes are fixt at B, C, D, E. Suppose the power at $M=1$, then the force on AF, FB is 1; on FG, GC, 2; on GH, HD, 4; on HI, IE, 8. But the weight P acts against HI, IE, and is therefore$=16$: and the power is to the weight, as 1 to 16. 198.

SCHOL.

In a single pulley, as fig. 39. if a given power at P was to be a weight or heavy body, which was to raise some other weight W, there will be the greatest motion generated in W, in any given time, when $W=\frac{2}{5}P$. 39.

And in a combination of pullies, as fig. 42. if a weight P was to raise another weight W; and if velocity of W : velocity of P :: $\frac{2}{5}P : W$; then W will be the weight which will acquire the greatest motion in a given time, by that given power P. 42.

Ex.

FIG. 199.

Ex. XVII.

Let DE be a *boat* rowed by oars; and let ABC be one oar. Here the power acts at A, and the pin B will be the fulcrum; and the force at C, acting againſt the water, is that which gives her motion. Let the power at A be 1; then the force at C, by which the boat is moved, is $\frac{AB}{BC}$. Whence the longer AB, or the ſhorter BC is, ſo much more power there is at A to move her forward.

Therefore long oars have the diſadvantage of loſing power. Yet the oars may be too ſhort, as well as too long. For if they be very ſhort, the motion of the boat will allow little time to ſtrike, and they will have but ſmall force to act againſt the water with, in ſo ſmall a time, as well as from the ſlow motion of the end C; which is a diſadvantage on the other ſide.

Ex. XVIII.

200. Let FR be *a boat or a ſhip*, AS a ſail. Suppoſe a plumb-line drawn through the center of gravity of the ſection of the ſhip and water; and another line BO, parallel to the horizon and to the axis or keel of the ſhip, and to paſs through the center of preſſure or reſiſtance of the ſhip, which ſhe has by the water in her motion. Let this interſect the former plumb-line in O. Through C the center of gravity of the ſail, draw CD perpendicular to the ſail; and CB perpendicular to BO, and AS in the plane of the triangle CBD.

Then if DC be the force of the wind againſt the ſail AS, then DB is the force generating her progreſſive motion, and BC is the force lifting the ſhip upwards. Now the force DB, acting at C, in direction DB, endeavours to turn the ſhip round an axis paſſing through O, with a force which is equal to the abſolute force $BD \times$ by the diſtance CB, or $CB \times BD$; and this is the force by which her head is depreſſed. Likewiſe the force BC, in direction BC, endeavours to turn the ſhip round an axis at O, the contrary way; and that with the force $BC \times$ diſtance BO, or $BC \times BO$; and this is the force that raiſes her head. Therefore the force to raiſe her head is to the force to depreſs it, as $CB \times BO$ to $CB \times BD$, or as BO to BD.

Hence, if the point D fall before O, then the ſail endeavours to raiſe the ſhip's head; if it be behind O, it endeavours to ſink it. If it be in O, it will keep her ſteady. And the height

of

of the sail *AS* contributes nothing to her progressive motion; the same ratio of the absolute to the progressive force, remains still as *CD* to *DB*. FIG. 200.

Ex. XIX.

EF is a *cart* or *carriage*, *BD* a rub for the wheel *CAD* to pass over, *AB* the horizontal plane; *DB*, *AC* perpendicular, and *OD* parallel to *AB*. *C* the center of the wheel. Then the horizontal force required to pull the wheel over the rub *BD*, is as $\frac{DO}{CO}$. And the difficulty of going over rubs increases in a greater ratio than that of their heights. Also the higher the wheels, the more easily they pass over them; but then they are more apt to overturn. To draw the cart with the least power over the rub *BD*, it should not be drawn in the horizontal direction *AB* or *OD*; but in the direction *AD*. The advantage of high wheels is, that they pass the rubs most easily, and they have also less friction, and sink less in the dirt, and more easily press down an obstacle. But their disadvantage is, that they easily overturn; they also make cattle draw too high; for they can apply their strength best when they draw low and upward, as in the direction *AD*; which is the advantage of low wheels. Yet if the wheels are high, they may be made to draw low, by fixing the limmers or traces as far below the axle as you will, which will then be an equal advantage with low wheels. For the power not pulling at the wheel, but at the carriage, may draw from any part of it. There is another advantage in small wheels, that they are better to turn with. 201.

A *waggon* with 4 wheels is more advantageous than a cart with only 2 wheels, especially on sand, clay, &c. Narrow wheels and narrow plates are a disadvantage; the broader the wheels, the less they sink, and therefore require less draught, and also cut the roads less; yet they take up a great deal of dirt, which clogs the carriage. There is a great deal of friction in all carriages, as is evident by the force required to draw them upon plain ground. And for that reason, experience can only inform us, how much force is able to draw any carriage. To make the resistance as small as can be, axles of iron, running in brass boxes in the wheel naves, go the easiest.

The spokes in the wheel ought to be a little inclined outwards; that when a wheel sinks into a rut, the spokes (bearing then the greatest weight) may be nearly perpendicular to the horizon.

FIG. 201. The underside of the axle-tree, where the wheels run, ought to be nearly in a right line; if they slant much upward towards the ends, the wheel will work against the lin pin. Yet this causes the wheels to be further asunder at top than at bottom in the rut, because the ends of the axle-tree are conical; which is an inconvenience.

Ex. XX.

202. Suppose the *waggon FG* is moved forward, by a power acting within it. Which power turns the wheel *DE* by the spokes *AD*, *AD*, &c. and *DE* turns the wheel *IC*, which carries the waggon. Let the power at *A* be 1, then the force acting at *E* will be $\frac{DA}{DE}$; also if the power at *E* be 1, the force at *C* by which the waggon is moved, will be $\frac{BE}{BC}$. Therefore the power at *A*, to the force by which the waggon can be moved, is as 1 to $\frac{DA \times BE}{DE \times BC}$. Or the power is to that force, as $DE \times BC$ to $DA \times EB$. It will be the same thing, if instead of teeth, the wheel *DE* carries *EB* by a chain going round them. You must suppose the like wheels on the opposite side.

Hence if the absolute force to move the waggon without, be 1; the force within, applied at *A*, to move it, will be $\frac{DE \times BC}{BE \times DA}$.

Ex. XXI.

203. *ABCD* are the *sails of a windmill*, all alike inclined to their common axis, and facing the wind, and turning about in the order *ABCD*. *WC* the direction of the wind parallel to the axis *EH*. Since *WC* is perpendicular to *EC*, draw *CF* in the sail perpendicular to *EC*; then the angle *WCF* will be the angle of incidence of the wind upon the sail. Therefore the force of the wind to turn the sails about the axis *EH*, is as the square of the sine of the angle *WCF* × by its cosine. And the force acting against the mill, in direction of the axis *EH*, is as the cube of the sine of *WCF*. Now since the force of the wind to turn the sails round, is as $\overline{S.WCF}^2 \times$ cos. *WCF*; therefore when that force is the greatest, the angle *WCF* will be 54° : 44′.

And

And this is the most advantageous position of the sails to move them from rest; and would always be so, if the wind struck them in the same angle when moving as when at rest. But by reason of the swift motion of the sails, especially near the end *G*, the wind strikes them under a far less angle; and not only so, but as the motion at the end *G* is so swift, it may strike them on the backside. Therefore it will be more advantageous to make the angle of incidence *HCF* greater, and so much more as it is further from *E*. Therefore at the places *n*, *o*, *G*, the tangents of the angles ought to be nearly as the distances, *En*, *Eo*, *EG*. And therefore the sails ought to be twisted, so as at *r* to lie more sharp to the wind, and at *G* almost to face it. And by that means they will avoid the back wind. FIG. 203.

Ex. XXII.

GB is a common *sucking pump*; *GKL* the handle; *CD* the bucket; *E*, *F* two clacks opening upwards. When the end *L* is put down, the end *G* raises the sucker or bucket *CD*, and the valve or clack *F* shuts; and the water above the bucket being raised, the weight of the atmosphere is taken off the water underneath in the pump. Then the pressure of the external air in the pit or well *MN*, raises the water up the pump, opens the valve *E*, and ascends thro' the hole *B* into the body of the pump *DB*. Again, when the handle *L* is raised, the bucket *CD* descends, the valve *F* opens, and lets the water ascend through it, and the pressure of the water shuts the valve *E*, so that the water cannot return through *B*. Then whilst the end *L* is put down again, the sucker *CD* is raised again, together with the water above it, whilst more ascends through *B*. So that at every stroke of the handle, water is raised into the pump, till at last it flows through the pipe *H*. 204.

If the bucket *CD* be more than 30 or 32 foot from the surface of the water *MN* in the pit, no water will ascend above it; for the pressure of the atmosphere reaches no farther. Therefore it must be always within that distance, or this pump is useless for raising water.

The weight of water which the bucket lifts at each stroke is that of a column of water, whose height is *MH*, and its diameter that of the bore of the pump at *CD*, where the bucket goes. Therefore as *GK* to *KL* :: so the power applied at *L*, to that weight. Therefore it signifies nothing where the bucket is placed, as to the weight of water. If a leak happens in the barrel of the pump below the bucket *CD*, the air will get in and hinder the

FIG. 204. working of the pump: If above CD, only ſome will be loſt, therefore CD ſhould be placed low; but then it will be bad to come at to repair it.

The bucket, ſucker, or piſton, is to be ſurrounded with leather to fit exactly, and muſt move freely up and down in the barrel, and muſt alſo exactly fill it. Of valves or clacks, ſome are flat, made of leather; others are conical: and they muſt all fit very cloſe, and move freely. To balance the weight of water, the handle KL is commonly made heavy, as of iron, with a knob at the end L.

The bore of the pipe at B ſhould not be too ſtrait; the wider it is, the more freely the water aſcends, and the eaſier the pump works. Likewiſe the longer ſtroke the pump makes, the more water is raiſed by the ſame power; there being leſs water loſt, by the valves ſhutting.

Calculation of a common pump.

Suppoſe LK, 3 feet; KG, 8 inches.

$h = HM$ the height from the water in yards:

Then the diameter of the bore at D will be $= \sqrt{\frac{100}{h}}$ inches.

And a ſingle perſon will raiſe $\frac{80}{h}$ hogſheads of water in an hour.

In many pumps for common uſe, it is not neceſſary to draw a great quantity of water; and then a ſmaller bore will ſerve, as 3 or 4 inches; which will make the pump go ſo much the lighter.

Ex. XXIII.

206. If a man ſitting in the ſcale E, be in equilibrio with a weight in the ſcale A; and if he thruſt againſt the beam CB, with a ſtick or otherwiſe, in direction ED, and by that means thruſts out the ſcale E to the poſition BE. Then the man in the ſcale E will over-balance the other ſcale A, and raiſe the weight. For let EL be perpendicular to FB; then the force at E to turn the ſcales is to the contrary force at F, as CL to CF or CB. For it is the ſame thing, as if E was ſuſpended at L.

And when the perpendicular obſtacle GH hinders the ſcale from going out; and the center C is always kept ſteady. Yet the ſcale E will ſtill preponderate. For let ED be the force acting againſt D; this is equivalent to the two forces EB, BD, acting at

at E and D. The force BD tending to or from the center, does nothing. But the force EB at E, acting at the distance CB, its power to bring down the scale E is $CB \times BE$. And the same force acting at D, its power to push up the scale is $CD \times BE$. And their difference $DB \times BE$ is the absolute force to thrust down the scale. And this force is to the whole thrusting force DE, as DB to DE. And if D were on the other side of C, the force would still be $DB \times BE$, or $\overline{CB+CD} \times BE$. FIG. 205.

But if the scale E was not moveable about B, as if it were tied by the cord DE; then no force acting against any part of the beam FB, could have any effect to destroy the equilibrium.

Ex. XXIV.

Suppose a man A standing upon the plank CB, supported only at the end C, and pulling the end B towards him by the rope EB, in order to keep himself and the plank from falling. 207.

Imagine the man and the plank to be one body; then the action and re-action in direction EB, destroy one another, and his pulling does nothing. It would therefore be in vain for him to endeavour to support himself by that force; for both he and the plank must fall down together towards B, by their own weight.

Ex. XXV.

CD is a machine with two wheels fixed to an axis DF, round which goes a cord $GDFE$. There is a power at E endeavouring to draw the machine towards E, in a direction parallel to the horizon HO, by the cord EF, going under the axis DF. In the radius AH of the wheel, take AB equal to the radius of the axle DF, towards H, because the string goes below it. Then the force to move the machine, is the same as if the string was fixed at B; where H is the fulcrum, A the weight. Then the force to move the machine towards E, with the given power E, will be as BH. Therefore it would be in vain, by pulling at the string, to endeavour to make the body roll towards D, the contrary way. But if DF was greater than the diameter of the wheel, that is, if B falls beyond H, then the force drawing towards E, would move the body towards D the contrary way. 208.

If the direction of the power DE be elevated above the horizon, as fe; then the machine could approach or recede, till the direction of the string ef fell upon the point of contact H, and there it would rest.

Ex.

FIG.

Ex. XXVI.

209. *AB* is an *artificial kite*, kept up by the wind blowing in direction *WC*, by drawing the ſtring *AIBHI*, fixed at *A* and *B*. The kite will gain ſuch a poſition, that *HI* produced will paſs through the center of gravity of its ſurface at *C*. Draw *CO* perpendicular to *BA*, and *DO* perpendicular to the horizon *HO*. Then *OC* is the direction of the force of the wind acting againſt the kite; and the force of the wind to keep her up, is as the ſquare of the ſine of the angle *ACW* or *COD*. Now if *DO* repreſent the given weight of the kite, *CO* will be the force of the wind acting againſt her, and *CD* the force pulling at the ſtring. The tail *EF* (with a bullet *F* at the end) being always blown from the wind, keeps her head always towards the wind.

As the direction of the thread always paſſes through *C*, therefore the angle *ACH*, and conſequently *HCO*, will always be the ſame at all altitudes. And ſhe can never aſcend ſo high, till the angle of altitude *CHO* be equal to *ACH*. And hence it follows, that the leſs the angle *HCO* is made, the higher ſhe will riſe. And likewiſe the greater the wind is, or the lighter the kite, *cæteris paribus*, the higher ſhe will riſe.

210. After a like manner a machine as *ab* may be contrived, to keep at the top of a running water, being held by the ſtring *dc* tyed to a ſtone and ſunk to the bottom: *ab* is a thin board, *b* a piece of lead to ſink the end *b*, but the whole muſt be lighter than water, *cd* an iron pin fixed at *C*. Or the machine may have a looſe tail at *b*, heavier than water, as in the kite.

Ex. XXVII.

211. If *AB* is a machine to be moved by a power acting at *C* out of the machine, in direction *DC*. *DF*, *GI* two leavers within the machine, moveable about the two fixed fulcrums *E*, *H*.

Call the power at *C*, 1; then the force at *F* to move the leaver *GI*, is $\frac{DE}{EF}$. Then if the force at *F* be 1, that at the obſtacle *I* out of the machine is $\frac{GH}{HI}$. Therefore if the power at *C* be 1, the force acting againſt the obſtacle at *I*, or which is the ſame thing, the force urging forward the machine towards *B*, is $\frac{DE\times GH}{EF\times HI}$. But the power at *C* draws back the machine with the

4 force

force 1. Therefore the abſolute force urging forward the machine is $\frac{DE \times GH}{EF \times HI} - 1$ or $\frac{DE \times GH - EF \times HI}{EF \times HI}$. FIG. 211.

Note, if the force at F be 1, the force againſt H is $\frac{FI}{HI}$. But this is not the force urging forward the machine, but to tear her in pieces, or to ſeparate the fulcrums E, H, from one another.

If there had been three leavers, and the power at D; the third had been directed towards K, the way the machine goes; then the power 1 muſt be added to the force at I, and the whole is the force urging forward the machine.

Hence, if the abſolute direct force to move the machine be 1, the power applied at D, which is able to move it, will be $\frac{EF \times HI}{DE \times GH - EF \times HI}$. But if the power at D act within the machine, this power could only be $\frac{EF \times HI}{DE \times GH}$; ſince there is then no force to be deducted for drawing back the machine.

Ex. XXVIII.

DABH is a *wooden bridge*. *AC*, *AD*, *AB*, *BH*, *BO*, beams of timber. *DE*, *EL*, *SR*, *RH*, braces to ſtrengthen the angles *A*, *B*. The ſtreſs upon any of the angles, is, *ceteris paribus*, ſo much greater, as the angle is greater. But the ſtrength on any angle *A*, is as the perpendicular *AP*. 212.

Ex. XXIX.

AB a *ſailing chariot*. *CDEF* horizontal ſails, ſo contrived that the ſails *D* facing the wind may expand, and thoſe going from the wind may contract. The ſails are turned about by the wind coming from any coaſt. Theſe ſails turn the axis and trundle *GH*. And the trundle turns the wheel *IL* by the cogs in it. Therefore the chariot may move in any direction. *R* is a rudder to ſteer with. 213.

Suppoſe the chariot to go againſt the wind. Let *D* be the center of preſſure of the two ſails *C*, *D*, the wind blows on. And let the power, (that is the force of the wind acting againſt the ſails) be 1, then the force acting againſt the teeth in *IL*, is $\frac{GD}{OH}$. And this force being 1, the force at *L* is alſo 1. Therefore

FIG. 213. fore the power at D to the force at L, is as 1 to $\frac{GD}{OH}$; or as OH to GD. Now ſince the maſt is ſtrained by the power falling on the ſails, therefore by this power OH, the chariot is urged backward. And by the force at L which is GD, it is urged forward. Let R be the force of the wind upon the body of the chariot, together with the friction in moving. Therefore if GD is greater than the radius $OH+R$, the chariot will move forward againſt the wind; if leſs, backward. But if they be equal, it will ſtand ſtill.

Ex. XXX.

214. FG a *chariot* or waggon to ſail againſt the wind. S the ſails of a windmill, turning in the order 1, 2, 3. As the ſails go round, the pinion A moves B, and the trundle C moves D, which has both teeth and cogs. D by its teeth moves E; and the trundle E fixt to the axle-tree carries round the wheels H, I, which move the waggon, in direction HG.

The ſails are ſet at an angle of 45°, ſo the force to turn them, and the force in direction of the axis, will be equal. This waggon will always go againſt the wind, provided you give the ſails power enough, by the combination of the wheels. But then her motion will be ſo much ſlower.

Ex. XXXI.

215. Let AB be part of a *rope*, *cd*, *cd*, &c. the particular ſtrands running about in a ſpiral manner. Let FH be the axis of the rope, the angle GEH or HFK the obliquity of the ſtrands. Draw KH, GH parallel to FG, FK, and draw GEK. Then the tenſion of the rope in direction HF, is to the ſtreſs on all the ſtrands in direction FG; as FH to $FG+FK$ or $FG+GH$, that is, as EF to FG. Therefore the abſolute force by which the rope is ſtretched, is to the ſtrain or ſtreſs upon all the ſtrands, or upon the twiſted rope; as FE is to FG; and ſo is the length of any part of the rope, to the correſpondent length of a ſtrand.

Hence ropes the leaſt twiſted are ſtrongeſt and bear the moſt weight; and the harder they are twiſted the ſooner they will break. And for the ſame reaſon if they be double twiſted, they will be weaker ſtill. But as it is very difficult to make all the fibres pull equally without twiſting, and impoſſible to make a rope hold together without it. Therefore it is neceſſary,

it

it have as much, as to prevent the fibres from drawing out; FIG.
and a ſmall degree will not much impair its ſtrength. A rope 215.
conſiſting of ſeveral ſtrands, is thicker when twiſted, than when untwiſted.

Ex. XXXII.

ABC a *ſyphon* or crane. If the ſhorter end *AB* be immerſed 216.
in a veſſel of water *AD*, then by applying the mouth to the end *C*, and ſucking till the water comes, it will continue to flow out at the end *C*, as long as that end is lower than the ſurface of the water at *D*. If there be a mouth-piece at *E*, then ſucking at *E* (whilſt the end *C* is ſtopt with the finger) will make the water flow, when the finger is taken off. And when the water is begun to flow, the hole at *E* ſhould be ſtopt up, or elſe the water will flow no longer, than till the ſurface at *D* be as low as *E*.

The reaſon of its flowing is this: the perpendicular height of the column of water *BC* being greater than that of *BD*; the preſſure at *C* is greater than at *D*; and the preſſure of the atmoſphere being the ſame at *D* and *C*; therefore the greater weight at *C* will make it flow out there; whilſt the preſſure of the atmoſphere at *D* forces more water up the tube *DB*; and ſo keeps it continually running as long as there is any water, and the end *C* continues lower than the ſurface at *D*. But if *C* is higher than *D*, the water will return back into *BD*. But if the height *DB* exceed the preſſure of the atmoſphere, which is 30 or 32 feet; then it cannot be made to flow out at the end *C*; or if there be a hole in the ſyphon higher than the ſurface at *D*, the air will get in, and the water will return through *BD*. Or if the ſyphon be very wide, the air will inſinuate itſelf into the end *G*, between the water and the tube, which will hinder it from running. To prevent which the end *C* may be immerſed into another veſſel of water, lower than the ſurface at *D*. If the ends of the ſyphon be turned up, as *F*, *G*, then the water will remain in the ſyphon, after it has done working, which in the other will all run out.

Ex. XXXIII.

CDLF a veſſel of water; *AB* a tube open at both ends, and 217.
about $\frac{1}{8}$ inch diameter. *AE* a quantity of mercury put into the tube. Then ſtopping the end *B*, let the other end *A* be immerſed deep enough in the water. Then opening the end *B*, the mercury will ſink ſo deep in the tube, till the height of

FIG. 217. the water AB be 14 times the height of the mercury AE: and then the mercury will be at reſt.

For the ſpecific gravities of water and mercury being 1 and 14, the column of water AB will be equal in weight with the column of mercury AE. Therefore the preſſures at A being equal, they will ſuſtain one another.

Ex. XXXIV.

218. A, B, are two *barometers*; *ed* is a tube, its bore ¼ or ⅓ inch diameter, at leaſt, cloſe at top, and communicating with the veſſel C, with mercury in it. C is open to the external air. The uſe of this inſtrument is to ſhew the weight of the atmoſphere, and its variations. This tube and veſſel with mercury, is put into a frame, and hung perpendicular. Near the top of the tube is placed a ſcale of inches, by which the height of the mercury in the tube is known, and likewiſe a ſcale for the weather. At the top of the tube above the mercury is a vacuum. Now the atmoſphere preſſing upon the ſurface of the mercury at C, keeps the mercury ſuſpended at the height d in the tube, which therefore will be higher or lower according to the weight of the atmoſphere. The height of the mercury in the tube is generally 28, 29, or 30 inches; ſeldom more. If any air get into the tube it ſpoils the machine. Leſt the quickſilver ſtick to the glaſs, it is good to drum a little with the fingers upon it, in making any obſervation.

Rules for obſervation of the weather.

1. The riſing of the mercury preſages fair weather. It riſes and ſtands higheſt in ſerene, ſunſhiny, droughty weather: and in calm froſty weather it generally ſtands high. In thick foggy weather it often riſes.

2. The falling of the mercury denotes foul weather. It generally falls or ſtands low, in rainy, windy, or ſnowy weather.

3. In windy weather the mercury ſinks loweſt of all; and riſes faſt after ſtorms of wind.

4. In very hot weather the falling of the mercury foreſhews thunder.

5. In winter, the riſing foretels froſt; and falling in froſty weather foretels thaw.

6. In continual froſt, the riſing preſages ſnow. At other times, it generally falls in ſnowy weather.

7. When

7. When the mercury riſes after rain, expect ſettled ſerenity; FIG.
if it deſcends after rain, expect broken ſhowery weather. 218.

8 When foul weather happens ſoon after the falling of the mercury, or fair weather after its riſing, expect but little of it.

9. In foul weather, riſing faſt and high, and continuing to 2 or 3 days, before the foul weather be quite over, expect a continuance of fair weather to follow.

10. In fair weather falling faſt and low, and continuing 2 or 3 days before the rain comes, expect a great deal of wet, and probably high winds.

11. Unſettled motion of the mercury denotes unſettled weather.

12. The greateſt height of the mercury is upon eaſterly and north-eaſterly winds.

13. The alterations are greater in northerly parts, than in the more ſoutherly; and there is little or no variation within the tropics.

Ex. XXXV.

ABE is an *artificial fountain*. *A* is an open veſſel, *B* a cloſe 219.
one; *E* may be made cloſe by ſtopping the hole *C*; theſe veſſels all communicate by the tubes *F*, *G*. The tube *F* reaches near the top of *E*, and the tube *G* near the bottom of *B*. Pour water into *C* almoſt to the top of the pipe *F*; and ſtop the hole *C*. Then pour water into *A* which will run down into *B*. Then open the cock *D*, and the water will ſpout up to the height of *AB* above *D*. For the air in *B*, *F*, *E* is condenſed by the weight of the column of water *AB*; and its preſſure on the water in *C*, is equal to the weight of that column; and will therefore make the water ſpout to that height above the water in *C*, nearly. But the pipe leading to *D* muſt be turned curve.

Ex. XXXVI.

AB is a *dart* or an *arrow*; at *A*, 3 or 4 feathers are placed 220.
nearly in planes paſſing through the arrow. If the feathers were exactly in this plane, the air could not ſtrike againſt the feathers, when the arrow is in motion. But ſince they are not ſet perfectly ſtrait, but always a little aſlant; whilſt the arrow moves forward, the air ſtrikes the ſlant ſides of the feathers; by which force the feathers are turned round, and with the feathers the arrow or reed. So there is generated a motion about the axis of the arrow; which motion will be ſwifter as they ſtand more aſlant. This motion is like the motion of the

FIG. 220. fails and axle of a windmill, turned round by the wind. The head *B* is made of lead or iron, and will therefore go foremoſt in the air; and the feathered end *A* the hindmoſt, as being lighter. An arrow will fly about 60 yards in a ſecond.

Ex. XXXVII.

221. *AB* is a veſſel which keeps its liquor till filled to a certain height; and if filled higher, lets it all run out. *EFG* is a crooked pipe, or crane open at both ends. If water be poured into the veſſel, it will continue in it till it riſes above *F*, and aſcend to the ſame height in the pipe *EF*. But riſing above *F*, the preſſure at *E* will make it run out through the pipe *EFG*, till the ſurface of the fluid deſcends as low as *E*. This is ſometimes called *Tantalus's cup*. The funnel *EFG* may be put in the handle of this cup, which will look neater.

Ex. XXXVIII.

222. *BC*, *CG* are two *bones* of an animal, moveable about the joint *FK*, by help of the muſcle *KD*. The joints of animals are either ſpherical or circular, and the cavity they move in is accordingly either ſpherical or circular. And the center of motion is in the center of the ſphere or circle, as at *C*. Let *W* be a weight hanging at *B*, and draw *CP*, *CK* perpendicular to *BW*, *KD*. Then if the weight *W* be ſuſpended by the ſtrength of the muſcle *KD*, it will be as *CK* : *PC* :: *W* : tenſion of the muſcle *KD*.

The bone *BC* is moved about the joint *FK*, by the ſtrength of the muſcle *KD*. For when the muſcle is contracted, the point *K* is moved towards *D*; and the end *B* towards *E*, about the immoveable center of motion *C*. The ſtrength of the muſcles is ſurprizingly great.

Borelli (in his Book, de Motu Animalium, Part I. *Prop.* XXII.) computes the force of the muſcles to bend the arm at the elbow; and ſays, a ſtrong young fellow can ſuſtain at arms end a weight of 28 lb. taking in the weight of the arm. And he finds the length of *CB* to *CK* to be in a greater proportion than that of 20 to 1. Whence he infers the ſtrength of theſe muſcles to be ſo great, as to bear a ſtretch at leaſt of 560 lb.

It is evident that all animal bodies are machines. For what are the bones but leavers, moved by a certain power placed in the muſcles, which act as ſo many ropes, pulling at the bones, and moving them about the joints? Every joint repreſenting

 the

the fulcrum or center of motion. What are all the vessels but tubes which contain fluids of different sorts, destined for the use or motion of the several parts of the machine? And which by opening or shutting certain valves, let out or retain their contents as occasion requires; or convey them to distant places, by other tubes communicating therewith. And therefore all these motions of an animal body are subject to the general laws of mechanics. FIG. 222.

Ex. XXXIX.

The motion of a *man*, walking, running, *&c.* will easily be accounted for. Let us first suppose a man sitting in a chair; he cannot rise from his seat, till by thrusting his head and body forward, and his feet and legs backward, the line of direction, or the perpendicular from the center of gravity, pass through his feet, as the base. Likewise when we stand upon our feet, the line of direction must fall between our feet; otherwise we cannot stand, but must fall down towards the side the center of gravity lies on. And when a man stands firm upon his feet, his legs make an isoceles triangle, the center of gravity lying between them. And then he is not supported by the strength of the muscles, but by the bones of the legs and thighs, which then stand in a right line with one another. 223.

When a man *AC* endeavours to walk, he first extends his hindmost leg and foot *S* almost to a right line, and at the same time bends a little the knee *H* of his fore leg. Thus his hind leg is lengthened, and his fore-leg shortened; by this means his body is moved forward, till the center of gravity *V* falls beyond the fore-foot *B*; and then being ready to fall, he presently prevents it, by taking up the hind-foot, and by bending the joints of the hyp, knee, and ancle, and suddenly translating it forward to *T*, beyond the center of gravity: and thus he gains a new station. After the same manner by extending the foot and leg *HB*, and thrusting forward the center of gravity beyond the foot *S*, and then translating the foot *B* forward, he gains a third station. And thus is walking continued at pleasure.

His two feet do not go in one right line, but in two lines parallel to one another. Therefore a man walking has a libratory motion from one side to the other; and it is not possible to walk in a right line.

Walking on plain ground is easy, pleasant, and performed with little labour. But in going up hill is very laborious, by reason of the great flexure of the joints required to ascend, and their

FIG. 223. their suffering more stress from the weight of the body in that position. Descending down hill is, for the same reason, more laborious than walking on plain ground, but not so bad as ascending.

The walking of birds is not unlike that of men; only their weight is intirely supported by the strength of the muscles; since their joints are always bent. Also their feet go in two parallel lines.

A man in walking always sets down one foot before the other be taken up: and therefore at every step he has both feet upon the ground. But in running he never sets one down till the other be up. So that at each step he has but one foot upon the ground, and all the intermediate time none. A good footman will run 400 yards in a minute.

Ex. XL.

224. When a *beast* stands, the line of gravity must fall within the quadrilateral made by his 4 feet. And when he walks, he has always 3 feet on the ground, and one up. Suppose he first takes up the hind foot *C*. Before he does this, by extending his leg backwards, he thrusts forward his body and the center of gravity; then taking up the foot *C* he moves it forward to *F*. Then he immediately takes up the fore-foot *B* on the same side, and carries it to *H*; then he takes up the hind foot *D*, and translates it forward; and then the fore-foot at *A*; then *F* again, and so on.

When he trots, he takes up two together, and sets down two together, diagonally opposite.

When he gallops, he takes up his feet one by one, and sets them down one by one; though some animals strike with the two fore feet nearly at once, and the two hind feet near at once; and have not above two feet on the ground at once. A good horse will run half a mile in a minute.

Animals with six or more feet, take up the hindmost first; then the next, and then the next in order, to the foremost, all on one side; and after that, all the feet on the other side in the same order, beginning at the last. If they were to take up the foremost first, the animals would move backward.

Ex. XLI.

225. *AD* is a *bird* flying in the air, by help of the wings *F*, *T*; and the tail *C*. The structure of their wings are such, that in striking

ftriking downward, they expand to their greateft breadth, and become almoft two planes, being fomething hollow on the under fide. And thefe planes are not then horizontal, but inclined, fo that the back part *K* is higher than the fore part *DFG*. But in moving the wings upward, to fetch a new ftroke, they go with the edge *DFG* foremoft, and the wings contract and become hollow. Their bodies are fpecifically lighter than men or beafts. Their bones and feathers are extremely porous, hollow, and light. The mufcles, by which their wings are moved downwards, are exceeding large, being not lefs than a fixth part of the weight of the whole body. When a bird is upon the ground and intends to fly, he takes a large leap; and ftretching his wings right from his body, he ftrikes them downwards with great force, by which they are put into an oblique pofition; and the refiftance of the air acting ftrongly againft them by the ftroke, impels them, and the bird, in a direction perpendicular to their planes: which is in an oblique direction, or partly upwards, and partly horizontally forward: the part of the force tending upwards is deftroyed by the weight of the bird; the horizontal force ferves to carry him forward. The ftroke being over, he moves his wings upwards, which being contracted, and turning their edges upward, they cut through the air without any refiftance, and being fufficiently elevated, he takes a fecond ftroke downwards, and the impulfe of the air moves him forwards, as before. And fo from one ftroke to another, which are only like fo many leaps taken in the air. When he has a mind to turn to the right or left, he ftrikes ftrongly with the oppofite wing, which impels him to the contrary fide. The tail acts like the rudder of a fhip, except only that it moves them upwards or downwards inftead of fideways; becaufe its plane is horizontal. If a bird wants to rife, he puts his tail in the pofition *LH*; or if he would fall, into the pofition *LI*. Whilft it is in the horizontal pofition *LC*, it keeps him fteady. A bird can by fpreading his wings continue to move horizontally for fome time, without ftriking. For having acquired a fufficient velocity, by keeping his wings parallel to the horizon, they meet with no refiftance; and when he begins to fall by his weight, he can eafily fteer himfelf upward by his tail; till his motion be almoft fpent, and then he muft renew it by two or three more ftrokes of his wings. When he alights, he expands his wings and tail full againft the air, that they may meet with all the refiftance poffible. The center of gravity of a bird is fomething behind the wings; to remedy which,

FIG. 225.

FIG. 225. which, they thruſt out their head and neck, in flying; which carries the center of gravity more forward.

It is impoſſible that ever men can fly by the ſtrength of their arms. For their pectoral muſcles are vaſtly too weak to ſupport ſuch a weight. For in a man they are not the 60th part of the reſt of the muſcles of the body: but in a bird they are more than all the others put together.

Some birds will fly 1000 yards in a minute.

Ex. XLII.

226. *AB* is a *fiſh* ſwimming; which he does by help of his fins and tail. A fiſh is nearly of the ſame ſpecific gravity as water; and moſt fiſh have a bladder *L*, which they can expand or contract, and ſo make themſelves lighter or heavier than water, in order to riſe or fall in it. The muſcular force by which the tail is moved is very great. The direct motion of a fiſh is by means of his tail *BCD*, moving from one ſide to the other, with a vibrating motion, which he performs thus. Suppoſe his tail in the poſition *FG*, being about to move it ſucceſſively to *H*, *I* and *K*; he turns the end *G* oblique to the water, which being moved ſwiftly through it in that poſition, the reſiſtance of the water acts obliquely againſt his tail, and moves him partly forward and partly laterally. The lateral motion is corrected the next ſtroke the contrary way; but the progreſſive motion is continued always forward. When his tail is arrived at *K*, he turns its obliquity the contrary way, that in moving back to *G*, it may ſtrike the water in the ſame manner as before. And thus he makes one ſtroke after another, and moves forward thereby as far as he pleaſes. The oblique poſition of his tail is moſtly owing to the elaſticity of his tail, which by bending, is put into that form by the reſiſtance of the water. They can exert a very great force with their tail; and which is neceſſary, to overcome the reſiſtance which their bodies meet with in the water. By help of the tail they alſo turn to one ſide, by ſtriking ſtrongly with it on that ſide, and keeping it bent, which then acts like the rudder of a ſhip. The fins of a fiſh ſerve to keep him upright, eſpecially the belly-fins *E*, which act like two feet; without them he would ſwim with his belly up; for his center of gravity lies near his back. His fins alſo help him to aſcend or deſcend, by expanding or contracting them, as he can with pleaſure, and ſo putting them in a proper poſition. His tail will alſo help him to riſe and fall, by inclining it obliquely, and turning it a little from an erect poſition,

tion, to one side. Fish can swim but slow, yet some of them will swim 70 or 80 yards in a minute; but they soon tire. FIG. 226.

Brutes can swim naturally, for they are specifically lighter than water; and require to have but a small part of their head out, for breathing. Also they naturally use their legs in swimming, after the same manner as they do when walking.

Birds swim very easily, being much lighter than water; and readily move themselves along with their web feet.

Men cannot swim naturally, though they are specifically lighter than water. For their heads are very large, and require to be almost all out of the water for breathing. And their way of striking has no relation to that of walking. Men attain the art of swimming by practice and industry. And this art consists in striking alternately with the hands and feet in the water, which like oars will row him forward. When he strikes with his hands, he neither keeps the palms parallel nor perpendicular to the horizon, but inclined. And his hands striking the water obliquely, the resistance of the water moves him partly upward, and partly forward. Whilst his hands are striking, he gradually draws up his feet; and when the stroke of his arms is over, he strikes with his feet, by extending his legs, and thrusting the soles of his feet full against the water. And while he strikes with his legs, he brings about his arms for a new stroke, and so on alternately. He must keep his body a little oblique, that he may more easily erect his head, and keep his mouth above water.

After the same manner may the motions, velocities, powers, and properties of any machine be explained and accounted for, by mechanical principles. I shall proceed to lay down a short description of several other machines, without being so particular in the calculation of their powers and forces. The mechanism of which being understood, will assist the invention of the practical mechanic, in contriving a machine for any use.

Ex. XLIII.

AB a machine to raise a weight, and stay it in any position. 227.
CD a roller turned by the handle *E*. To the roller is fixed the racket wheel *F*. *GH* is a catch made of metal, moveable about *H*, and forked at the end *G*, where it falls into the teeth of the wheel *F*. As the roller is turned, the weight is raised by the rope *IKL*. And the catch *G*, slides freely over the teeth of the wheel, till the machine is stopped, and then the catch *G* falling

FIG. 227. in between the teeth, keeps the wheel fixed there, that it cannot turn back again with the weight.

Ex. XLIV.

228. *KI* is a machine to raiſe or depreſs the leaver *GH*, moveable about an axis at *H*, and to keep it at any poſition: *bf* a ſtring fixed at *f* and *a*, and going in the order *abcdef*, through the holes *b*, *c*, *d*, *e*, of the runner *IK*, which being put up or down, raiſes or lowers the beam *GH*, at pleaſure. And more or fewer holes may be made in the piece *IK*, as occaſion requires.

Ex. XLV.

229. *CD* another machine to ſtay a weight in any poſition. This is only a cylinder of wood, upon which is cut a channel for the rope to go in. If the weight *B* be lifted up, and *A* pulled down, then *B* will remain in any given poſition, by the friction of the cylinder and rope. And there may be taken as many turns of the rope about the cylinder, as there is occaſion for.

Ex. XLVI.

230. *C* is a clock weight carrying the two wheels *A* and *B*. *D* the counterpoiſe. *F* a pulley. *ADBFA* an endleſs cord. When the weight is down, draw the cord *G*, till the weight *C* riſe to the top; then the catch *e* keeps the wheel *A* from turning backwards. This may be ſerviceable for other uſes, beſides moving a clock.

Ex. XLVII.

231. *ADB* a machine for reckoning the number of ſtrokes or vibrations made. *DH* is a wheel moving about a fixed axis, upon the neck of which axis goes a braſs ſpring *L*, to keep the wheel from ſhaking. *AB* a piece of wood or metal, cut away between *I* and *K* to receive the wheel. The plane of the piece *AIKB* is perpendicular to the plane of the wheel. *FG* are two ſtaples, to guide the motion of the piece *AB* back and forward. When the piece *AB* is moved from *A* towards *B*, the edge at *I* catches the tooth *C*, and ſliding along the edge, moves the wheel about in direction *CD*; this brings the tooth *E* to the edge *K*. And when the piece *AB* is moved back from *B* to *A*, the edge at *K* ſliding down the tooth *E*, moves the

wheel from *E* towards *H*; which brings another tooth before FIG. the edge *I*; so that at every motion of *AB* back and forward, 231. the wheel is moved the breadth of one tooth. And if the teeth be numbered, the index *M* will shew when the wheel has made one revolution.

Ex. XLVIII.

ABED a machine moving one circle within another, concentrical to it. *ABC* represents a flat ring of brass; and *abc* a smaller concentric ring lodged in a circular grove, turned within the larger, and kept in the grove by three small plates of brass *A*, *B*, *C*, fixed to the outward ring, and reaching over the edge of the inner one. Upon the inner ring is fixed a concentric arch of a wheel *de*, having teeth in it, which are driven round by the threads of an endless screw *DF*, turning in a collar at *E*, and upon a point at *F*, both fixed to the outward ring. By this mechanism, any point of the circle *abc*, may be set to a given point of the circle *ABF*, by turning the screw *DEF*. 232.

Ex. XLIX.

ABE is a *crane* for hoisting goods up. *AF* is a double wheel, 233. within which a man *A* walks, and by his weight raises the weight *W*, by help of the rope *FBEW*, which goes round the axis of the wheel at *F*. At *D* are two pullies, one vertical, and one horizontal; the vertical pulley facilitates the motion of the rope in hoisting the weight; and the horizontal one serves for the rope to run on, when the crane *CDE* and weight *W* are drawn aside, by the rope *GH*, in order to be lowered. *CDE* moves about the axis *BC*. At *E* is another pulley, for the rope to run on. If the rope *FBEI* go about a pulley at *I*, and be fixed with its end at *E*, the crane will lift twice the weight.

Ex. L.

AB a *sailing chariot*. This is driven by the wind, by help of 234. the sails *C*, *D*. *R* is the rudder. The wheels must be set at a greater distance, or the axle-tree made longer, than in common chariots, to prevent overturning. Sailing chariots are proper for large planes and champagne countries; and are said to be frequent in *China*.

FIG.

Ex. LI.

235. *BE* is a *ſmoak jack*. *AB* is a horizontal wheel, wherein the wings or ſails are inclined to the horizon. The ſmoak or rarified air moving up the chimney at *B*, ſtrikes theſe ſails, which being oblique, are therefore moved about the axis of the wheel, together with the pinion *C*, of 6 leaves. *C* carries the toothed wheel *D* of 120 teeth, all theſe are of iron. *E* a wooden wheel 4 or 5 inches diameter; this carries the chain or rope *F*, which turns the ſpit. The wheel *AB* muſt be placed in the ſtrait part of the chimney, where the motion of the air is ſwifteſt; and that the greateſt part of it may ſtrike upon the ſails. The force of this machine is ſo much greater, as the fire is greater. The ſails *B* are of tin, 6 or 8 in number, placed at an angle of $54\frac{1}{2}$ degrees.

Ex. LII.

236. An engine to make a *hammer ſtrike*. *W* the water wheel, 7 or 8 yards diameter. *D*, *D*, the floats. *BC* the axle, 3 or 4 feet diameter. *H* the hammer 3 or 4 hundred weight, moveable about the axis *OP*. *I*, *K* four coggs in the axis, lifting up the hammer as the axis goes round, that it may fall on the anvil *A*. *FG* a beam of wood acting as a ſpring, to give the greater force to the hammer. *MN* the courſe of the water down an inclined plane. *M* the place where it iſſues out. *LM* the perpendicular height of the water, 3 yards. All the machine except the water wheel is within the houſe.

237. A hammer may alſo be made to ſtrike thus; *A* is the hammer moveable about the point *C*. *G* the axle of a water wheel, in which axis are the pins *F*, *E*, &c. As the wheel and axle goes about from *F* towards *E*, the pins *F*, *E* thruſt down the end *B*, and raiſe the end *A* of the hammer. And when the end *B* goes off the pin, the hammer falls upon the anvil *D*.

Ex. LIII.

238. *I I* a crooked axis or elbow for the *ſuckers of pumps*. *IK* the peſtle or chain of the ſucker. Upon the axis is the lantern *EF*, which is turned by a great wheel, carried either by water, or men or horſes. The peſtles *IK* riſe and fall alternately, as the lantern *EF* goes about; and each gives one ſtroke of the pump for one turn of the lantern. Place pullies or rolls at *a*, *b*, *c*, *d*, for the chain *IK* to work againſt, when it goes out of its perpendicular

dicular position; by the obliquity of the motion of the cranks *I*, *I*. FIG. 238.

Ex. LIV.

ABCD a particular combination of *pullies*. *T*, *T*, *T* are posts to which the tackles are fixed. *S*, *S*, *S* are stays to keep them erect. If the power at *A* be 1, that at *B* is 3; at *C*, 9; and at *D* 27, where the weight is placed. 239.

Ex. LV.

A, *B* are two *bellows* going by water, and blowing alternately, but neither of them with a continual blast; *W* the water wheel. *DE* the direction of the water. *FG* the axis of the wheel; *a*, *a*, *&c.* 4 cogs of wood in the axis, forcing down the end of the bellows *A*. *bb*, *&c* 4 cogs forcing down the end of the bellows *B*. *LM*, *NI* two rods of iron fastened to the bellows and to the leaver *MN*, and moveable about the pins *M*, *N*. *SP* a piece of timber moveable about *S* and *P*. *OP* a beam serving for a spring, lying over the piece of timber *QR*. As the wheel and axle turns round, a cog *b* forces down the end of the bellows *B*, and makes it blow; this pulls down the end *N* and raises the end *M* of the leaver *MN*, which raises the bellows *A*. And when the cog *b* goes off, the bellows *B* cease blowing; and a cog *a* forces down the bellows *A*, and makes it blow; and at the same time raises the bellows *B*. And thus the cogs *a*, *b* alternately force down the bellows *A*, *B*, and make them blow in their turns. *H* is the hearth or fire. 240.

A pair of bellows may be moved by water thus; *A* is a water wheel, carried by the water at *W*. *CD* a rod of iron going on the crooked axle-tree of the wheel; *DF* a leaver moveable about *E*. *FG* a chain going to the bellows *B*. *I* a weight. As the wheel goes about, the ends *D* and *F* of the leaver *DF*, rise and fall; which motion raises the bellows, and the weight *I* carries them down again. 241.

Ex. LVI.

AB is a wheel with teeth, and a *roller* to draw up any weight. *H*, *H*, *H* the handles, which may be wrought by two or three men. 242.

But the easiest and simplest rollers for common use are such, as *C* and *D*. In these, as 30lb. is the weight to be raised, so 243. 244.
must

FIG. muſt the radius of the axle be to the length of the handle, for a man to work it.

30. If a given weight P raiſes another weight W, on ſuch a machine as fig. 30. it will generate the greateſt motion poſſible in a given time; when the diameters AB, EF, and weight W are of ſuch quantities, that $W = \frac{\frac{2}{5}P \times AB}{EF}$, or when $EF = \frac{\frac{2}{5}AB \times P}{W}$. For then the motion will be greater, than if any one (W, AB, or EF) be altered, the reſt remaining the ſame.

36. And in ſuch a machine as fig. 36; the greateſt motion will be generated in W; if you make, as velocity of W : velocity of P :: as $\frac{2}{5}$ P : to W.

Ex. LVII.

245. An engine to *drive piles*. A the rammer, drawn up by the rope BCD going over the pulley C. DN, DN ſeveral ſmall ropes for ſeveral men to pull at. M the pile. EF a brace and ladder to go up. The rammer A is bound at bottom with iron, leaſt it ſplit. And has two tenants on the backſide to keep it in the groves, made in the upright puncheons G, H, by which its motion is directed. The rammer is raiſed to the top, by men pulling at the ropes DN, and then letting go, all at once, it falls upon the head of the pile M.

Old piles are drawn out, by ſtriking gently upon their heads, whilſt they are pulled by a ſtrong rope ſtretched.

Ex. LVIII.

246. ACL a pair of *ſmith's bellows*. AL, BL, CL are three boards, the middle board BL divides the internal ſpace into two parts. In the middle board is a valve S opening into the upper part; and in the loweſt board is another valve T opening into the under part, The pipe P communicates only with the upper cavity. DE a leaver moveable about the axis GH. At I a weight is laid upon the upper board to make it fall. The bellows is fixed in the frame MK, by two iron pins, which are faſt in the middle board. And the pipe P lies upon the hearth. When the end E is pulled down by the rope EF, the end D is raiſed, and the rope or chain DR raiſes the lower board CL; this ſhuts the valve T and opens S, and the air is forced into the upper cavity, which raiſes the upper board, and blows through the pipe P. And when E is raiſed, the boards A and C deſcend, and the valve S ſhuts, and T opens. And the weight I forces the

the air ſtill out of the pipe, whilſt more air enters in at the FIG.
valve *T*; which, when *C* aſcends, is forced again through the 246.
valve *S* as before. And thus the bellows have a continual blaſt.

Ex. LIX.

An engine to raiſe water. *LMOI* a great horizontal wheel. 247.
ABP the axis, *P* the pevot or ſpindle it turns upon. *OQI* the waves of the great wheel. *QR* a ſmall wheel perpendicular to the horizon, and placed under the edge of the great wheel: this wheel is moveable about the center *C*, in the end of the leaver *EFC*, which is moveable about the center *D*; *EF* the arch of a circle, whoſe plane is perpendicular to the horizon, and in the plane of the wheel *QR*. *EG* the chain of a pump.

Whilſt the great wheel is turned by the leaver *NA*, from *O* towards *I*, the wave *Q* preſſes down the wheel *QR*, and raiſes the end *E*, which draws up the water in the pump *G*. But when the deepeſt part of the wave is paſt the wheel *QR*, the wheel then riſes up into the hollow *S*, and then the chain *EG* deſcends, till the next wave raiſes it again. And thus every wave makes a ſtroke of the pump.

The wheel *QR* is placed there only to avoid friction, and ſo that a perpendicular to its plane may paſs through *AB*. If the number of waves be odd, and another pump wheel and leaver be placed diametrically oppoſite, on the other ſide of the great wheel, then theſe acting by turns will keep the motion uniform, and the power at *N* will always act equally.

Ex. LX.

BFG a *capſtain*, to draw great weights. *BC* the axis, which 248.
is driven about by men acting at *A*, *A*, by help of the leavers *AB*, *AB*. Here muſt only be 3 or 4 ſpires of the rope *DCE* folded about the axis *BC*; for the axis could not hold ſo much rope as there is ſometimes occaſion for. And to hinder the rope from ſlipping back, a man conſtantly pulls at *E* to keep it light. And the axis is made conical, or rather angular at the bottom *C*, to keep the rope from going any lower, whilſt the capſtain goes about.

Ex. LXI.

AL is a *jack* to lift great weights. *E* is a pinion upon the 249.
axis *FG*, *GC* a toothed wheel, and *D* a pinion upon the ſame

axis,

FIG. axis, working in the teeth of the rack *AB*. The whole is incloſed in a ſtrong caſe *KL*, all of metal. The handle *GHI*
249. goes on the axis *FG* on the backſide of the caſe.

When a weight is to be lifted, the forkt end *A* is put under the weight; then turning the handle *HI*, the pinion *E* moves the wheel *GC*, with the pinion *D*; and *D* raiſes the rack *AB*, with the weight.

Ex. LXII.

250. An engine to raiſe and let fall two weights with contrary motions ſucceſſively, whilſt the moving power acts always one way. *GH* a great horizontal wheel. *N*, *M* two lanterns, ſo placed on the axis *AB*, that the great wheel can only work one of them at once. When the cog wheel *GH* is turned by the leaver *LI*, it turns the lantern *M*, and raiſes the bucket *E*, whilſt *F* deſcends. Then *E* being raiſed, move forward the axis *AB*, that the lantern *M* may leave the wheel, and *N* come to it. Then the great wheel moving the ſame way as before, will now work upon *N*, and turn the axis the contrary way, and raiſe the bucket *F* whilſt *E* deſcends. Which done, move the axis back towards *A*, and you will again riſe the bucket *E*, and ſo on.

This may alſo be performed by placing the lanterns *M*, *N*, ſo that the great wheel may work them both at once; but making them moveable about the axis *AB*, then there muſt be a pin to faſten either of them to the axis; ſo that firſt one lantern, and then the other, being thus fixt to the axis, whilſt the other is looſe, the buckets *E*, *F* will aſcend and deſcend alternately.

Ex. LXIII.

251. A *mill for iron work*. *AB* the ſlitting mill, *CD* the plate mill. *SP* the clipping mill. *E*, *F* are two great water wheels. After the water is paſt the wheel *E*, moving in direction *QW*, it comes about to the wheel *F*, in direction *XY*. The water wheel *E*, with the lantern *G* on the ſame axis, carries the ſpur wheels *M* and *H*, with the cylinders *B* and *D*. And the wheel *F* with the lantern *I*, carries the wheels *N* and *K*, with the cylinders *A* and *C*. The cylinders *A* and *B*, as alſo *C* and *D* run contrary ways about. And the cylinders *A* and *B* are cut into teeth, for ſlitting iron bars. *C*, *D* are 8 inches diameter; *A* and *B* about 12. And theſe cylinders may be taken out and others put in, and may be brought nearer to, or farther from one

one another, by help of screws, which screw up the sockets where the axles run. The axles of *N*, *I*, *K* lie all in one horizontal plane. And so does *M*, *G*, *H*. But the cylinders *A*, *B*, and also *C*, *D*, lie one above another. FIG. 251.

For making the plates; if a bar of iron be heated and made thin at the end, and that end put in between the cylinders *C*, *D*, whilst the mill is going; the motion of the cylinders draws it through, on the other side, into a thin plate. Likewise a bar of iron, being heated and thinned at the end, and put in between the toothed cylinders *A*, *B*; it is drawn through on the other side, and slit into several pieces, or strings. And then if there be occasion, any of these strings may be put through the plate mill with the same heat, and made into plates.

OPQ is the sheers for clipping bars of cold iron into lengths. *V* a cog in the axis of the water wheel. *OP* one side of the sheers made of steel, and moveable about *P*. The plane *LPR* is perpendicular to the horizon. When the mill goes about, the cog *V* raises the side *OP*, which as it rises clips the bar *TQ* into two, by the edges *SP*, *RP*. All the engine, except the water wheels *E*, *F*, is within the house.

Ex. LXIV.

AFC a *windmill* to frighten birds from corn or fruit. This is made of wood. The sails *F*, *F* a foot long, and their planes inclined to the axis *BC*, 45 or 50 degrees. The piece *B* goes upon the end of the axis *BC*, and is pinned fast on, and the sails and axis turn round together; and the axis goes through the board *AD*, and is kept from flying out of the hole, by the piece *B* pinned fast. The whole machine is moveable about the perpendicular staff *AG*, by which means the wind turns the mill about the axis *AG*, till the plane *AD* lies directly from the wind; and then the sails face it. At *S* is a spring to knack as it goes about; and the like on the other side. 252.

Ex. LXV.

An *anemoscope*, to show the turnings of the wind. *CD* is a weather cock of thin metal, fixt fast to the long perpendicular axis *DF*, which turns with the least wind upon the foot *F*, and goes through the top of the house *RS*. To this axis is fixt the pinion *A*; which works in the crown wheel *B*, of an equal number of teeth. The crown wheel is fixt on the axis *PI*, on the end of which the index *NS* is fixt. The axis *PI* goes 253.

 through

FIG. 253. through the wall *LM*, againſt the wall is placed the circle *NESW*, with the points of the compaſs round it. Then if the vane *CD* be ſet to the north, and at the ſame time the index *SN* fixt on the axis *PI*, to point at *N*. Then however the wind varies, it will turn the vane *CD*, and pinion *A*; and *A* turns the wheel *B* with the index *N*; ſo that the index will always be directed to the oppoſite point of the compaſs to the vane *DC*; or to the ſame as the wind is in.

Ex. LXVI.

254. *DEF* is a *rag-pump*, or *chain-pump*. *EF* the barrel, *CD* the roller. *GH* an endleſs chain, to which are fixt ſeveral leather buckets *I*, *I*, hollow on the upper ſide that aſcends. *AB* the handle. The uſe of this is to cleanſe foul waters from dirt and rubbiſh. The roller is ribbed to hinder the chain from ſlipping, in working. When the roller is turned, it draws up the chain through the pump, with whatever is in the water, and diſcharges it at the top. Inſtead of the roller *CD*, a wheel like a trundle may be uſed, called the rag wheel.

Ex. LXVII.

255. A *dyer's and fuller's mill*. *A* the great wheel carried about by horſes. This turns the trundles *B*, *C*, *D*; together with *E*, *F*, *G*. Then *E* turns the cog wheel *H*, with the axis *IK*, and the croſs pieces *L*, *L*; 1, 1, &c. are pullies or rollers. *MN*, *MN* wooden beaters, turning upon an axis paſſing through *N*, *N*. Whilſt the axis *IK* turns about, the end *b* ſlides along the pulley 1; and falling off, the part *M* ſtrikes againſt the cloth in the trough at *O*, *O*. The lantern *F* carries the cog-wheel *P*, and the cranks *Q*, *Q*; which work the pumps *T*, *T*, by help of the leavers *RS*, moveable about *a*. The trundle *G* carries the cog-wheels *V* and *W*, and *W* carries the trundle *X*, with the piſton *Y* that grinds the indigo in the veſſel *cd*; from whence it flows to the veſſel *Z*. The ends *m*, *m*, &c. of all the axles, run in pieces of timber going croſs the mill, and faſtened to one another and to the walls of the houſe.

Ex. LXVIII.

256. A machine to empty ſtanding waters. This is no more than a large pipe or ſyphon *ABC*, being extremely cloſe and tight that no air can get in.

If

If the pool of water *DE* is to be emptied over the hill *DHG*; let the pipe be placed with its mouth *A* within the water *DE*, and the mouth *C* within the water *FG*, if the pipe be very large. Then ſtop up *A* and *C*, and fill the pipe with water by the cock *B* at the top. Then ſtopping the cock *B* very cloſe, open *A* and *C*; and the water will flow through the pipe from *DE* into *FG*, which may run over at *F*, at a ſmall height above *C*, and go away. FIG. 256.

Note, the end *C* muſt always be lower than *A*, and the height of the top *B* above *DE* muſt not exceed 11 yards; for if it do, the water will not flow. If the pipe be very ſtrait, the end *C* need not be immerſed in the water; but if large it muſt; or elſe the air will inſinuate itſelf into the pipe at *C*, and hinder the flux of the water.

Ex. LXIX.

EFGH is a coal ginn. *E* the cog-wheel 11 feet diameter, and 72 cogs: this carries the trundle *F*, near 2 feet diameter, and 12 rounds, together with the roll *G*, 4 feet diameter. *AH* is the ſtart 20 feet long. The axis *AB* runs upon the kevy-ſtock *C*. There are two croſs trees *IK*, at the top, through which the axis *AB* goes. Theſe croſs trees are ſupported by four poſts *KL* at the four corners. When the coals are to be drawn up out of the pit, two horſes are yoaked at *H*, and go round in the path *OQD*, and draw the wheel about. And whilſt the loaded corf *N*, is drawn up to the top of the ſhaft *M*, by the rope going round the roll; the empty one at the other end of the rope, is deſcending to the bottom. And the loaded corf *N* being taken off, and an empty one put on, the horſes are turned, and made to draw the contrary way about, till the other corf comes to the top loaded: and ſo as one corf aſcends, the other deſcends alternately. A corf of coals weighs about 5 hundred weight, and contains about $4\frac{1}{2}$ buſhels. A pit is 40 or 50 fathom deep. And 50 fathom of the rope weighs about 3 hundred weight. 257.

Ex. LXX.

A *worm jack* for turning a ſpit. *ABC* the barrel round which the cord *QR* is wound. *KL* the main wheel of 60 teeth. *N* the worm wheel of about 30 teeth cut obliquely. *LM* the pinion of 15 or 16. *O* the worm or endleſs ſcrew, on which are two threads or worms going round, and making an angle with the 258.

FIG. axis of 60 or 75 degrees. *X* the ſtud, *Z* the loop of the worm
258. ſpindle. *P* a heavy wheel or fly to make the motion uniform. *DG* the ſtruck wheel fixt to the axis *FD*. *S*, *S* ſeveral holes in the frame, to nail it to a board, which is to be nailed againſt a wall; the end *D* going through it *HI* the handle, going upon the axis *ET*, to wind up the weight when down. *R* are fixt pullies, *V* moveable pullies with the weight. The axis *ET* is fixt in the barrel *AC*, and this axis being hollow, both it and the barrel turn round upon the axis *FD*, which is fixt to the wheel *KL*, turning in the order *BTA*; but cannot turn the contrary way, by reaſon of a catch nailed to the end *AB*, which lays hold of the croſs bars in the wheel *LK*.

The weight, by means of the cord *QR*, carries about the barrel *AB*, which by means of the catch, carries the wheel *KL*, which carries the nut *LM* and wheel *N*, which carries the worm *O* with the fly *P*. Alſo the wheel *LM* carries the axis *FD* with the wheel *DG*, which carries the cord or chain that goes about the ſpit head (a wheel like *DG*) which turns the ſpit. The more pullies at *R* and *V*, the longer the jack will go; but then the weight muſt be greater.

The catch lies between the end *AB* of the barrel and the wheel *KL*, and is thus deſcribed: *ff* the barrel, *n* the main ſpindle: *dr* a tumbler moving eaſy on the center pin *a*, faſtened to an iron plate, nailed to the barrel: *b* a collar of iron, turning a little ſtiff on the ſpindle; from this proceeds the tongue *bc*, paſſing through the hole *c* in the tumbler: *r* the catch of the tumbler. Now whilſt the barrel with the catch is turned about, in the order *efg*, upon the axis *n*; the collar is drawn about by the tongue *bc*, which tongue acting backwards, turns the tumbler about the center *a*, and depreſſes the catch *r*. But the barrel being turned the contrary way, the tongue then acts towards *d*; this depreſſes the end *d*, and raiſes the catch *r*, which then takes the croſs bars of the main wheel, and ſtops the barrel. This catch would alſo ſerve for a clock, and is better than a ſpring catch, becauſe it makes no noiſe in winding up.

Note, the jack need not be placed ſo, that the axis *FD* be parallel to the ſpit; but any way it can conveniently. For it is no matter whether the chain croſſes or not.

Ex. LXXI.

259. *DAF* the *hydroſtatical bellows*. *AB*, *EF*, two flat boards of oak; the ſides *AE*, *BF*, of leather, joined very cloſe to the top and

and bottom, with ſtrong nails. *CD* a pipe ſcrewed into the piece of braſs, in the top at *C*. FIG. 259.

If a man blows in at the pipe *DC*, he will raiſe a great weight laid upon the board *AB*. Or if he ſtands upon the board *AB*, he may eaſily blow himſelf up, by blowing ſtrongly into the pipe *DC*. If water be poured in at *D*, till the bellows and pipe be full, the preſſure upon *AB* within, will lift as much weight upon the top *AB*, as is equal to a cylinder of water, whoſe baſe is *AB* and height *CD*.

Ex. LXXII.

ADM is a *water-mill* for grinding corn. *A* the *water-wheel* 16 feet diameter, *BC* its axis. *D* the *cog-wheel*, with 48 cogs. *O* a *trundle*, with 9 rounds; *LI* its axis. *M*, *N* the *ſtones* 6 or 7 feet diameter. The lower ſtone *N* is the *lyer*, being fixt immoveable upon beams of wood; and the upper ſtone is the *runner*, and is faſtened to the *ſpindle LI*, by a piece of iron called the *rind*, fixt in the lower ſide of the ſtone, to go ſquare upon the ſpindle; between which and the ſtone, there is room left for the corn to fall through upon the lower ſtone. The ſpindle goes through the lower ſtone, and is made ſo tight with a wooden *buſh*, as to turn round in it eaſily. The upper ſtone, with the ſpindle *LI*, is ſupported on the end *I*, upon a horizontal beam of wood *FE*, called the *bridge*: the end *F* being fixt, and the end *E* lying upon the beam *HG*, fixt at *G*, called the *brayer* or *bearer*. The end *H* is ſupported by the *lifting-tree HK*, by help of a wedge at *K*. By this means, the upper ſtone may be raiſed or lowered. For if *KH* be raiſed with the leaver *Kk*, the end *E*, the axis *LI*, with the ſtone *M*, and the piece *GH* are all raiſed, and may be fixed there by the wedge *K*. Thus the ſtones may be ſet as near or far off as you will. The lower ſtone is broader than the upper ſtone, and is feathered, or cut into ſmall channels, to convey the flour out; and is encloſed with boards all around as *ab*, cloſe to the lower ſtone, and above the edge of the under one, to keep the meal in. And through one ſide of the boards is a hole called the *mill-eye*, through which the meal runs out into a *trough*. 260.

The ſurfaces of the mill-ſtones are not flat, but conical; the upper one an inch hollow, the under one ſwells up $\frac{3}{4}$ of an inch: ſo the two ſtones are wider about the middle, and come nearer and nearer towards the outſide; which gives room for the corn to go in, as far as $\frac{2}{3}$ of the radius, where it begins to be ground. The upper ſtone has a dancing motion up and down,

FIG. 260. down, by the ſpringing of the bridge, which helps to grind the corn. The flour, as ſoon as made, is thrown to the outſide, by the circulation of the ſtone and the air, and driven out at the mill-eye. The quantity of flour ground, is nearly as the velocity and weight of the ſtone. The ſtone ought not to go round above once in a ſecond, for bread corn.

The corn is put into the *hopper S*, which falling down into the *ſhoe TV*, runs into the hole at the top of the ſtone *M*. The axis *LM* is made with 6 or 8 angles; which, as it turns about, ſhakes the end *V* of the ſhoe, and keeps the corn always running down. The axis *LM* may be taken off. *PQ* is the direction of the water, which acting againſt the floats *R*, carries about the water-wheel *A*, and cog-wheel *D*, which cog-wheel carries the lantern or trundle *O*, and the upper ſtone *M* that grinds the corn.

Sometimes one water-wheel *A*, carries 2 pair of ſtones, and then 2 cog-wheels as *D*, are put into the axis *BC*, which carry two trundles with the ſtones. Otherwiſe the cog-wheel *D* carries a trundle *O*, and ſpur-wheel; which ſpur-wheel carries 2 lanterns with the ſtones, one lantern on each ſide the wheel. Or ſometimes, the ſame cog-wheel *D* carries another lantern and cog-wheel, whoſe axis is parallel to the horizon; and this cog-wheel carries another lantern with the ſtones. And the trundle is ſuch, as to make the blue ſtones, or thoſe that grind wheat flour, go near twice as ſwift as the grey ſtones do. In theſe caſes, when one pair of ſtones is to ſtand ſtill, there is either a looſe rung to be taken out of its lantern, or elſe the bridge *EF* is ſhifted towards *H*, till the lantern *O* be clear of *D*.

The diameter of the water-wheel *A* muſt not be too large; for then it will move too ſlow; nor too little, for then it will want power. When a mill is in perfection, the velocity of the floats, wings, or hands *R*, upon the water-wheel muſt be $\frac{1}{3}$ the velocity of the ſtream.

The higher fall the water has, the leſs of it will ſerve to carry the mill. In an underſhot mill, where the water comes underneath the wheel, it is brought by a narrow channel called the *mill race*. The water is kept up in the mill dam, and let out by the *penſtock*, when the mill is to go: and the penſtock is raiſed or let down by help of a leaver. The penſtock being raiſed opens a paſſage to the water, 10 or 12 inches wide, through which it flows to the wheel. And when the mill is to ſtop, the penſtock is let down, and the orifice ſtopt.

When the water comes underneath the wheel, it is called an *underſhot mill*. But if it comes over the wheel (as in fig. 282.) it

it is called an *overfal* or *overfhot mill.* This requires lefs water than an underfhot mill; but there is not convenience in all places to make them. The water is brought to the wheel of an overfal mill, by a trough; which is turned afide, to throw the water off the wheel, when the mill does not go. FIG. 260.

A *breaft mill* is that where the water is delivered into boxes, at about the height of the axis of the wheel, and moves the wheel by its weight. This requires more water than either of the other forts.

A good overfal mill will grind $2\frac{1}{2}$ or 3 bufhels of corn in an hour; and in that time requires 100 hogfheads of water, having 10 or 12 feet fall.

Ex. LXXIII.

AB is a trap to catch vermin, made of boards. *GH* a piece of wood fufpended over the bar *IL*, by the leaver *DE*, moveable about *D*; and the thread *FE*, tied to the ftart *CK*. *lmB* a piece of flat wood, moveable about *lm*, and lying on the bottom, whofe end *B* comes through a hole in the fide, in which is a catch to take hold of the end *K* of the ftart, when the trap is fet. When the vermin go into the trap, they tread upon the board *lmB*, on which a bait is laid, which puts down the end *B*, and the ftart *CK* flies up. This gives liberty to the rod *DE* to rife up; then the piece of wood *GH* falls down, and knocks them on the head. If two pieces of board were nailed on the ends *G*, *H*; to reach below the piece of wood *GH*, the trap would take the vermin alive. 261.

ABE is another *trap*, the end *B* is wire; and the end *A* flides up and down in two groves in the fides. When the trap is fet, the end *A* is fufpended by the thread *CD*, tied to the rod *DI*, moveable about *O*, the end *I* being held by the crooked end of the wire *IS*, moveable about *R*, the end *RS* going within the trap. A bait is put on the end at *S*, and the end *E* of the trap being open, the vermin goes in and pulls at the bait *S*, this pulls the catch *I* from off the end of the leaver *ID*, which lets the end *A* fall down, and the vermin is taken. 262.

Ex. LXXIV.

An engine for moving feveral *faws* for the fawing of ftones, &c. *ILLI* is a fquare frame, perpendicular to the horizon, moving in direction *LL*, in gutters made in the fixed beams *AM*, *CB*; and running upon little wheels. *IL* two rods of iron fixed 263.

at

FIG. 263. at *I* and *L*. *op* two hands of iron running along these rods; to these are fixed the saws *S*, *S*. *HIK* is a triangle fixed to the axis of a great wheel. As the wheel and triangle go about from *H* towards *I*, the point *I* acting against the piece *G*, moves the frame towards *MB*, together with the saws *S*, *S*. When *I* is gone off, the angle *K* acts against the piece *F*, and moves the frame back again. Then *H* acting against *G*, moves it forward; and so the saws are moved back and forward, as long as the wheel turns round. As these saws work by the motion of the engine, the hands *op* descend. The parts *F* and *G* ought to be made curve; and little wheels may be applied at the points of the triangle *HIK*, to take away the friction against *F* and *G*. The axle of the wheel may be made to carry more triangles, and work more saws, if the power is strong enough.

Instead of the triangle *HIK*, the frame may be moved by the two pieces *ab*, *cd*; going through the axis, across to one another. So that *ab* may only act on *F*, and *cd* on *G*. *F* being only in the plane of *ab*'s motion, and *G* in that of *cd*. So that *F* never falls in the way of *cd*, nor *G* in the way of *ab*.

Ex. LXXV.

264. *A* is an *eolipile*. This is a hollow globe of brass, with only a very small hole at the mouth. Take it by the handle *B*, and set it on a fire till it is heated; then plunge it in cold water, and the air in it which was rarified, will be condensed; and water will go into it, till it be about half full. Then if it be set on the fire, the water will turn into vapour by the heat, and will blow out at the mouth with great violence; and continue so till the water is spent.

Ex. LXXVI.

265. *ABD* is a *hygroscope*. *BC* is an index hung by the (therm) string *AB*, the point *B* hanging over the center of a circle, which is divided into equal parts. The string *AB* twists and untwists by the moisture or dryness of the air. By this means the index *BC* turns about, and shews the degrees of drought or moisture, on the circumference *DC*.

Ex. LXXVII.

266. *A windmill*. *AHO* the upper room, *HOZ* the under one. *AB* the *axle-tree*, going quite through the mill. *STVW* the *sails* covered

covered with canvafs, fet obliquely to the wind, and going about in the order *STVW*, their length about 6 yards, and breadth 2. *CD* the *cog-wheel*, of about 48 cogs, *a*, *a*, *a*; which carries the *lantern EF*, of 8 or 9 rounds, *c*, *c*; and its axis *GN*. *IK* the *upper ftone* or *runner*; *LM* the *lower ftone*. *QR* the *bridge*, fupporting the axis or *fpindle GN*. The bridge is fupported by the beams *cd*, *XY*, wedged up at *c*, *d*, and *X*. *ZY* the *lifting tree* ftanding upright: *ab*, *ef*, leavers whofe centers of motion are *Z*, *u*: *fghi* a cord with a ftone *i*, for a balance, going about the pins *gh*. The fpindle *tN* is fixed to the upper ftone *IK*, by a piece of iron called the *rind*, fixed in the under fide of the ftone. The upper ftone only turns about, and its whole weight refts upon the bridge *QR*, and turns upon a hard ftone fixed at *N*. The trundle *EF* and axis *Gt* may be taken away; for it fixes on the lower part at *t*, by a fquare focket, and the top runs in the edge of the beam *w*. Putting down the end *f* of the leaver *fe*, raifes *b*, which raifes *ZY*, which raifes *YX*, and this raifes the bridge *QR*, with the axis *NG*, and the upper ftone *IK*; and thus the ftones are fet at any diftance. The lower ftone is fixed immoveable upon ftrong beams, is broader than the upper, upon which boards are placed round the upper at a fmall diftance, to confine the flour from flying away; and the flour is conveyed through the tunnel *no* down into a cheft. *P* is the *hopper* into which the corn is put, which runs along the *fhoe* or fpout *r* into the hole *t*, and fo falls between the ftones where it is ground. The axis *Gt* is fquare, which fhaking the fpout *r* as it goes about, makes the corn run out: *rs* a ftring going about the pin *s*, which being turned about, moves the fpout nearer or further from the axis, and fo makes the corn run in fafter or flower, according to the wind. And when the wind is great, the fails *S*, *T*, *V*, *W* are only part of them, or one fide of them, covered; or perhaps only a half of two oppofite fails *T*, *W* are covered. Towards the end *B* of the axle-tree is placed another cog-wheel, trundle, and ftones, with exactly the fame apparatus as before. And the axle carries two pair of ftones at once. And when only one pair is to grind, the trundle *EF* and axis *Gt* is taken out from the other: *xyl* is a *girth* of pliable wood, fixed at the end *x*; and the other end *l* tied to the leaver *km* moveable about *k*. And the end *m* being put down, draws the girth *xyl* clofe to the cog-wheel, and by this means the motion of the mill is ftopt at pleafure: *pq* is a ladder going into the higher part of the mill. The corn is drawn to the top, by means of a rope going about the axis *AB*, when the mill is going.

FIG. 266.

FIG. In mills built of wood, the whole body of the mill turns
266. round to the wind, on a tampin or perpendicular post. But in those of stone, only the upper part turns; the roof is the surface of a cone; there is a wall plate of wood upon the top of the wall: in this a channel is cut quite round, in which are several brass rollers. The roof has a wooden ring for its base, which exactly fits into this channel; and the roof is easily moved round upon the rollers, by help of a rope and windlass.

In the wooden mill, 1 is the mill house, which is turned about to the wind by a man, by help of the leaver or beam 2. 3 is a roller to hoist up the steps 4.

Concerning the position and force of the sails, see Ex. 21, before.

Ex. LXXVIII.

267. *AB a force pump.* *C* the piston fixed to the rod *EC*, moveable about *E*. *DF* the handle moveable about *D*. *a*, *d* two clacks or valves opening upwards. The piston *C* must move freely up and down in the barrel, and exactly fill it, that no air get in. 'Tis made close by circular pieces of leather cut to fit the barrel, and screwed close between pieces of brass. This pump acts by pressing down; for when the handle *F* is raised, it raises the piston *EC*, and the water rises from *H*, opens the valve *d*, and goes into the barrel, at the same time the valve *a* shuts. But when *F* is put down, the piston *C* pressing upon the water, shuts the valve *d*, and opens *a*, and forces the water, that has been raised, through the pipe *BG*. The piston *C* must not be above 30 feet from the water in the well.

268. *LM* is *another force pump*, or a lifting pump. *N* the bucket. *a*, *b*, *c* valves opening upward. This pump is close at the top *S*, and the small rod of iron plays through a hole made tight with leather. This pump acts by forcing upwards; for when the handle *P* is put down, it lifts up the bucket *N*, the pressure shuts the valve *b*, opens *c*, and forces the water in the barrel *NS* along the pipe *QR*. At the same time the valve *a* opens, and lets in more water from *M* into the barrel. And when *P* is raised, *N* descends, the valves *a*, *c* shut, and *b* opens, and lets more water pass into the bucket *N*, through the upper part. And when the bucket *N* is drawn up again, the water is forced along the pipe *QR* as before. This pump is the same as a lifting pump, only there is added the valve *c*, which is not absolutely necessary. No hole or leak must be suffered below the piston or bucket; for air will get in, and spoil the working of the pump. And the bucket must always be within 30 feet of the water.

In these pumps the bore at *H* or *M*, through which the water rifes, fhould not be too ftrait; the wider it is, the fafter the water afcends. Nor fhould the pipe *BG* or *QR*, that difcharges the water, be too ftrait; for then the pump will work flower, and difcharge lefs water in a minute, or require more force to work it. For the calculation of a pump, fee Ex. XXII. FIG. 268.

There are feveral forts of *valves* ufed in pump work; as *T*, *V*, *W*, that at *T* being made of two pieces of flat leather, is called a *clack*. Thefe at *V*, *W* are made conical, or of any indented figure, and fit exactly into a hole of the fame fhape. At the bottom of the valve is put a pin acrofs it, to hinder its flying quite out of the hole.

Ex. LXXIX.

AB a *hydrometer*, to meafure the denfities of liquors, efpecially fpirituous liquors. This is a hollow ball of glafs, *B*, partly filled with quickfilver or fhot; and hermetically fealed at the top *A*, when made of a due weight, by trials. The fmall tube *AB* is divided into equal parts, and graduated at equal diftances. And thefe divifions noted to which it finks in different fluids of the beft forts; which points muft be taken as ftandards to compare others with. Then if the hydrometer be immerged in any fluid, and the point to which it finks in the furface be marked, it fhews the denfity of it, and its goodnefs. For it finks deepeft in the lighteft liquor: and the lighteft liquids are the beft. 269.

Ex. LXXX.

AB is a *thermometer*, to meafure the degrees of heat. *B* is a glafs ball with a long neck *AB*. The ball and part of the neck is filled with fpirit of wine tinged red with cochineal; and the end *A* is fealed hermetically; in the doing of which, the end of the tube *A*, the fpirit and included air, are heated, which rarifies the air and fpirit; fo that when the end *A* is fealed, and the tube cools, the fpirit contracts, and there is a vacuum made in the top of the tube. And therefore the fpirit expanding and contracting by heat and cold, has liberty to rife and fall in the tube. This ball and tube is enclofed in a frame, which is divided into degrees. Then as the top of the fpirit rifes or falls, the divifions will fhew the degrees of heat or cold. Thefe divifions are arbitrary, and therefore two thermometers will not go together, or fhew the fame degrees of healt and cold; except they be made to do fo by graduating them both alike by obfervation. 270.

FIG. This is commonly put in the same case with the barometer, fig. 218.

270. There are other sorts of thermometers. *CD* is a ball with a long neck open at the end *D*, partly filled with tinged spirit of wine, and put with the open end into the vessel *D*, near the bottom; which vessel is half full or more, of the same spirit. The top of the tube *CE* is air. So in warm weather, when the air in *C* is rarified by heat, it presses the spirit down into the bucket *D*, and as the point *E* descends, the divisions being marked, shew the degree of heat; or when it ascends, the degrees of cold. But this sort is affected with the pressure of the atmosphere, and therefore is not so true.

Ex. LXXXI.

271. *DA* is an *artificial fountain.* *AE* a strong close vessel of metal, *AB* a pipe reaching near the bottom of the vessel, and soldered close at *A*. *F*, *A* two cocks. If the cocks be opened and water poured in at *A*, till the vessel be about half full. Then stop the cock *F*, and with a syringe inject the air at *A*, till it be sufficiently condensed within the vessel. Then stop the cock at *A*, and take away the syringe. Then as soon as you open the cock at *A*, the compression of the air at *C* will force the water up the tube *BA*, and spout up to the height *D*; and a little ball of cork may be kept suspended at the top of the stream *D*.

But an artificial fountain is most easily made thus; take a strong bottle *G*; and fill it half full of water. Cork it well, and through the cork, put a tube *HI* very close, to reach near the bottom of the vessel. Then blow strongly in at *H*, till the air in the bottle be condensed; then the water will spout out at *H* to a great height.

Any of these fountains placed in the sun-shine, will shew all the colours of the rainbow; a black cloth being placed behind.

Ex. LXXXII.

272. *CpD* is *Archimedes's water screw.* This is a cylinder turning upon the axis *CD*. About this cylinder there is twisted a pipe, or rather several pipes *no*, *pq* running spiral ways from end to end. This cylinder is placed higher at one end *D*, than at the other. And its use is to screw up the water from the lower end to the higher. *AB* is a river running in direction *AB*. *a*, *b*, *c*, *d* several floats fixed to the cylinder. *EF* the surface of the water. Since the cylinder stands in an inclined position, the upper floats

a, *b*

a, *b* are set out of the water, and the under ones *c*, *d* within it. FIG.
So that the water acts only upon the under ones *c*, *d*; and turns 272.
about the cylinder in the order *a*, *b*, *c*, *d*. By this motion the water taken into the spiral tubes at the low end, is by the revolution of the cylinder, conveyed through these pipes, and discharged at the top into the vessel *G*. If *AB* is a standing water, there is no occasion for the floats *a b c d*. And then the cylinder is to be turned by the handle at *D*. Instead of the pipe, a spiral channel may be cut round the cylinder, and covered close with plates of lead. The closer these spiral tubes are, the more water is raised, but it requires more force. Also the more the cylinder leans, the more water it carries, but to a less height.

Ex. LXXXIII.

AL is a *rolling press*, for copper-plate printing. *DE*, *FG*, 273.
two wooden rollers, of about 12 or 16 inches diameter, running upon the ends of two strong iron axles, that go quite through them, and are fixed in them. To the axis of the upper one *DE*, is fixed the handle *BAC*. These rollers run in brass sockets, and must run very true upon their axles, and may be brought nearer, or set further from one another, by wedges, in the frame at *P*, *R*. *HIK* is a flat table or plank, going in between the rollers, and sliding freely upon the frame. *LM* the frame. *NO* a shelf to lay the paper upon. When the press is used, the upper roller is folded round with flannel, that every part of the print may take an equal impression; and a paper bottom, spread upon the table *HIK*, where the plate is to lie, to prevent the indenting of the plank; then warming the plate well over a charcoal fire, and rubbing it with the sort of ink proper for it, and laying it upon the paper bottom, on the table at *H*. Take the printing paper, and laying it carefully upon the plate, and turning the handle *CAB*, the motion of the roller *DE* turns the roller *FG*, and draws the table through between the rollers, together with the plate and paper; and the paper is printed.

Note, the paper must be thoroughly wetted in a trough; and after it has laid about a day or two, it is then to be passed through a screw press, to squeeze the water out, and then it is fit for printing.

The ink made use of for printing copper-plates, is made of the stones of peaches and apricots, the bones of sheeps feet and ivory, all burnt; this is called *Francfort* black. This must be well ground with nut oil, that has been well boiled; and then

FIG. 273. then it is fit for uſe. But the beſt ink is ſaid to be brought from *Holland*.

Ex. LXXXIV.

274. The *fire engine* to raiſe water. *LL* is a *great beam* or leaver, about 24 feet long, 2 feet deep at leaſt, and near 2 feet broad. It lies through the end wall *W* of the engine houſe, and moves round the center *a*, upon an iron axis. *CC* a hollow *cylinder* of iron, 40 inches diameter, or more, and 8 or 9 feet long; *P* the *piſton* ſuſtained by the chain *LP*. *F* the *fire-place* under ground; *BB* the *boiler* 12 feet diameter, which communicates with the cylinder, by the hole 2, and throat *pipe E*, 6 or 8 inches diameter. The boiler is of iron, and covered over cloſe with lead: in this, the water is boiled to raiſe a ſteam. 45 is the *regulator*, being a plate within the boiler, which opens and ſhuts the hole of communication 2; this is fixed on the axis 34 coming through the boiler, on which axis is fixed the horizontal piece *b*3, called the *ſpanner*; ſo that moving *b* back and forward, moves the plate 45 over the hole 2, and back again. *bl* is a horizontal rod of iron, moveable about the joint *b*. *xyedl* a piece of iron, with ſeveral claws, called the *wye*, moving about the axis *de*, in a fixed frame. The claw *sl* is cloven at *l*; and between the two parts, paſſes the end of *bl*, with two knobs to keep it in its place. *AA* is the *working beam*, in which is a ſlit, through which the claws *xy* paſs, and are kept there by the pin *q* going between them. *DDD* is a leaden pipe, called the *injection pipe*, carrying cold water from the *ciſtern S*, into the cylinder *CC*, and is turned up at the end within the cylinder. *f* the *injection cock*, to which is fixed the iron rod *fg*, lying horizontal. The end *g* goes through a ſlit, in the end of the piece *rg*, and on the end is a knob ſcrewed on, to keep it in. *pcbrg* a piece of iron with ſeveral claws called the *eff*, moveable about the axis *bc*. The claw *rp* goes through the ſlit in the beam *AA*, and is kept there by the two pins *c*, *n*: the claw *rg* goes over the piece *gf*: as the piece *gf* is moved back and forward, the injection cock *f* opens and ſhuts. 1, 1 &c. are ſeveral holes in the beam *AA*, that by ſhifting the pins, ſerve to ſet the pieces *p*, *x*, *y*, higher or lower, as occaſion requires. *N* is the *ſniffing clack*, ballanced by a weight, and opening outwards, to let out the air in the cylinder, at the deſcent of the piſton. In ſome engines a pipe goes from it to convey the ſteam out of the houſe. *G* is a leaden pipe, called the *ſinking pipe*, or *eduction pipe*, going from the cylinder to the *hot well H*; it is turned up at the end, and has a valve opening upwards; this carries away the

the water thrown in by the cold water pipe, or injection pipe. FIG.
t is the *feeding pipe*, going from the hot well to the boiler, to 274.
supply it with water, by a cock opening at pleasure. *i*, *i* are two *gage pipes*, with cocks, one reaching a little under the surface of the water in the boiler, the other a little short of it. By opening these cocks, it is known when there is water enough in the boiler; for one cock will give steam, and the other water; they stand in a plate, which may be opened, for a man to go into the boiler, to clean or mend it. *m* is the *puppet clack*; from this a wire comes through a small hole, to which is fixed a thread going over a pulley, with a small weight at it; the weight on the clack *m* is about a pound for every square inch. *YZ* the *steam pipe* going from the clack, out of the house. When the steam in the boiler is too strong, it lifts up the puppet clack *m*, and goes into the steam pipe *YZ*, by which it is conveyed away; otherwise the boiler would burst. *KK* a pipe carrying water from the cistern *S*, into the cylinder, to cover the piston to a good depth. *I* a cock opening to any wideness, that the water may run in a due quantity; *M* a hole to let it out, through a pipe, into the hot-well *H*, when there is too much. *VVV* a *force pump*, with a bucket and clack, and two valves opening upwards. This pump is close at the top *R*, and being wrought by the leaver *LL*, it brings water out of a pit, into the cistern *S*. *Q* the *pit* where water is to be raised. *X*, *X*, the *spears* which work in wooden pumps within the pit. The cylinder is supported by strong beams as 7, 8, going through the engine house; 6, 6 is the first floor; 7, 7 the upper floor. At *O*, in the end of the beam *CL*, there are two pins, which strike against two springs of wood, fixed to two timbers, lying on each side the great leaver *LL*; these pins serve to stop the beam, and hinder the piston coming too low in the cylinder.

When the engine is to be set to work, the water in the boiler must be made to boil so long, till the steam is strong enough; which is known by opening the cocks *i*, *i*. Then the hole 2 is opened, by moving the spanner *b3* by hand; then the steam is let into the cylinder, which lets that end of the beam *LC* rise up; this raises the working beam *AA*, moves the eff *prg*, which moves *gf*, and opens the cold water cock *f*: at the same time is moved the wye *xyl*, which draws *lb*, and shuts the hole 2. The cock *f* being open, the cold water rushing into the cylinder, is thrown up against the piston, and descending in small drops, condenses the hot rarified steam, and makes a vacuum under the piston. Consequently the weight of

FIG. 274. of the atmoſphere, preſſing upon the piſton, brings down the end *LC*, which raiſes the other end *LQ*, which works the pumps *X*, *X*. As the end *LC* deſcends, the working plug *AA* deſcends, and moving the eff, *prgf*, and the wye, *xyslb*, ſhuts the cold water cock *f*, and opens the hole 2, and the ſteam goes into the cylinder, which takes off the preſſure of the atmoſphere; and the end *LQ* deſcends by the weight of the ſpears *X*, *X*; and the end *LC* aſcends as before, which opens *f*, and ſhuts 2. So by the motion of the beam *AA* up and down, the cock *f*, and hole 2, ſhut and open alternately: and by this means of condenſing and rarifying the ſteam by turns, within the cylinder, the leaver or beam *LL* conſtantly moves up and down; by which motion, the water is drawn up by the pumps, and delivered into troughs within the pit, and carried away by drifts or levels. At the ſame time, the motion of the beam *LL* works the pump *VRV*, and raiſes water into the ciſtern *S*.

When the engine is to ceaſe working, the pins *n*, *o* are taken out, and the cold water cock is kept cloſe ſhut, while the end *LC* is up.

The diameter of the pumps within the pit, is about 8 or 9 inches; and the bores of the pumps where the ſpears *X*, *X*, work, ſhould be made wide at the top; for if they be ſtrait, more time is required to make a ſtroke, and the barrels are in danger of burſting. Likewiſe if water to be raiſed from a great depth at one lift, the pumps will be in danger of burſting; therefore it is better to make 2 or 3 lifts, placing ciſterns to receive the water.

292. The ſpears or rods, that work in the pumps, conſiſting of ſeveral lengths are joined thus; each piece has a ſtud *a*, and a hole *b*; which are made to fit; and the ſtuds of one being put cloſe into the holes of the other, and the iron collar *g* drove upon them to the middle *d*, they are firmly fixt together.

274. There is never made a perfect vacuum in the cylinder; for as ſoon as the elaſtic force of the ſteam within is ſufficiently diminiſhed, the piſton begins to deſcend. The vacuum is ſuch, that about 8 lb. preſſes upon every ſquare inch of the piſton, or in ſome engines, not above 6 lb. This engine will make 13 or 14 ſtrokes in a minute, and makes a 6 foot ſtroke; but the larger the boiler is, the faſter ſhe will work. A cubic inch of water in this engine, will make 13340 cubic inches of ſteam; which therefore is 15 times rarer than common air. But its elaſtic force within the boiler is never $\frac{1}{10}$ ſtronger or weaker than common air; if ſtronger, it would force the water out of the feeding pipe.

This

This engine will deliver 300 hogfheads of water in an hour, to the height of 60 fathom, and confumes about 30 bufhels of coals in 12 hours. FIG. 274.

In fome engines there is a different contrivance to open and fhut the regulator, which is performed thus; as the beam QQ afcends, it raifes $G5$, turns the wye $5GCED4$ about the axis AB; and the weight C falling towards B, the end E ftrikes a fmart blow on the pin L, and drives the fork FL towards L; which draws the fpanner PO towards L, and fhuts the regulator. And when the beam QQ defcends, a pin in it puts down the end 4, and turns the axis AB, and the weight C defcending towards 5, throws the end D of the wye againft L, which moves PO, and opens the regulator: the fpanner PO fliding upon the horizontal piece O. There is a cord *ron*, fixt at r, n, and the top of the wye O, to hinder it from going too far on each fide. 293.

Likewife, for opening and fhutting the injection cock; inftead of the pieces rg, gf of the eff (fig. 274.) fome engines have quadrants of 2 wheels H, I, with teeth, which moving one the other, opens or fhuts the cock f, of the injection pipe K.

In fome engines there is a catch, held by a chain fixt to the great beam; and this catch holds the eff from falling back, and opening the cold water cock; till the rifing of the beam pulls the catch up by the chain, and then the eff falls.

A calculation of the cylinder and pumps of the fire engine.

If it be required to make an engine to draw any given number of hogfheads of water in an hour, from f fathom deep, to make any number of ftrokes in a minute, by a 6 foot ftroke. Find the ale gallons to be drawn at 1 ftroke, which is eafily found from the number of ftrokes being given. 274.

Let $g =$ *number of ale gallons to be drawn at* 1 *ftroke.*

$p =$ *pump's diameter.* } in *inches.*
$c =$ *cylinder's diameter.* }

Then $p = \sqrt{5g}$,

And fuppofing the preffure of the atmofphere on an inch of the pifton, to be 7 lb.

Then $c = p\sqrt{\frac{2.614f}{7}} = \sqrt{\frac{13.07fg}{7}}$.

Note, if inftead of 7, you fuppofe the preffure of the atmofphere to be l pounds; and inftead of a 6 foot ftroke to make an r feet ftroke; then $p = \sqrt{\frac{6}{r} \times 5g}$, and $c = \sqrt{\frac{13.07fg}{l}}$.

FIG.

Ex. LXXXV.

275. *AB* is the *water engine* to quench fire. *D*, *E* are two *pumps* 5 inches diameter, having each a clack *a*, *b* opening upwards. *CO* a large copper *air veſſel* 9 inches diameter. This veſſel ſtands upon a ſtrong plate *kw*, 5 or 6 inches above the bottom of the cheſt *NM*. *SY* is a braſs pipe coming through the end of the cheſt at *S*; and at *Y* it divides into two cavities going under the copper pot *CO*, to the two pumps *E*, *D*. The cavity *YW* leads to the pump *E*. And directly above this cavity at *W*, there is another cavity *y*, communicating with the pump *E*. And above the cavity *y* is placed the valve *r*, opening upwards into the copper pot *CO*, from this cavity. There are the like cavities belonging to the pump *D*; the firſt going to the valve *a*; the other from the pump to the valve *s* of the copper pot. Theſe cavities are made of hollow pieces of braſs ſcrewed faſt together. *Z* is a cock, through which are two paſſages, one along the pipe *SY*, and another at the ſide of the pipe into the cheſt *NM*. This cock, by turning the handle *ce*, opens one paſſage and ſhuts the other, as there is occaſion. *Xx* a leather pipe to be ſcrewed on the end *S*, to draw water out of a well or river.

PR is the *conduit pipe* reaching near the bottom of the veſſel *CO*, and ſoldered cloſe into the top of it. At *R* and *Q* are ſcrews, ſo that the pipe may be turned in any direction by the man that holds it. And at *V* a copper pipe muſt be ſcrewed on, or elſe a long leather one, which being flexible, is carried into rooms and entries. *HI* an iron axis, to which the iron leavers *FG*, *LK* are fixt. This axis moves in ſockets about *H*, *I*, which are ſcrewed hard down. *FK*, *GL* two wooden handles fixt to the leavers to work them by. *gh*, *pl* are two arches of iron fixt on the axis *HI*. *fd*, *mn*, the ſhanks of the piſtons, being two ſtrong rods of iron. *fg*, *ht*, *lq*, *mp* four iron chains fixt at *f*, *g*; and *h*, *t*; and *l*, *q*; and at *m*, *p*. At *f* and *m* are ſcrews to ſcrew the chains tight: theſe chains work the pumps. For when *FK* is put down, the chain *fg* pulls down the rod of the piſton *fd*. And when *FK* is raiſed, the chain *th* pulls it up again. And the ſame way the chains *lq*, *mp* raiſe and depreſs the piſton *mn*. In ſome engines there are two arches, like *hg*, *tp*, fixt near the end *I* of the axis, and chains at them; from the ends of which, as alſo from *t* and *q*, two boards are ſuſpended. Theſe boards ſerve for treadles for men to ſtand upon, to help to work the engine.

The

The veſſel *CO* and two pumps are incloſed in a cheſt *AN*, FIG.
and the whole machine moveable on wheels. The fore axle- 275.
tree turning on a bolt in the middle, for the conveniency of turning to either ſide. But there are a great many forms of theſe engines. In ſome the leaver lies croſs over, in others length-ways; in ſome there is no chain work, but only pins for the piſtons to move upon.

When the engine is to play; if it is by the water in a river, *&c.* the pipe *Xx* muſt be ſcrewed on at *S*, and the end *x* put into the water. But if water is to be fetched, it muſt be poured into the cheſt *M*, which runs through the holes *T*, into the body of the engine *N*. Then turning the cock *ceZ* to open the proper communication; the handles *FK*, *GL*, muſt be moved up and down by men; by which means water is drawn into the pumps *E*, *D*, and forced into the veſſel *CO*, and out of the pipe *PR*. For the piſton *mm* being raiſed, the water is drawn along the cavity *ZYW*, through the valve *b* into the pump *E*; and when *mn* is depreſſed, the valve *b* ſhuts, and the water is forced into the cavity *y*, through the valve *r*, and into the pot *CO*; which cannot return for the ſhutting of the valve *r*, when the piſton *mn* riſes again. And the like for the other pump *D*. Since the piſton of one pump goes down whilſt the other goes up, the water is forced by turns into the veſſel *CO*, by theſe two pumps; ſo that there is always water going in. And the air confined at top of the veſſel at *C* being condenſed, will preſs the water up the pipe *PRQV*, and make it flow with a continual ſtream. If the water in *C* be compreſſed into half the ſpace, it will force the water to 30 feet high.

In ſome engines, there is another pipe as *PR*, coming through the copper pot, and through the ſide of the engine, and theſe two pipes may play both at once, if there is occaſion. And if not, the end of one is ſcrewed up.

Ex. LXXXVI.

A ſhip. This is the nobleſt machine that ever was invented. 276.
It is ſo compounded, and conſiſts of ſo many parts, that it would require a whole volume to deſcribe it. Some of the principal parts are theſe.

A the hull.
B the bow.
C the forecaſtle.
D the main deck.
E the ſtern.
F the ancient ſtaff and enſign.
G the poop-lantern.
H the rudder.

I the

FIG. 276.

I the bowsprit.
K the fore-maſt.
L the main-maſt.
M the mizen-maſt.
N top-maſts.
O top-gallant maſts.
P ſails
Q yards.
R vanes.
S the jack.
T the pendant.
V ſtays.
Vv main-ſtay, &c.
W ſhrowds.
X main-top-maſt back-ſtay, &c.
Y the crane-line.
Z the anchor, to which the cable is fixt.

1. Main lifts.
2. Fore-braces.
3. Main-top-ſail-ſheets.
4. The fore-tacks.
5. Fore-top ſail clew-lines.
6. Main-top-ſail-leetch-lines.
7. Fore-bunt-lines.
8. Mizen-bow-lines.
9. Main-top ſail-halliards.

And the like for the reſt of the ſails.

Moſt of theſe ropes are for hoiſting the ſails, and ſetting them in a proper poſition to the wind. For the wind always acts perpendicularly upon the plane of any ſail; and urges the ſhip in direction of that perpendicular. And by the help of the rudder *H*, ſhe is made to keep any direction required. For if the rudder be put about to any ſide; the water (as the ſhip moves along) will act violently againſt it, and drive the ſtern the contrary way, or her head the ſame way, as the rudder. A ſhip with a fair briſk wind will ſail 8 or 10 miles an hour.

That any one ſail may have the greateſt force to move a ſhip forward, it muſt be ſo placed between the point of the wind and the ſhip's way, that the *tan.* of the angle it makes with the wind, may be twice the *tan.* of the angle it makes with the ſhip's way.

When the rudder is ſet to an angle of $54\frac{1}{2}$ deg. with the keel, it has the greateſt force to turn the ſhip, and make her anſwer the helm.

301. Becauſe the figure of a ſhip is the cauſe of her going well or ill, and of making more or leſs way through the water; I ſhall here give the conſtruction of the fore part of a veſſel, that will move through the water with the leaſt poſſible reſiſtance.

Let *DdAcC* be the *water line*, or horizontal ſection of the water and the hull of a ſhip, *AB* 30 feet, *CD* the greateſt breadth 20 feet, *BC* 10 feet. *AeE* the ſtem and part of the keel. Then the following table ſhows the length of every ordinate, as *bc*, taken at the diſtance *Ab*, or 1, 2, 3, &c. feet from *A*; by which the curve *AcC* is determined.

Length

FIG. 301.

length of *AB* in feet	length of *bc* in feet	length of *AB* in feet	length of *bc* in feet
1	0.90	16	6.36
2	1.48	17	6.64
3	1.96	18	6.92
4	2.39	19	7.19
5	2.79	20	7.46
6	3.17	21	7.73
7	3.54	22	7.99
8	3.89	23	8.25
9	4.22	24	8.51
10	4.55	25	8.76
11	4.87	26	9.01
12	5.18	27	9.26
13	5.48	28	9.51
14	5.78	29	9.76
15	6.07	30	10.00

The practice is thus: having made $AB = 30$ feet, and accordingly divided it into 30 equal parts; at the feveral points of divifion, erect perpendiculars to *AB*, equal to the lengths given in the 2d col. of the table. The curve *AcC* drawn through the ends of all the ordinates, is the figure of the fhip on each fide.

The curve *AeE* which the ftem and keel make, muft be the fame curve as *AcC*; if the depth *BE* is fuppofed equal to *BC*; and the ordinates, *be*, *BE* muft all be drawn perpendicular to *AB*. But if the depth *BE* be taken greater or lefs than *BC*; then the ordinates muft be taken greater or lefs in proportion.

Again if *CDE* be the fection of the fhip, made perpendicular to the axis *AB*, or horizontal plane *CAD*; and *ced* be any other fection parallel to it; then whatever the curve *CED* is, all the fections *ced* muft be fimilar to it.

If a fhip is required to be built either greater or lefs than this; then it is only taking a greater or lefs length inftead of a foot, and dividing it decimally, and ufing it inftead of a foot, to meafure off the lengths as in the table.

Likewife if it was required to have the breadth to be greater or lefs than is here affigned, whilft the length remains the fame; then it is only taking a proportionally greater or lefs line, inftead of a foot, and fetting off the ordinates *bc* by that.

And

FIG. 301. And thus the requisites may be altered at pleasure, still retaining the general construction.

If any ship carpenter thinks fit to build a ship according to this model, it will be found to move faster through the water, than any other ship of the same length, breadth, and depth, and of a different form. The form of the curve is truly represented by the curve *AcC*.

But it must be observed, that the curve at *C*, the broadest part is not perpendicular to the ordinate *BC*, but makes an angle of about 76 degrees: to avoid this, it will be proper to produce *AB* a little further, and turn the side *AC*, at *C*, round in a curve, as quick as possible. Or else make the 2 or 3 last perpendicular ordinates, something less than in the table, that the part of the curve at *C* may be in a parallelism with *AB*, as it ought; because *C* is the broadest part.

But though the form here given is the most proper for sailing fast; yet perhaps it may not be so commodious as the common form, upon other accounts, as for the stowage of goods, &c. Yet privateers and ships of war made to pursue the enemy, ought to be built as near this form as they can conveniently. For it is a matter of great moment, either to have it in our power to come up with a ship we are able to take, or else to fly, and escape from one of superior force.

That a ship may steer well, the water ought to come freely and directly to the rudder; and therefore she must not be too short from the midship to the stern; and towards the stern she must rise well, and be built very thin below, lessening gradually to the stern-post. Likewise she must draw considerably more water abaft than afore. To carry a good sail, and also to avoid rowling, she must be made pretty deep in the hold. As to the dead work, or upper part of the ship, that may be left to the fancy of the builder, or contrived to answer such conveniences as may be wanted.

Ex. LXXXVII.

277. *AT* an *air pump*. *C*, *D* two brass cylinders, 2 or 3 inches diameter, and a foot high, having two valves at the bottom opening upwards. *t*, *t* two pistons working in the cylinders, having two valves also opening upwards. *FF* a handle going upon the axis of the wheel or lantern *E*, which wheel by the teeth, moves the racks *G*, *G*; and by them the pistons, within the cylinders or pumps. *AB* a table or plate supported by the pillars *I*, *I*, *I*, *I*. *H* the receiver of glass, which by the hol-

low pipe of brass *nooo*, called the swan's neck, communicates FIG.
with the cylinders by means of a hollow brass pipe *PQ*; into 277.
which the swan's neck passes. *rrs* a mercurial gage, being a glass tube standing in the bucket of mercury *s*, and communicating with the pipe *no*. *K* a cock under the table *AB* to let in air into the pipe *no*, and so into the receiver, when there is occasion.

When the air is to be drawn out of the receiver *H*, a wet leather is placed over the plate, and upon that the receiver *H*. Then raising the right hand *F*, the piston *t* of the barrel *D* is raised, which takes off the weight of the atmosphere; consequently, the air passes out of the receiver *H*, through the swan's neck *no*, and through the hollow brass *PQ*, through the valve into the cylinder *D*. Then the right hand *F* put down, the valve at the bottom of the cylinder *D* shuts, and the air passes through the valve at *t*: at the same time that the left hand being raised, draws the air from the receiver, through *noP*, through the valve into the cylinder *C*. So that by the motion of the handle *FF* up and down, the air is at length drawn out of the receiver *H*, by the pumps *C*, *D*. And the rarity of the air within the receiver, is known by the height of the mercury in the tube *rs*, which is known by the graduated frame. An absolute vacuum can never be perfectly made. For when the spring of the air is so weak, as not to be able to lift up the valves at the bottom of the cylinders, no more air can be drawn out.

The handle *F* is lately made to turn always one way; thus *A* 278.
is a crank turned by the handle *F*. *NN* the leader or sword going over the pin *I*, in the wheel *E*. Whilst the crank *A* is rising, it raises the side *S* of the wheel, and when the crank descends, it thrusts down the same side *S* of the wheel *E*. So the racks are alternately raised and depressed as the crank goes about.

There are several sorts of glasses made use of for the air 279.
pump. As *A* a receiver open at top, covered with a brass plate and oiled leather at *D*, and kept down by the cross piece *EF*, screwed down upon the pillars *BC*, which pillars are screwed into the table *AB* of the air pump.

H a receiver open at top, with a plate and collar of wet 280.
leathers *K*, through which goes the slip wire *GI*, so tight as to let no air in. This serves to lift any thing up by the hook *I*.

MP is a transferrer. *N* is a plate and leather, on which stands the receiver *M*. *NP* a hollow tube going through the plate. *O* a cock to open or shut the passage. The cock *O* being open, and the air exhausted by the pump, and then the

cock

FIG. 280. cock being ſhut, the receiver and pipe may be taken away from the air pump, the vacuum remaining in *M*.

L a receiver cloſe at the top; with infinite others of like ſort.

Ex. LXXXVIII.

281. *London-bridge water-works.* *AB* the axis of the water-wheel *CD*; which wheel is 20 feet diameter, and the axis 3 feet, and 19 feet long. *E*, *E* 26 floats $1\frac{1}{2}$ foot broad. *G* a ſpur wheel fixed to the axis *AB*, 8 feet diameter, 44 cogs of iron; this moves the trundle *H* $4\frac{1}{2}$ feet diameter, and 20 rounds; *HI* its iron axis. *IK* a quadruple crank of caſt iron 6 inches ſquare, each crank being a foot from the axis. The crank is faſtened to the axis at *I*, by help of a wedge going through both, which cauſes the crank to turn. *L*, *L* four iron ſpears belonging to the cranks, and fixed to the 4 leavers *MN*, 3 feet from the ends; which leavers are 24 feet long, moving on centers in a frame of wood. *P*, *P* four force pumps of caſt iron, wrought by four piſtons or rods, *NP*. Theſe pumps are 7 or 8 inches diameter, having valves opening upward. *O* a hollow trunk of caſt iron, to which the pumps are cloſe fixed. *Q* a ſucking pipe going into the water. *R*, *R* four hollow pipes, 7 inches diameter, and cloſe fixed to the lower part of the pumps; theſe pipes are cloſe ſcrewed to the hollow iron trunk *S*, into which 4 valves open. *T* a pipe communicating with the trunk *S*, through which the water is forced to any height. There are alſo four forcers placed at the ends *M*, *M* of the leavers *M*, *N*, and working in four more pumps, to which belong other trunks and pipes, *O*, *Q*, *R*, *S*. At *B* the other end of the axis, there is placed exactly the ſame work as at *A*; ſo that the great wheel *CD* works 16 pumps. There is alſo a machine made of cog-wheels and trundles, contrived to raiſe the great wheel as the tide riſes. The great wheel will go at any depth of water; and as the tide turns, the wheels go the ſame way with it; but ſtand ſtill at high and low water.

As the great wheel is carried about by the tide, it carries round the ſpur wheel *G*, which carries the trundle *H* with the cranks *IK*, which by the ſwords *L*, move the leavers *MN*. When the end *M* is pulled down, *N* is raiſed with the piſton *NP* in the pump *P*, by which means the water is drawn out of the river through the pipe *Q*, into the pump *P*; and when *NP* deſcends, the valve ſhuts, and the water is forced through the pipe *R*, through the trunk *S*, and along the pipe *T* into the town. And when *N* riſes up again by the motion of the cranks, the

the valve in S shuts, and that in the pump opens, and more water rises through the pipe Q into the pump P. And as the cranks stand every way, there is always water rising in some of the pumps; and some always forcing through R, S, T. When the tide is strongest, the great wheel goes 6 times round in a minute. This engine is said to raise 30 or 40 thousand hogsheads of water in a day. FIG. 231.

Ex. LXXXIX.

The pile engine for Westminster-bridge. A the great cog-wheel fixed to the great shaft D. MO, a trundle and fly turned by the cog-wheel; this is to prevent the horses from falling when the ram is discharged. B the drum or barrel on which the great rope is wound. C a less barrel on which the rope L is wound, carrying the weight N. The use of this is to hinder the follower from falling too fast. The barrels BC are moveable about the great axis D. The cog-wheel and barrel B are fixed together by the bolt F, going through the cog-wheel into the barrel. EI is a leaver moveable about 1, going through the great shaft D; this lifts up the bolt F, the end E being made heavier by a weight; by which means it locks the barrel B to the great wheel A. KI the forcing bar going into the hollow axis of the great shaft D; this rests upon the leaver EI. XY a great leaver moveable about 3, the end X being heavier, which with the end Y presses down the bar KI, thrusts down the end of the leaver at F, and lets the bolt F descend, to unlock the barrel C. Z a rope fixed at X, and going up through the guides at R. GK a crooked leaver moveable about 2, the roller at the end G being pressed with the great rope, forces the end K against the catch at K, and hinders the bar KI from ascending. When the rope H slackens, the spring 7 forces the end K from the catch, and the bar KI ascends. H the great rope going round the barrel B, over the pulley P, up to the top and over the pulley Q, then down to the follower, where it is fixed. T the ram that drives the piles. S the follower, in which is fixed the tongs IV, moveable about the center. VV the guides between which the ram falls. At the inside of the guides at R, where they are fastened together, there are two inclined planes. At the bottom of the follower is a slit, to receive the handle 6 of the ram T, to be taken up by the tongs IV. a, b, c, d timbers for horses to draw at, in direction $abcd$. 283.

As the horses go round, the great rope H is wound about the barrel B; and the follower S, and ram T are drawn up, till the

FIG. 283. tongs come between the inclined planes, which squeezing the ends 4 4 together, opens the end 5, and lets the ram fall down. Then the follower *S* taking hold of the rope *Z*, raises the end *X*, and depresses the end *Y* of the leaver *XY*, which thrusts down the bar *KI*, which thrusts down the end *FI* of the leaver *EI*, with the bolt *F*, and unlocks the barrel *B*, which turning about the axis, the follower *S* descends by its weight, till it comes to the ram *T*; and the end 5 of the tongs slips over the handle 6 of the ram. Then the rope *H* slackens, and the spring 7 forces the end *K* from off the catch at top of the bar *KI*, and lets the bar rise, and the weight *E* raises the bolt *F*, and locks the barrel *B* to the wheel *A*; and the horses [illegible] going about, the end 5 of the tongs takes hold of the handle 6, and the ram *T* is taken up as before.

All this machine is placed upon a boat, which swims upon the water; and so is easily conveyed to any place desired.

Ex. XC.

284. *GH* a *blowing wheel*. *AB*, *CD* an iron cross; to this is fixed the circle of iron *EF*. To these are fixed 12 leaves *I*, *I*, *I*, which reach no nearer the center than the iron circle. 1, 1, 1 are holes, through which the air passes into the cavities between the leaves. There is the same cross and iron circle on the other side, but without any hole. Through the center of both sides is put an iron axis and fixed there, and on the further end a handle is put to turn it by. This wheel is enclosed in a case, which just touches the edges of all the leaves. But the rim or out edge *KK* is at a small distance from the ends of the leaves. On this side or flat of the case, there is a hole left against the holes 1, 1, to let the air through; the other flat is close. *LM* is the sucking pipe, being a tube fixed upon the case, so as to communicate with the cavities, by the holes 1, 1. *G* is the blowing pipe, and is another wooden tube communicating with the inside of the case. The axis turns upon two concave pieces of metal fixed to the case, the handle being on the back side of the figure. *abcd* is a thin ring of wood fastening the leaves all together; and the like on the other side. On these rings are put two circles of blanketing to go close to the case, and also upon the iron circle *EF*.

The frame being fixed, and the handle turned about in the order *BCAD*. The motion of the leaves moves the air very swiftly to the outside, which being confined by the rim, is forced in a tangent along the tube *G*; whilst new air ascends along the

the sucking pipe *LM*, passes through the hole in the frame, and through the holes 1, 1, into the cavities between the leaves; and so thrown out of the wheel, through the blowing pipe *G*. FIG. 284.

If the pipe *LM* be continued to the place where any foul air is, it may soon be thrown out through the tube *G*, and dispersed abroad. Or if the tube *LM* communicate with the fresh air, and *G* with any close room, fresh air may presently be injected into the room.

Ex. XCI.

AB an *artificial fountain* to play with either end up. *A* and *B* two cavities; *FO*, *KB* two open pipes, fixed to the basons at *K* and *O*. *GHI* and *CDE* two curve tubes open at both ends. When the fountain stands on the end *A*, pour water in at *O*. Then turning the fountain like an hour glass upon the end *B*, the water will descend through the pipe *CDE*, and spout out at *E*. The air passing up the pipe *OF* to give it liberty. The water falling down upon the bason *EK*, runs through the pipe *KB*, into the cavity *B*. And the fountain being turned, the water will descend through *GHI*, and spout out at *I* as before. And so being turned, it will play a-fresh as often as you will. 285

Note, while the jet *E* is playing, if the end *O* of the pipe *FO* be stopped with the finger, the jet will cease playing; which being taken off, it will begin again; and so may be made to play or stop at pleasure.

Ex. XCII.

AF a *water barometer*. *AD* is a small tube open at both ends, cemented in the neck of the bottle *CE*. Then the bottle being a little warmed to drive out some of the air; the end *A* is immerged in water tinged with cochineal, which goes into the bottle as it cools. Then it is set upright, and the water may be made to stand at any point *B*, by sucking or blowing at *A*. This is a very sensible barometer; for if it be removed to any higher place, a very small decrease in the air's gravity, will make the water rise sensibly in the tube. This may be made use of to find the level of places. But it is subject to this inconvenience, that it is a thermometer as well as a barometer, the least alteration of heat raising the water in the tube. To prevent which, it must be enclosed in a vessel of sand; and then the air included in the bottle, will retain the same degree of heat, at least for a small time. 286.

FIG.

Ex. XCIII.

287 *ADOF* is a *jet d'eau.* *AB* the reservoir where the water is kept. *CDIO* the pipe of conduct, which conveys water from the reservoir. *O* the cock, or adjutage, being a small hole in a thin horizontal plate, fixed upon the end of the pipe, through which the water flows. *OF* the jet of water, spouting up through the hole *O*, which descends again in the streams *FE* and *FH*. *OF* the height of the jet. *AG* the horizontal height of the water in the reservoir. If the part *LIO* of the pipe of conduct be buried under the surface of the water *KH*, and be invisible; the jet will seem to rise out of the water *KH*, as in many artificial fountains.

The adjutage is sometimes made conical, but the best sort for spouting highest, is a thin plate with a hole in it. The bore of the adjutage ought to increase with the height of the reservoir, and the larger the adjutage, the higher the jet will go, provided the pipe of conduct be large enough to supply it with water. Pipes of conduct ought not to be made with elbows, but to turn off gradually in a curve as *DIO*. The diameters of pipes of conduct ought at least to be 5 or 6 times the diameters of the adjutage, or else it will not spout so high. If a reservoir be 50 feet high, and the adjutage half an inch; the pipe of conduct should, at least, be 3 inches; or if the adjutage be an inch, which is better, the pipe of conduct must be 6 inches. And in these cases the jet will rise to the greatest height it can have. In general the diameter of the adjutage ought to be nearly as the square root of the height of the reservoir. And if you would have the velocity in the pipe of conduct to be the same at all heights of the reservoir, that the friction may not increase too much; then the square of the diameter of the pipe of conduct must be as the cube of the diameter of the adjutage. When water is carried a great way through pipes, the friction of the pipes will diminish its velocity, and the jet will not rise so high.

A jet never rises to the full height of the reservoir. If the height be 5 feet 1 inch, the jet will only rise to 5 feet; thus the jet *OF* wants the space *FG* of the height of the reservoir. And the defect *FG* is as the square of the height of the reservoir *OG*. But smaller jets fall short more than in that proportion, being more retarded by the resistance of the air. The greatest jets never rise 300 feet high; for the velocity is so great, that the water is dissipated into small drops, by the resistance of the air. If a ball

a ball of cork or light wood be laid at *F*, it will be suſpended by the pillar of water, and play there without falling. FIG. 287.

Ex. XCIV.

AGE is a *compound ſteelyard*, for weighing vaſt weights. *IG*, *CK* two leavers moveable about *B* and *E*. *LE*, *MB* two fixed pieces. *AC* a croſs bar ſupporting the end *C*, and moveable about the pins *A* and *C*. The center of gravity of *IG* and *AC* is in *B*; and of *CK* and the hook *DN*, in *E*. *H* the weight to be weighed; *F* the counterpoiſe moveable along the graduated leaver *BG*. The machine is hung upon the hooks at *L*, *M*. Here the power *F* is to the weight *H*, as $AB \times DE$ to $CE \times BF$. 283.

Ex. XCV.

ABC is a *horſe mill* to grind corn; *C* the *ſpur wheel* having 72 cogs; *B* the *lanthern* of 7 rounds; *A* the *hopper*, *E* the *ſhoe*; *F*, *G* the two *mill-ſtones*. *H* a leaver or *arm* 8 feet long, going into the axis *D* of the great wheel; *I* the traces to which one or two horſes are yoaked. As the horſe goes about in the path 1 2 3, he draws the arm *H*. which turns the great wheel *C*, and this drives the trundle *B* with the upper ſtone *F*, which grinds the corn; the corn is put into the hopper *A*, and falling into the ſhoe *E*, runs through a hole at top of the ſtone *F*, and in between the ſtones where it is ground. *KL* is the upper floor. The whole is within a houſe. 294.

Ex. XCVI.

AB a *lifting ſtock*, ſet perpendicular; its uſe is to raiſe a great weight. *LO* is a ſlit going through it, in which there moves the leaver *CD*. *II*, *KK* two ſets of holes; into which goes the pins *G*, *H*. When the weight *W* is to be raiſed, it is hung on the hook and chain at the end *D* of the leaver. And the pin *G* being put into the firſt hole *I*, and the end *C* being put down, the other end with the weight is raiſed, and then the pin *H* is put into the ſecond hole *K*, under the leaver; then the end *C* being raiſed to *E*, the pin *G* is put into the ſecond hole *I*. Then *E* being put down to *C*, and the end *F* raiſed, the pin *H* is put into the third hole *K*. Thus the leaver and pins being thus ſhifted from hole to hole, the weight *W* is by degrees raiſed up. 295.

Ex.

FIG.

Ex. XCVII.

296. A *bob gin*, for raiſing water. *AB* a large water-wheel carried by the water *W*. *C* and *D* two cranks, upon the axis, on each ſide the wheel, lying contrary ways. *EF*, *GH*, two pieces of timber moving about on the cranks *C* and *D*, and alſo moveable at the joints *F*, *H*, upon two pins. *FI*, *HK* two beams, moving on the axes *L* and *M*. *I*, *K* two arches with chains fixed to them, by means of which the pumps *O*, *N* are wrought. When the water-wheel goes about, one crank as *C* pulls down the bar *EF*, together with the end *F* of the beam *FI*, and at the ſame time raiſes the end *I*, which draws the water up in the pump *O*. In the mean time the other crank *D* is raiſing the end *H*, and depreſſing the end *K*. When by motion of the wheel, the crank *C* begins to aſcend, the end *I* begins to deſcend, and the end *K* to aſcend. So that one beam goes up whilſt the other goes down, and there is always one pump working.

Ex. XCVIII.

297. A *gunpowder-mill*. *AP* the water-wheel; *B* its axis. *RPS* the water-courſe. *E* a ſpur-wheel carrying the two drums *C*, *D*, and the rollers *CF*, *DH*, to which they are fixed. *a*, *a*, *&c.* 10 or 12 pins, or cogs fixed in either roller, equally on all ſides. *b*, *b*, *&c.* 10 or 12 peſtils, 10 feet long, and 4 or 5 inches broad, armed with iron at the bottom; in theſe peſtils are pins fixed to anſwer the pins *a*, *a*; which lift them up as the rollers turn round. *m*, *m*, *&c.* are wooden mortars, into which the peſtils fall, each mortar will hold about 20 lb. of paſte. *OQ*, *IK*, *LN* are timbers through which the peſtils work, and ſerve to keep them direct.

The materials being put into the mortars *m*, *m*; as the mill goes about, the pins in the rollers take up the peſtils *b*, *b* by their pins, and when theſe pins go off, the peſtils fall into the mortars *m*, *m*, and beat the ingredients to a paſte. And as theſe cogs are placed on all ſides the circumference of the rollers, there will be always ſome peſtils riſing, and others falling in a regular order.

Ex. XCIX.

298. A *crane* or engine to raiſe a great weight, and keep it in any poſition. *AB* a double wheel for a man to walk in; *CD* a ſpur-wheel

wheel upon the same axis. *E*, *F*, *G* are three wheels also fixed upon one axis, of which *G* is of wood, and *E* is moved by *CD*. *HI* is a catch moving on the pin *I*, this falls in between the teeth of the wheel *F*. *KLM* a half ring of iron, in which is a grove, going upon the edge of the wooden wheel *G*. *NM* a piece of timber fixed to the ring at *M*, and to the leaver *PN*, and is moveable about the pins *M*, *N*. The leaver *PN* is moveable about the center *O*. *QR* a wooden rod, reaching to the catch *IH*. *PST* a string fixed to the leaver at *P*. *VXIV* the rope going about the axis of the great wheel to raise the weight. FIG. 298.

When the great wheel *AB* goes round, together with *CD*, the rope *VXIV* raises the weight *IV*. The wheel *CD* drives *E*, together with *F* and *G*; and the end of the catch *IH* slides freely over the teeth of the wheel *F*; and the motion being stopped, the catch *IH* acting against the teeth of *F*, hinders the wheel *F* from turning back, and so keeps the weight *IV* suspended. But if you pull at the string *T*, it raises the leaver *PO*, and thrusting the rod *QR* against the catch, raises it out of the teeth of the wheel *F*, and lets the weight *IV* descend. But lest it descend too fast, the leaver *PO* is to be raised higher, by pulling at the string *TS*, and this depresses the end *ON* of the leaver, and draws down the piece *NM*, together with the ring *KLM*, which ring being drawn close against the wheel *G*, stops the motion, or regulates it at pleasure.

Ex. C.

An engine for drawing water. *A* the cog-wheel, 10 feet diameter; *B* its axis, running in the frame *FFFF*, and on the foot *Z*. *C* a trundle 3 feet diameter. *K* its axis 15 or 20 feet long, running in the stocks *G*, *G*. *D*, *D* two cranks of iron on opposite sides of the axis, and 2 feet long. *OP*, *QR* two beams moving upon an axis in the frame *SSSS*. *PD*, *RD* two rods of wood or iron, reaching from the beams to the cranks, moveable about *R* and *P*; and turning round on the cranks *D*, *D*. *I*, *I* two rods of iron, fixed to two chains that go over the arches *O*, *Q*; and to two pistons that work in the pumps *x*, *y*. *E* the tiller to which the horses are yoaked; 1 2 3 4 the path in which the horses go round. *H*, *H*, the surface of the earth. The wheel *A*, and trundle *C* are in a pit; the axis *K* under ground; and the cranks *D*, *D*, are in a pit. 299.

When the horses, walking in the ring 1 2 3 4, draw about the cog-wheel, by the tiller *E*; this turns the trundle *C*, with the

FIG. 299. cranks D, D; and the rod PD being drawn down, pulls down the end of the beam P; and raises the other end O, with the rod Ix; and draws the water out of the pump x. In the mean time, the other crank raises the rod DR, with the end R of the beam; and the other end Q, with the rod I descends; and the piston goes down into the pump y. But as the wheel A goes about, the rod RD is pulled down, and QI rises up, and draws water out of the pump y, whilst OI and the piston descends in the pump x. Thus whilst one piston goes up, the other goes down, and there is always one pump discharging water.

Instead of two cranks, one may have three or four cranks, at equal distances round the axis, and these will move three or four beams QR, and work three or four pumps. But beams of timber should be put between every two working beams, OP, QR, for the axles to run in.

Ex. CI.

300. *AEL a twisting mill to make thread or worsted.* B a cog-wheel 3 feet diameter, of 33 or 34 teeth. C the drum of 4, 6, or 8 rounds, going on the square end of the axis of the cog-wheel. D a spur-wheel, 2 feet diameter, and 30 or 32 teeth; this is fixed to the reel E. The reel consists of 4 long pieces of wood, 6 or 7 feet long, 3 of which are fixed in the cog-wheel D, and are also fixed to one another by cross bars going through the axis of the reel; the fourth long piece of wood which composes the reel, is not fixed in the cog-wheel, but may be set nearer or further from the axis, by help of the pins 1, 1, 1. F a drum of 12 rounds, carried by the cog-wheel B; these rounds are fixed into the barrel G, of 1 foot and 6 or 8 inches diameter. $MNOP$ a fixed frame 6 or 7 feet long, and 4 feet broad. 22, 22, &c. are whorles, carried round by the leathern belt $IKLIH$. These whorles run in iron sockets at the bottom of the frame, and are kept in their places by the snecks 3, 3, fixed to the upper side of the bottom part of the frame; upon the spindles of these whorles are put the bobbings, with the thread or worsted. The spindle and whorle is represented at *a*, the bobbing at *b*, the bobbing with the worsted on it at *c*. The length of the whorle and spindle is 10 or 11 inches, length of the bobbing 6 or 7 inches; diameter of the whorle where the belt runs, about an inch; diameter of the bobbing at top $1\frac{1}{2}$ inch; at the smallest part $\frac{2}{3}$ of an inch; these are for worsted. The whorles may be taken out of the snecks at pleasure, and they are kept in these snecks by a feather put across the slit through two holes. The bobbings they

they use for thread are represented at *d*; *e* is a piece of lead which goes upon the top of the spindle to keep down the bobbing: *f*, *g* are two wires fixed in it, for the thread to run through, from the bobbing to the reel, the diameter of the whorle about half an inch. The number of snecks, spindles, and bobbings on one side of the engine is 20 or 24, that is 40 or 50 in all. 4, 4, &c. are wires in the upper part of the frame, for the thread to run through from the bobbings, the number of wires are equal to the number of spindles. Also in the horizontal beam *QR* are the same number of wires, 5, 5, 5, &c. to direct the thread to the reel. *n*, *n* are rollers for the edge of the belt to move over. 6, 6 are two hanks upon the reel. When the belt grows slack by stretching, the frame *MN* is drove back, by means of a wedge *δ*, and so kept at a greater distance from the roller *G*. FIG. 300.

The trundle *C* may be taken off, and others of more or fewer rungs put on, as occasion requires, by lifting the end *T*, of the axis of the reel, out of its socket; and the finer the thread, the fewer rungs it must have. The circumference of the reel *DE* for worsted is 4 feet, 4 inches; for thread is 5 feet, 5 inches.

When the thread or worsted is wound upon the bobbings, by the help of a wheel, they are put upon the spindles, as *c*; and then put within the belt *IKL* under the snecks 2, 2; then the handle *A* being turned, carries the cog-wheel *B* about, which drives the drum *F*, and the barrel *G*; the barrel *G* moves the belt in direction *IKL* about the frame *MN*, which resting on the whorles 2, 2, moves them, and the whorles and bobbings very swiftly about. In the mean time, the drum *C*, is turned round by the axis of the cog-wheel *B*, and *C* carries about the spur-wheel *D* and the reel, with a slow motion. So the threads being put through the wires 4, 5, and fixt to the reel, these threads will be wrapt about the reel, and make the hanks 6, 6; as many as there are bobbings. When the hanks are of sufficient bigness, they must be taken off the reel, which is done by pulling out the pins 1, 1, and then one side of the reel will fall in, and the hanks slackt, and may be taken off one after another, by lifting the end *V* of the axis out of its socket.

The double yarn, &c. is to be wound tapering on the bobbings, as at *c*, making it broadest at the low end, otherwise it will not come freely off the bobbings, without breaking.

The frame work consists of perpendicular beams fixt in others lying horizontal, as described in the figure, the breadth from *A* to *V* being 9 or 10 feet. The lower part of the frame

FIG. 300. *MN* confifts of two elliptical pieces, cut out of boards, and fet at about a hand's breadth diftance one above the other, with pieces of wood between. In the lower (which is broader than the other) are the fockets, in which the bottom part of the fpindle of the whorles move; in the upper, the fnecks are fixt. The part *OP*, in which are the wires, is an elliptical piece like the under ones, and fixt thereto by 4 perpendicular pieces or pillars of wood. All the reft will be plain from the figure.

Ex. CII.

303. *AEKF is a clock.* The different forms and conftructions of clocks are almoft as various as the faces of thofe that make them. The following is a common 8 days clock. *KF* is the moving part; *AE* the ftriking part.

The work contained between 2 brafs plates is as follows; *F* the *firft* or *great wheel* of 96 teeth; *G* the *fecond wheel* of 60 teeth, its pinion *g* of 8 leaves; *H* the *third wheel* of 56 teeth, its pinion *h* of 8 leaves; *I* the *balance wheel* of 30 teeth, its pinion *i* of 7 leaves; and *K* the *balance.* Likewife *A* the *great wheel* of 78 teeth; *B* the *pin wheel* of 48 teeth, *b* its pinion of 8 leaves; *C* the *hoop wheel* of 48 teeth, *c* its pinion of 6 leaves; *D* the *warning wheel* of 48 teeth, and its pinion *d* of 6 leaves; *E* the *fly*, *e* its pinion of 6 leaves.

The ends *R*, *R*, of the *arbors* of the wheels *A*, *F* come thro' the face of the clock, and thefe arbors are fixt in the *barrels* *P*, *P*, of 6 or 7 inches circumference; and on thefe barrels, the therm *ftrings Tt* are wound, which go round two *pullies* with the weights, that carry the wheels about. Thefe two barrels are moveable round about within the wheels, but are kept from turning back, by the *catch S* and its *fpring*, and the *racket wheels* *Q* fixt to the barrel. The weights are wound up by help of the *winch* or handle 11. In the rim of the wheel *B* are 8 *pins*, which as the wheel goes round, thruft back the end 5 of the *hammer O*, and when it goes off the pin, the *fpring* 7 makes the hammer *O* ftrike againft the *bell N*.

302. The wheel *C* has a *hoop* upon its rim, which is cut away in
303. one place, to let the end 2 of the *detent* fall in. In the rim of the wheel *D*, there is a pin which ftops againft the end *x* of the arm *wx*, and hinders the wheels turning about.

On the axi *op* is fixt the two *pieces vs*, and the detent 1 2. On the axis *qr* is fixt two *pieces wx* and the *lifter* 3; and on the end *r* of that axis, which comes through the fore plate, is put the *lifter* 10, and pinned faft on.

8 The

The *arbor* of the wheel *G* comes through the *face*; upon this arbor between the face and *fore plate*, is put the wheel *z* of 20 teeth, its arbor being hollow, and under the wheel is put the brafs fpring *l*, with the concave fide upward, this fpring having a fquare hole in it, to fit the fhoulder of the arbor of *G*. The wheel *Z* of 40 teeth turns upon a fixt pin or axis, and is driven by the wheel *z*. The dial wheel *f* of 48 teeth, is put with its hollow *focket* upon the arbor or *focket* of *z*, then the face being put on, their ends come through it, and the *hour hand k* is put upon the fquare end of *f*, and the *minute pointer W*, upon the end of *z*, the wheel *z* being thruft down to bend the fpring, and then a pin put in to keep it there; the pinion of *Z*, called the *pinion of report*, has 8 teeth, and drives the wheel *f* and the hour hand. Now the fpring *l* keeps the wheel *z* pretty tight upon the axis of *G*, fo that *G* will carry it about along with it. And if the minute pointer be thruft about, it will force about the wheel *z*, and alfo *Z*, and likewife *f* with the hour pointer. FIG. 302. 303.

The arbor of the wheel *A* goes through the *back plate*; upon it, behind the plate is put the wheel *V* (or *pinion of report*) of 28 teeth, and pinned there. The double wheel *XY* is carried by *V*, and turns upon a pin fixt on the back of the plate. The wheel *X* has alfo 28 teeth, and the *count wheel Y* is divided into 11 parts of unequal lengths, according to the ftrokes the clock is to ftrike at every hour; part of this wheel is reprefented at *s*. A flender fpring is put on with this wheel to keep it tight. This part may be made more fimple, by leaving out the wheels *V*, *X*; and putting *Y* upon the axis of *A* inftead of *V*; but it muft be put on the contrary way.

The arbor of the balance wheel *I* comes through the fore plate, almoft to the face, and through a hole in the face is put the hollow focket of the *fecond pointer* 12; and this fhows the feconds by a fmall circle divided into 60 parts. And the face is alfo divided into two circles, fhowing hours and minutes.

The *pendulum* hangs on the fixt piece of brafs *M*, by a button at top, and a thin piece of brafs going into a flit at *M*, and a flat piece of brafs goes into the *fork L*, fo that if the pendulum moves, it muft move the rod *KL*, and balance *K* along with it.

The *pallats* 8, 9 of the balance *K*, are fo formed, that the under fide of 8, and upper fide of 9, where the teeth of the wheel *I* act, are polifhed planes; and made floping, fo that a tooth fliding along the under fide of the pallat 8, will force

 the

FIG. the balance *K* to the left hand; and a tooth sliding along the
202. upper side of the pallat 9, will force it to the right.
5. 3. The work is put together, by setting the teeth together that are markt in the wheel *B* and in the pinion *c*, and likewise in the wheel *C* and pinion *d*. Then the minute pointer is put on the arbor of *z*, mark to mark; and the hour pointer the same way on the arbor of *f*. And the wheels *z*, *Z*, *f* are set to one another according to their marks.

The weights hanging upon the wheels *A*, *F*; and the pendulum made to vibrate, the wheel *F* drives *G*, which drives *H*, which drives *I*; then whilst the pendulum vibrates to the right, a tooth slips off the pallat 9, and in its return to the left, a tooth slips off the pallat 8, then on the right another goes off 9, and so on alternately; and the weight causing the teeth to act against the pallats of the balance, keeps the pendulum in motion; and the wheel *I* goes round in a minute.

As the wheel *G* goes round it carries about *z*, with the minute pointer once round in an hour; *z* drives *Z*, which drives *f* once round in 12 hours. Whilst the wheel *z* goes round, the pin *m* raises the lifter 10, which lifts up the piece 3, and the arm *wx*; the piece 3 raises the detent 1 2, together with *vs*; the end 2 of the detent being raised above the hoop, the wheel *C* moves about, and by the oblique figure the end of the detent 2, it raises the end of the detent higher, and also raises *s* out of the notch of the count weel. Then the wheel *D* turns round, till the pin in the rim stops at the end *x*, which hinders the motion. But as the wheel *z* goes further about, the lifter 10 falls down off the pin, together with the piece *wx*, and latch 3, which suffers the wheel *D*, and the rest to turn round; and the pin wheel causes the hammer to strike so often, till the end *s* falls into a notch of the count wheel, and then the detent 2 falls into the vacancy in the hoop, and locks the work; which continues so till the next hour, that the piece 10 is raised again; and then she strikes as before; the wheel *C* goes round every stroke of the clock; but she strikes 1 more every succeeding hour, because the teeth between the notches are made longer and longer in the count wheel; and it turns round once in 12 hours.

General rules in all clocks.

In the striking part, the pin wheel being divided by the pinion of the hoop wheel, the quotient shews the number of pins in the pin wheel.

If

If 78 be divided by the number of pins, the quotient shews the revolutions that the pin wheel makes for one revolution of the count wheel. FIG. 302. 303.

The hoop wheel divided by the pinion of the warning wheel, must be a whole number.

In the moving part, the *train* is the number of beats the clock makes in an hour; which is 3600, if she beats seconds: in this case the balance wheel must have 30 teeth.

If G turns round once in an hour and shews minutes; then the quotient of G divided by the pinion of H, multiplied by the quotient of H divided by the pinion of I, and that multiplied by twice the teeth in I, must be equal to the train. And if she beats seconds, then the product of the two quotients must be 60.

If also G shows the hours, then the quotient of f divided by the pinion of Z, multiplied by the quotient of Z divided by z, must be 12.

From the great wheel to the balance, the wheels drive the pinions; but to the dial wheel, the pinions drive the wheels; the former quickens, the latter lessens the motion.

Any wheel being divided by the pinion that works in it, shews how many turns that pinion hath to one turn of the wheel. As if the pinion be 5 and the wheel 60, it is set down thus,

5) 60 (12 times. Or thus $\frac{60}{5} = 12$ times.

The teeth of several wheels and pinions that work in one another, are set down thus,

4) 36 (9 times
8) 80 (10 times
6) 54 (9 times
5) 40 (8 times

Or thus $\frac{36}{4} \times \frac{80}{8} \times \frac{54}{6} \times \frac{40}{5}$.

In the former way, the number on the left hand of any wheel is the pinion that it drives; and the number over it is the pinion on the same axis. In the latter way the several fractional quanties represent the quotient.

Any wheel and the pinion it drives, will have the same motion with another wheel and pinion, when their quotients are equal.

FIG. equal. Thus a wheel of 36 drives a pinion of 4, all the ſame
302. as a wheel of 45 does a pinion of 5; or a wheel of 90 a pi-
303. nion of 10.

In any motion you may uſe one wheel and one pinion, or elſe ſeveral wheels and ſeveral pinions, provided they all give the ſame motion. Therefore when a number is too big to be cut in one wheel, you may divide it into two or more quotients.

In a wheel and pinion that work in one another, their diameters muſt be as the number of teeth in each. And the diameter muſt be meaſured, not to the extremity, but to the middle of the tooth, or where they act.

303. The excellency of clock-work, conſiſts in forming the teeth truly, and to fit the notches exactly without ſhaking, and to play freely; the teeth muſt be cut into the form of cycloids, which reſembles the ſhape of a bay leaf.

A clock goes exacter as the pendulum is longer, and the bob pretty heavy, and to make but ſmall vibrations; and for more exactneſs, to play between two cycloidal cheeks; and the longer the arms $K8$, $K9$, the eaſier the clock goes. The length of a ſecond pendulum is 39 $\frac{2}{10}$ inches. See the theory of pendulums in Prop. XL. XLI. LVIII.

The pallats 8, 9, are here formed after the common way: but there is another way of forming them. From the center of motion θ, deſcribe two ſmall arches $\alpha\beta$, and $\delta\varepsilon$. Theſe ſmall lines or planes $\alpha\beta$, and $\delta\varepsilon$, and alſo the working ſide of the tooth $\lambda\mu$, muſt all range to $\varkappa$ the center of the balance wheel. And the ends of the pallats $\alpha\gamma$, and $\delta\eta$, muſt range a little to the right hand of the center θ. Then the teeth of the balance wheel will fall alternately on the ſides $\alpha\beta$, and $\delta\varepsilon$. And any tooth, whilſt it acts againſt $\alpha\beta$, or $\delta\varepsilon$, will have no effect in moving the pendulum; but lies *dead*, till it makes its *eſcape* off the angle α or δ; and then in moving along the plane $\alpha\gamma$, or $\delta\eta$, it forces the pendulum to the right or left.

But the conſtruction will be better thus; take $\theta\tau$, $\theta\phi$ each equal to $\frac{1}{3}\,\alpha\delta$. From the center τ deſcribe the arches $\alpha\beta$, $\delta\varepsilon$; and let the ends $\alpha\gamma$, $\delta\eta$, range to ϕ. Or perhaps it may anſwer the end as well, to deſcribe $\alpha\beta$, and $\delta\varepsilon$, from the center θ; and let the acting ſide of the tooth range (not to $\varkappa$, but) to υ the outſide of the circle deſcribed with the radius $\varkappa\upsilon = \frac{1}{3}\,\varkappa\mu$.

The inconvenience of any of theſe conſtructions, is, that the pallats are too thick, and can hardly find room to fall in between the teeth of the balance wheel.

Ex.

FIG. 304.

Ex. CIII.

ABC is a *cutting engine* to cut the teeth of clock wheels. *AC* an iron plate $2\frac{1}{2}$ feet long, and 3 or 4 inches broad *EE* another plate fixt 4 or 5 inches lower. *G* a *slider*, sliding along a groove in the end *C*: this is made of several plates of iron fixt to one another with screws, and fitting closely to the edges of the plate, and to the sides of the groove, and likewise to the upper and under side of the plate; this is to cause it to move truely along the groove when forced forward or backward, by the screw at *I* and its handle; for this screws through *C*, and turns round in a collar in the end *G*. The end of this slider turns up perpendicular; to this is fixt the part *F* by a pin *K*, which goes square into this part, and through a round hole in *F*; so that the part *F* can turn about the screw pin *K*, and may be fixt by turning the nut 2 with the key 9, which nut screws upon the end of the pin.

B is a brass wheel of 96 teeth, carrying the pinion *D* of 12 leaves; these move between the cheeks *LL*, *MM*; which are joined by the cross bars *N* and *P*; these cheeks and their machinery turn round on the axis *LM*, in the part *F*. *f* is the cutting wheel, whose edge is nothing but a file to cut the teeth, as it goes about; this goes upon the arbor of the pinion *D*. There are a great number of these cutting wheels, of different shape and bigness; which may be taken off the arbor and others put on; these parts are described at *a*, *b*, *c*, *d*; *a* the pinion on its arbor, *b* the cutting wheel going upon the arbor which is octagonal, and fits it exactly, having the sides marked that are put to each other, *c* a hollow piece which goes on the same axis; and the nut *d* screws on the end of the arbor, to keep *c* and the wheel *b* fast on. The ends of the arbor are hardened steel, and pointed; and this arbor runs between the cheeks *LM*, through which cheeks there goes 2 screws, with holes to receive the points of the arbor; and these screws are set to a proper distance, by screwing them in or out by help of a key *p*, going on square upon the end, and then the screws are lockt there, by the nuts *O*, *O*. *rs* is a spring, fixt with one end to the under-side of the cross-bar *N*, and the other end *s* lying upon the plate *AI*, and this spring raises the part *LLMM*, when the notch is cut *tu* is a screw pin going through the bar *P*, its end *u* rests upon the plate *AI*, and hinders the wheels from descending lower. 1, 1 are two screw pins, which screw through *LM*, by help of the key *p*, and go with their points

FIG. 304. points into *F*, which has two holes to receive them; these pins are lockt to *L*, *M* by turning the two nuts, which also screw upon the pins. These screw pins, nuts, and cheeks, all turn round together in the holes in *F*.

H is the *dividing plate*, being a brass circular plate 15 or 16 inches diameter. This plate is fixt to a hollow brass axle *Q* an inch in diameter; and this axle goes through the two plates *AI*, *EE*; and both the wheel and its axle turn about together; the lower plate cannot be seen, but is represented at *R*. Near the edge of this plate, there are 24 concentric circles, each divided by points into a certain number of equal parts, *viz.* 366, 365, 360, 118, 100, 96, 92, 90, 88, 84, 80, 78, 76, 72, 70, 68, 64, 62, 60, 58, 56, 54, 52, 48. The use of these is to divide a revolution into any number of equal parts, according to these different circles.

ei is an arbor going through the hollow axle *QR*, with the shoulder *b* against the top of that axis, then the nut *n* is screwed upon the end *i*, to keep it fast. *m* is the wheel to be cut into teeth; there is a hole made in the center, just to fit the part *ge*, which being put on, and the piece *l* above it, they are then screwed hard down with the nut *k* going on the end *e*. Then if the wheel *H* be turned round, it carries about with it the wheel *m*. There are several arbors *ei*, for fitting different wheels *m*.

wy is a moveable index; it turns about a nail, as a center in the end *w*, there being a slit in it, to let the bottom of the screw *x* pass through as it moves. *y* is moveable back and forward, and may be fixt any way by the two screws. *z* is a steel point, which moves along the circumference of any circle you require, from one point to another. *T* is the winch to turn the wheel *B*; 2 is the handle to pull down the machinery near the plate.

To use this machine. An arbor *ei* proper for the wheel *m*, which is to be cut, being put through the axis *Q*, and screwed fast, as appears at *R*; and then the wheel *m* and the parts *k*, *l* put on above *Q*, and screwed fast. Loosen the screw *x*; and, moving the index *wy* till the steel point *z* fall in the circle, containing the same number of parts, the wheel *m* is to be divided into, there screw it fast with the screw *x*. Then putting on the cutting wheel *f* proper for the work, turn the handle and screw *I*, and drive the machinery with the wheel *f* towards *Q*, till the edge of *f* lie just over the edge of the wheel *m* to be cut; there fix it by the handle *F*; and turn the wheel *H* till *z* falls into some point of the circle; then take hold of the handle *S*, and pull it down, till *f* falls against the edge of *m*; then hold-

I ing

ing it there with one hand, turn the winch *T* with the other; which carries about *B*, and this drives *D* with the cutter *f*, and this motion cuts a notch in the edge of *m*, and when it is deep enough, the pin *m* (properly ſet) ſtops at the plate *AC*, and hinders it from going further. Then let go *S*, and the ſpring *rs* raiſes up the wheels, *&c.* This done, pull about the wheel *H*, till *z* fall in the next point of diviſion; then draw down *S*, and turn the machine as before, till you have made another cut deep enough. And thus you muſt proceed, till *z* has gone thro' all the points of diviſion in the circle, and then your wheel is cut into its proper number of teeth. FIG. 304.

When the number of teeth wanted to be cut anſwers to none of the circles, take ſuch a circle as can be divided by your number, and if the quotient be 2, 3, 4, *&c.* then you muſt ſet *z* to every 2d, 3d, 4th point, *&c.* ſkipping the reſt. As if you want 21 teeth, take the circle 84, which divided by 21 gives 4; ſo that you muſt ſet *z* to every 4th tooth only, and ſo cut it.

A crown wheel may be cut the ſame way; but then the center of the wheel *f* muſt be brought over the edge of the wheel to be cut, and there fixed. Alſo oblique teeth may be cut in a wheel after the ſame manner; but you muſt firſt eaſe the ſcrew *K*, and then turn the cutting frame about *K* as an axis, till the cutter *f* have a proper degree of obliquity, and there ſcrew faſt the pin *K*, by the nut 2, and proceed as before.

After the teeth are cut with this engine, they are ſtill to be wrought into their proper form, with files ſuitable for the buſineſs; and this the workman muſt do by hand.

Ex. CIV.

EH is a *glazier's vice*, for drawing window lead. *PG*, *QH* two axles, running in the frame *KL*, *ML*. *C*, *D* two wheels of iron caſe-hardened, $1\frac{1}{2}$ inch broad, and of the thickneſs of a pane of glaſs; theſe wheels are fixed to the axles, and run very near one another, not being above $\frac{1}{16}$ of an inch diſtant; acroſs their edges are ſeveral nicks cut, the better to draw the lead through. *E*, *F*, two pinions of 12 leaves each, turning one another, and going upon the ends of the axles, which are ſquare, and kept faſt there by the nuts *P*, *Q*, which are ſcrewed faſt on with a key. *A*, *B* two cheeks of iron, caſe-hardened, and fixed on each ſide to the frame with ſcrews; theſe are cut with an opening where the two wheels meet, and ſet ſo near the wheels, as to leave a ſpace equal to the thickneſs of the lead; ſo that between 305.

FIG. 305. the wheels and the cheeks there is left a hole, of the form reprefented at *N*, which is the fhape of the lead when cut through. The frame *KLML* is held together by crofs bars going through the fides and fcrewed on: and a cover is put over the machine to keep out duft; and it is fcrewed faft down to a bench, by fcrew nails *LL*.

When it is ufed, the lead to be drawn is firft caft in moulds, into pieces a foot long, with a gutter on each fide. Take one of thefe pieces, and fharpen one end a little with a knife, and put it into the hole between the wheels; then turning the handle *I*, the lead will be drawn through, of the form defigned.

Ex. CV.

306. *AC* a *water-mill* for grinding corn, without either trundle or cog-wheel. *BC* is the arbor, or axis of the mill; this is a cylindrical piece of wood, about two feet diameter; *GHIKLMN* is a leaf or wing of wood, whofe breadth is about the radius of the arbor; this runs fpiral-wife round the arbor from bottom to top, afcending in an angle of about 35 degrees; it muft every where ftand upright on the furface of the arbor. Inftead of one you may ufe two of thefe fpiral leaves, efpecially if they be narrow. This arbor and its fpiral leaf, turns round upon a pivot *P* at the bottom; and at the top *B*, it has a fpindle which goes through a plank, and is fixed to the upper mill-ftone *D*, which turns round with it; fo that the arbor has little or no friction. *QRST* is a hollow cylinder, made of ftone or brick, to enclofe the arbor and its leaf; and whofe infide is walled as near as poffible, juft to fuffer the leaf to turn round without touching; fo that no water can efcape between the leaf and the wall; and confequently it can only run down the declivity of the leaf; its top is reprefented by the circle *QBTV*. *RWS* is an arch to let the water out at the bottom, to run away; and big enough to go through to repair the engine. *F* is the trough that brings the water; *D*, *E* the two mill-ftones. *A* the hopper and fhoe. The arbor and its leaf may be cut altogether out of the folid trunk of a tree; or elfe the leaf may be made of pieces of boards, nailed to feveral fupporters of wood, which are to be let every where into holes made in the body of the arbor, fo as they may ftand perpendicular to its furface; and all fet in a fpiral. And the fpiral is made on this confideration; that for every 10 inches in the circumference of the axis, you muft rife 7 inches in the length. But at the top *G*, it will be better to

to rise faster, so as to have its surface almost perpendicular to the stream. FIG. 306.

When the mill is to go, the corn is put into the hopper at *A*, which runs down the shoe, through the mill-stone *D*. And the spout *F* being opened, the water falls upon the oblique leaf *GHIK*, and by its force turns the axis *BC* about, and with it the stone *D*, and grinds the corn.

Ex. CVI.

DBF is the *arch of a bridge*, which shall sustain itself, and all the parts of it in equilibrio. Such an arch will be stronger than any other, because an arch that can sustain itself, will more easily sustain an additional weight, than an arch that cannot sustain itself, but only by the cohesion of the mortar. This arch *DBF* is a semicircle, whose center is *R*, and vertex *B*; and the wall *ATta* must be so built, that the height *AT*, in any place *A*, must be as the cube of the secant of the arch *BA*, which will cause it to run upwards towards *D*, in the form of the curve *tST*. But as this form is not commodious for a bridge, the construction may be performed thus. In any place of the arch, as *A* let the superincumbent part *AT* be built of heavier materials, than at *B*; in proportion of the cube of the secant of the arch *BA*; for the parts near *B*; but in something less proportion in the parts towards *A* and *D*. And the right line *GSg* being drawn, will nearly terminate the top of the wall. But as materials cannot well be procured for this purpose, the following way may be used. 307.

With the radius *BR* describe the arch *DBd*, of 90 degrees; *DB*, *Bd* being each 45. And if *BR* consist of 100 parts, make *BS*, 16. Draw the right line *GSg*, perpendicular to *SBR*: and the arch *DBd* shall support the wall *DGgd* in equilibrio in all its parts. If the arches *DB* and *Bd* be made each 60 degrees, and the height *BS*, 7 parts; and the right line *GSg* drawn; then the arch *DBd* will equally support the wall *DGgd* in all parts: but then the materials made use of about the places *a*, *a*, ought to be only about half the weight of those at *B* and *D*. And these are the principal cases in which a circle is serviceable, for making an arch stand in equilibrio. 308.

Another equilibrial arch is from the catenary. Make the *latus rectum BS*, 100 equal parts; *BR*, *AR*, *RF* each 159; describe the catenary *ABF*, through the points *A*, *B*, and *F*. Then *ABF* will be an arch which will support the wall *AGgf* in equilibrio, in every point of it. The fault of this arch is, that by reason 309.

 of

FIG. 309. of the height *BS*, there is too much weight of wall upon it, which will endanger the ſinking the piers, except the foundation be very good; and it likewiſe raiſes the bridge too high.

310. Another arch of equilibration is this; make *SB*, *BR*, *AR* of any lengths at pleaſure; draw the right line *GS* parallel to the horizon; and to the aſſymtote *GS*, draw the logarithmic curve *AB*: which may be done thus; draw *AG* perpendicular to *GS*; divide *SG* into any number of equal parts, and as many points of diviſion as you have, find ſo many mean proportionals between *SB* and *GA*; ſet theſe from the reſpective points in *SG* downwards, in lines drawn through theſe points parallel to *SR*; and theſe will give ſo many points, through which the curve *BA* is to be drawn; and the curve *bF* is drawn the ſame way; between theſe, the pier *BD* is placed with a tower upon it. The only fault this arch has, is, that the water way is diminiſhed by the pier *BD*; and as many arches, ſo many ſupernumerary piers there will be.

311. I ſhall now ſhew how to deſcribe an arch clear of all theſe inconveniencies. Make *BR*, *AR*, *RF*, each equal to 30 feet; *BS*, $3\frac{1}{2}$ feet. Draw *AG*, *Fg* parallel to *RS*. Divide *SG*, *Sg* into 30 equal parts or 30 feet; through all the points of diviſion, draw lines parallel to *SR*, as *TC*. Then upon each of theſe lines, ſet off from *SG* downwards; the number of feet you find in the following table, reſpectively, as *TC*; then *C* will be in the arch. Do the ſame for the ſide *Sg*. Then the curve *FBCA* drawn through all theſe points *C*, will be the arch required. The curve is eaſily drawn through theſe points, by help of a bow held to every three points; or rather to four or five points at once; which may eaſily be done by two or three perſons holding it.

value

FIG. 311.

value of ST in feet.	value of TC in feet and dec. parts.	value of ST.	value of TC.	value of ST.	value of TC.
0	3·500	11	5·754	21	14.014
1	3·517	12	6.231	22	15 417
2	3·568	13	6.769	23	16.970
3	3·653	14	7·372	24	18.687
4	3·774	15	8.047	25	20.585
5	3 931				
6	4·127	16	8.799	26	22.682
7	4·362	17	9.636	27	24·999
8	4·639	18	10.567	28	27·557
9	4 961	19	11.600	29	30 381
10	5·332	20	12·745	30	33·500

If the thicknefs of an arch at top, *BS* be fuppofed to be 3 feet, 4 feet, or 5 feet, *&c.* It will require a different curve to be conftructed; but this feems to be ftrong enough for the bignefs of the arch; efpecially, if built of ftrong, found ftone. Here $3\frac{1}{2}$ feet is affigned for the thicknefs of the arch; but it muft be made 2 or 3 inches lefs, on the account of the parapet wall, for this adds weight to the whole. Alfo if the top *GS* is not exactly horizontal, but is 2 or 3 feet lower at *G* than at *S*; the thicknefs *BS* ought to be 2 or 3 inches lefs upon that account; or if higher at *G*, two or three inches more: but thefe niceties make no fenfible difference in practice.

This curve differs from the catenary (in fig. 309.) For at the vertex *B* it is lefs curve than the catenary; and towards *A* it is more curve. The curvature at *B* in this arch is very near that of a circle, whofe radius is *BR*. And the curvature increafes from the vertex *B*, and is at leaft about *C*.

At the points *A*, *F*, where the arch fprings, it rifes at an angle of 73° : 1′, above the horizon.

If an arch is required to be either greater or lefs than this, it is no more than taking any other equal parts inftead of feet; and fetting off all the lines by thefe equal parts.

In this fcheme I have drawn a circle to fhew the difference between that and this arch. The like I have done in fig. 309, and 310. Whence it appears, that a circle circumfcribes all thefe arches of equilibration; and confequently a circle is too curve at the lower parts, or at the hanch of the arch.

If

FIG. 311. If any architects or builders of churches or bridges, ſhall pleaſe to make uſe of this curve here conſtructed (fig. 311.) for the form of an arch; they will find it the ſtrongeſt arch poſſible to be made, for theſe given dimenſions. And where many thouſand pounds are laid out in building a ſingle bridge, it is certainly worth the pains to ſeek after the form of an arch, which ſhall be the ſtrongeſt poſſible, for ſupporting ſo great a weight. And it is very ſurprizing that no body has attempted it. Inſtead of that, all people have contented themſelves with conſtructing circular arches; not knowing that different preſſures againſt the arch, in different places, require different curvatures, which does not anſwer in a circle where the curvature is all alike. A circle, it is true, is very eaſily deſcribed, and that may be one reaſon for making uſe of it: but ſurely, the deſcription of the curve here given, is very eaſy by the foregoing table, and can create no difficulty at all. If there be any difficulty in the practice, it is only in cutting the ſtones, of a true curvature, to fit the arch exactly in all places; but this is eaſily managed with a little care, by taking proper dimenſions; obſerving that every joint muſt be perpendicular to the curve in that point.

A circle, or any other curve, where the curvature is not properly adapted to the weight ſuſtained, is not capable of ſuſtaining ſo vaſt a weight; but muſt in time give way, and fall to ruin; except the mortar happen to be ſo ſtrong as to keep it together. On the contrary, the arch here deſcribed, ſuſtaining every where a quantity of preſſure proportional to its ſtrength, will never give way, ſo long as the piers, which are its baſes, ſtand good; but by virtue of its figure, will ſtand firm and unſhaken, as long as the materials the arch is made of, will laſt.

As to the piers, their thickneſs may be $\frac{1}{5}$, $\frac{1}{6}$, or $\frac{1}{7}$ the width of the arch, according to the firmneſs of the ground they are to ſtand on. They muſt be conſiderably broader than the bridge, reaching out on each ſide into the water, being built with ſharp edges to divide the ſtream. At the bottom they muſt be well fenced with ſterlings for their ſecurity. The outermoſt pier muſt be built far backwards, to ſuſtain the oblique preſſure of the arch, which has nothing elſe to butt againſt; otherwiſe the pier or buttreſs will yield to the preſſure of the arch, and the arch will break.

Another conſtruction.

In the former conſtruction, I made the height *BS* to be only $3\frac{1}{2}$ feet. But as that may be reckoned too weak for an arch of 60 feet wide, like Weſtminſter bridge, where the height above the arches is 8 or 10 feet. Therefore I have here

here given a new table for conſtructing the arch of a ſtrong bridge, calculated upon the ſame principles as the former, being 7 feet, all the other dimenſions remaining the ſame. This arch riſes from the pier at an angle of 70° 20′. In the former I ſet off all the points of the curve from the line *SG*; in this I ſet them off from the line *BEE*, which is a tangent to the top of the arch at *B*. FIG. 311.

The conſtruction is thus. Having drawn the line *EBH* through the top *B* of the arch parallel to the baſe *AF*, take from the table col. 1 any length, and ſet it from *B* in the line *BE*, as to *L*, and from *R* to *I*, and draw the line *LI*. Then from col. 2. of the table, take the correſpondent length of *LC*, and ſet it from *L* to *C*, in the line *LI*; then *C* is a point in the curve. And thus all the other points of the arch muſt be found, and then a curve drawn regularly through them all, gives the form of the arch.

The TABLE.

value of *BL* in feet.	value of *LC* in feet.	*BL*	*LC*	*BL*	*LC*
1	0.021	16	6.251	25½	19.252
2	.086	17	7.173	26	20.263
3	.194	18	8.183	26½	21.316
4	.346	19	9 285	27	22.412
5	.543	20	10.488	27½	23 553
6	.787	20½	11.129	28	24.741
7	1.078	21	11 798	28½	25.978
8	1.419	21½	12.495	29	27 266
9	1.811	22	13.223	29½	28.605
10	2.258	22½	13 981	30	30.000
11	2.761	23	14.772		
12	3.324	23½	15.596		
13	3.951	24	16.455		
14	4.644	24½	17.349		
15	5.409	25	18.281		

Ex. CVII.

QCF is the *weighing engine*, at the turnpikes, for weighing road waggons. *CD* is a ſtrong beam of wood moving about the center *I*. *EF* a ſteel yard, moveable about the center *H*, and 312.

FIG. 312. and ſuſpended at *D*, by the iron hook *DH*. *PA* ſeveral iron chains ſuſpended at a hook, moveable about the center *P*; *PH* is about 3 or 4 inches, *HF* about 10 or 12 feet. The 4 chains at *A* are to put round the waggon. *F* a leaden weight fixed at the end *F*, whoſe weight is about $1\frac{1}{2}$ hundred weight. *G* a moveable weight of $\frac{1}{2}$ of a hundred weight, this is moved along the graduated beam *HF*, at pleaſure. *KNL* a ſcaffold to walk on. *CS* is a chain hanging at *C*, and fixed to the braſs pulley *S*. Round this pulley goes the rope *MSR*, whoſe end *M* is fixed to the croſs bar *QQ* of the frame *QQTT*. In this frame, the wheels and axles 1, 2, 3 move round; being turned by the handle *B*, fixed to an iron wheel or fly. Theſe wheels and trundles are iron, the trundles contain 11 teeth, the wheels about 60. The rope *R* is wound about the wooden axle 3, being 5 or 6 inches diameter. At the end of the axle oppoſite to *B*, is another handle to be uſed upon occaſion. The frame *QT* is fixed faſt in the ground, that it may not be pulled up. The beam *CD* and ſteelyard *EF*, move between the cheeks *KZ* and *NX*, which ſerve to guide them, and likewiſe ſtrengthen the frame they move in, which frame is tied together with ſeveral braces, as *NO*, *VO*, *&c.*

When any waggon is to be weighed, the 4 chains *A* are hooked round it, and a man turns the handle *B*; which, by turning the wheels, winds the rope about the axis 3, which pulls down the end *C*, which raiſes the end *D*, of the leaver *CD*. The end *D* raiſes the ſteelyard *EF*, with the chains *A*, and the waggon; and a man upon the ſcaffold *NL*, moves the weight *G* till it be in equilibrio; and the diviſions of the beam ſhew how much the waggon is above 60 hundred weight.

In ſome engines the beam *CD* is wanting; inſtead of which there are two blocks and pullies, the upper one faſtened to a croſs beam *ZX*; the lower is hooked to the piece *DH*; and the rope goes from the top block, to the axis 3; but in this caſe, the axis of the wheels are parallel to the ſide of the machine, and not perpendicular, as they are in the former; and then there is but one wheel and pinion, each of iron, the wheel of 110 or 120 teeth, and pinion 11.

Laſtly, in ſome machines that likewiſe want the beam *CD*; the wheels and axles 1, 2, 3 are placed in the top of the machine, above *ZX*; where being turned round, they raiſe the beam *EF*, either by a rope going from it, or by blocks and pullies.

FIG. 313.

Ex. CVIII.

HRK is a *large organ* *IIIH* the *sound board*; this is composed of two boards; the *upper board*, or *cover HHH*; and the *under one III*, which is far thicker than the upper one. Each of these is made of several planks laid edge-ways together, and joined very close. In the under side of the under board, there are several channels made, running in direction *LL*, *MM*, *&c.* continued so far, as is the number of stops in the organ; and coming almost to the edge *IIK*. These channels are covered over very close, with leather or parchment, all the way, except a hole which is commonly at the fore end next *HK*, upon which a valve or puff is placed. These channels are called *partitions*. When this flap or valve is shut, it keeps out the air, and admits it when open. On the upper side of the under board, there are likewise cut several broad, square gutters, or channels, lying cross the former, but not so deep as to reach them; these lie in direction *LN*, *PQ*, *&c.* And to fit these channels, there are as many wooden *sliders* or *registers*, *f*, *f*, *f*, *&c.* running the whole length; and these may be drawn in or out, at pleasure. The number of these is the same as the number of stops in the organ.

IKKK the *wind chest*; this is a square box, fixed close to the under side of the under board, and made air tight, so that no air can get out, but what goes through the valves, along the partitions.

V, *V* are the *valves* or *puffs* which open into the wind chest; and are all enclosed into it, and may be placed in any part of it, as occasion requires. One of these valves with the spring that shuts it, and wire that opens it, is represented apart, on the left hand.

C, *D*, *E*, *F*, *&c.* are the *keys* on which the fingers are laid, when the organ plays. These keys lie over the horizontal bar of wood *W*; in which are stuck as many wire pins *z*, *z*, on which the keys are put; and the keys move up and down upon this bar, as a center. 3 is another bar, against which the keys fall when put down; on this also are several wires, going through the keys to guide them; and on this bar a list is fastened, to hinder the knocking of the keys against it.

Now the keys are made to communicate with the valves several ways, as I shall now describe. *s*, *s*, *s*, are the *key rollers*, moving on the pivots *t*, *t*; these rollers lie horizontally one above another, and one at the end of another; of such a length, as to reach from the valve to the key. *a*, *a*, *a*, arms

 or

FIG. 313. or leavers fixed to the key rollers; *w*, *w* the *valve wires* fixed to the arms *a*, *a*, and to the valves *V*, and going through the holes *b*, *b*, in the bottom of the wind chest. *b*, *b*, *b* arms fixed likewise to the key rollers. *d*, *d*, *d* the *key wires*, fixed to the arms *b*, *b*; and to the keys *C*, *D*, *E*. Now putting down the end of any of the keys *C*, *D*, *E*; it pulls down the arm *b*, by the wire *d*, which turns the roller *s* about, with the arm *a*, which pulls down the wire *w*, which opens the valve; which is shut by the spring, as soon as the key is let go. In this construction, there must be a worm spring fastened to the key, and to the bar *W*, on the further side, to keep the end 5 of the key down.

Another method of opening the valves is this. *xy*, *xy*, are slender leavers moveable upon the centers 1, 1. 5*x*, 5*x* are wires going from the far ends of the keys, to the ends *x* of the leavers. *yV*, *yV*, other wires reaching from the ends *y*, of the leavers, through the holes *b*, to the valves *V*. So that putting down the key, *C*, *D*, &c. raises the end 5. which thrusts up the end *x* of the leaver, by the wire 5*x*; this depresses the end *y* of the leaver; which pulls down the wire *yV*, and opens the valve *V*.

A third way of opening the valves is this. At the end of the key 6, is a leaver 8, 9, moving upon the center 7. This with the key makes a compound leaver. From the end 9, there is a wire goes to the valve. Now putting down the end 6 of the key, raises the end 8, which depresses the end 9 of the leaver 8 9, and pulls down the wire, and opens the valve. I have only drawn one of these in the scheme, and but a few of the others, to avoid confusion.

R, *R* are the *rollers* to move the sliders, by help of the arms *cf*, *cf*, which are fixed horizontally in these rollers. *ke*, *ke* are leavers also fixed in the rollers. *le*, *le* are the handles, which lie horizontally, and pass through the holes *l*, *l*, and are fastened to the leaver *ke*, being moveable about a joint at *e*.

Now any handle *lp* being drawn out, pulls the end *e* towards *l*, which turns *Rk* about, along with the arm *cf*; and the end *f* pulls out the slider *fg*. And when *p* is thrust in, the arm *cf* likewise thrusts in the slider *fg*.

Upon all the several rows of holes which appear on the top of the upper board, are set upright so many rows of pipes; *X* is a *flute pipe* of wood, *Z* a *flute pipe* of metal, *Y* a *trumpet pipe* of metal. The pipes pass through holes made in boards, placed above the upper board, to keep them from falling.

The

The pipes are made to communicate with the wind cheſt after this manner. When any ſlider *fg* is drawn out, holes are bored through the upper board, through the ſlider, and through the under board, into the partition below: ſo that any pipes, placed upon theſe holes, will then communicate with the partition; which, by its valve, communicates with the wind cheſt. But when the ſlider is thruſt in, the holes in the ſlider do not ſtand againſt the holes, in the upper and under boards; and the communication is ſtopt, ſo that no wind can get to the pipe. FIG. 313.

qT, *qT* are the *bellows*, which muſt be two at leaſt. *q*, *q* the *wings*; *O*, *O* the *handles*, moving upon the fixt axes *nn*, *nn*. Each of theſe bellows conſiſts of two boards; the under board is fixt immoveable. In this there is a *valve r* opening inwards, and a tube leading to it, called the *conveying tube*. There is alſo a hole in this under board, from which a tube leads to the *portvent*, which is a ſquare tube 24, riſing upwards, and is inſerted into the under ſide of the wind cheſt at 2. And in the tube leading to the portvent, there is a valve which opens towards the portvent; which ſuffers the air to go up the portvent, but none to return. All the bellows are conſtructed after the ſame manner. Now the handle *O* being put down, raiſes the upper board *T*, and the air enters through the valve *r*; and when the handle is let go, the weight of the upper board *T*, (which carries 3 or 4 lb. to every ſquare foot) continually deſcending, drives the air through the portvent to the ſound board. And as one pair of bellows, at leaſt, is always deſcending, ſince they work alternately, there will be a conſtant blaſt through the portvent.

In chamber organs, there is but one pair of bellows, which conſiſts of 3 boards, in nature of a ſmith's bellows; and ſo has a continual blaſt.

All the inner work is hid from ſight, by the face of the inſtrument ſtanding upon 36.

As many partitions *LL*, *MM*, &c. as there are in the ſound board; ſo many valves *V*, *V*, rollers, *s*, *s*, or elſe ſo many leavers *xy*, or 89, and their wires, and that is juſt as many as there are keys *A*, *B*, *C*, *D*, &c. And there are generally 61 keys, with flats and ſharps; reaching from *G* to *G*, the compaſs of 5 octaves. But the ſcheme could not contain them all. Likewiſe there are as many handles *l*, *l*, &c. rollers, *R*, *R*, &c. ſliders *f*, *f*, &c. as there are different ſtops upon the organ. And it muſt be obſerved, that, between the ſliders *f*, *f*, &c. there are as many ſliders, on the right hand; and the ſame number of handles and rollers, which cannot be expreſſed in

 this

FIG. 313. this ſcheme. And other rows of pipes placed between *LN*, *PQ*, &c. And towards the middle of the organ, the leaſt pipes are placed, and the leaſt partitions; the greateſt being on the outſide.

There are many ſtops in ſome organs, but generally 10 or 12 on each hand; theſe are ſome of them, diapaſon, principal, fifteenth, twelfth, tearce, cornet, trumpet, french horn, vox-humana, flute, baſſoon, cremona, &c. and a contrivance to ſwell the notes of ſome of the ſtops.

When this noble inſtrument is to be played upon, put down the handle *O* of the bellows, this raiſes the upper board *T*, and cauſes the air to enter in at the valve *r*. Then that handle being let go, the other handle *O* is put down; during this time, the board *T* of the firſt, deſcending, and ſhutting the valve *r*, drives the air through the other valve, up the portvent into the wind cheſt. Then drawing out any handle, as the flute ſtop *pl*, this draws out the ſlider *fg*, and all the pipes in the ſet *LN* are ready to play, as ſoon as the keys *C*, *D*, *E*, &c. are put down. Therefore putting down the key *D*, by laying the finger upon it, opens the correſpondent valve *mV*, and the air enters thro' it, into the pipe *X*, and makes it ſound. In the ſame manner, any other pipe, in the ſet *LN*, will ſound, when its key is put down. But no pipe in any other ſet *PQ* will ſpeak, (becauſe the communication is ſtopt) till its ſlider *f* is drawn out by the correſponding handle *l*.

Pipes are made either of wood or metal; ſome have mouths like flutes, others have reeds. The ſmalleſt pipes are made of tin, or of tin and lead. The ſound of wooden and leaden pipes is ſoft. Short pipes are open, and the long ones are ſtopt; the mouths of large ſquare wooden pipes are ſtopt with valves of leather. Metal pipes have a little ear on each ſide of the mouth, to tune them, by bending it a little in or out. Whatever note any open pipe ſounds, when the mouth is ſtopt it will ſound an octave lower: and a pipe of twice its capacity will ſound an octave lower.

314. It will not, I think, be foreign to my deſign, if I give a ſhort account of the method of tuning organs, or harpſicords. But I muſt firſt premiſe ſomething concerning the ſcale of muſic. It is known from undoubted experiments, that if *AL* be a ſtring of a muſical inſtrument, and if the ſtring be ſtopt ſucceſſively at *Y*, *s*, *q*, *p*, *o*; and $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$ of it be made to vibrate, it will ſound an eighth, a fifth, a fourth, a greater third, and a leſſer third, reſpectively.

Now

Now as the difference between the fourth and fifth is accounted a whole tone, whereof there are 6 to make up the octave; therefore we shall have $\frac{3}{4} - \frac{2}{3}$, or $\frac{1}{12}$ for the difference of the strings, that are to sound a note one above the other, whereof the greater is $\frac{3}{4}$. Consequently if the string is 1, that difference would be $\frac{1}{9}$. Therefore $\frac{8}{9}$ of any string, will sound a note higher; and $\frac{8}{9}$ of this second, would sound 2 notes higher than the first; and $\frac{8}{9}$ of this third, would sound 3 notes higher than the first, &c. and this being 6 times repeated to make up an octave, we shall have $\frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} \times \frac{8}{9} = \frac{1}{2}$, as it would be if this note was exact, but that product is less, being but .4933; and therefore $\frac{8}{9}$ is too small, and $\frac{1}{9}$ too great for a tone. And 6 of these notes do not exactly make up an octave. FIG. 314.

After the same way, if we take the difference between the fourth and lesser third for a whole tone ($\frac{5}{6} - \frac{3}{4}$), we shall get $\frac{9}{10}$ of the string for a whole note, but this will be found to be too great, being .5314 instead of .5; therefore $\frac{1}{10}$ is too small for a note.

If we try by half notes, we shall still be no better. The lesser third, the greater third, and the fourth differ by half a note; of which there ought to be twelve in the octave. In the former case we get $\frac{24}{25}$ for the length of the string, in the latter $\frac{15}{16}$; and $\frac{1}{25}$ or $\frac{1}{16}$ for the length of half a note. The first is far too little, and the latter as much too big.

As none of these notes or half notes will make up an octave; so neither will any number of thirds, fourths, or fifths, make one or more octaves. A lesser third contains 3, a greater third 4, a fourth 5, and a fifth 7 half notes. Therefore 4 lesser thirds, or 3 greater thirds make an octave; and 12 fourths should make 5 octaves; and 12 fifths, 7 octaves. But if this was so, then we should have $\left(\frac{5}{6}\right)^4 = \frac{1}{2}$, $\left(\frac{4}{5}\right)^3 = \frac{1}{2}$, and $\left(\frac{3}{4}\right)^{12} = \left(\frac{1}{2}\right)^5$ (that is $\left(\frac{3}{4}\right)^{\frac{12}{5}} = \frac{1}{2}$), and $\left(\frac{2}{3}\right)^{\frac{12}{7}} = \frac{1}{2}$. But never a one of these is so. And hence we may conclude, that no scale, made up of these notes, or half notes, or any combinations of them, or of thirds or fifths, &c. can be perfectly exact.

Now to contrive a scale to answer as near as possible all the requisites. Let *AL*, *Am*, *An*, *Ao*, &c. to *AY* or $\frac{1}{2}$ *AL*, be 13 geometrical proportionals. Then these strings *AL*, *Am*, *An*, &c. will sound all the half notes in the octave, gradually ascending. Therefore if *AL* be put $= 1$, $AY = \frac{1}{2}$; *Am*, *An*, *Ao*, &c. being so many mean proportionals between 1 and $\frac{1}{2}$, will be found, as set down in the following table.

Cords.

FIG. 314.

cords.	strings	notes equally ascending.	pure concords.	errors
ground	*AL*	1.00000	1.00000	0
b second	*Am*	.94387		
※ second	*An*	.89090		
les. third	*Ao*	.84090	.83333	*b* $\frac{1}{13}$
gr. third	*Ap*	.79370	.80000	※ $\frac{1}{13}$
fourth.	*Aq*	.74915	.75000	$\frac{1}{100}$
※ fourth	*Ar*	.70711	....	.....
fifth	*As*	.66742	.66666	*b* $\frac{1}{100}$
les. sixth	*At*	.62996	.62500	*b* $\frac{1}{13}$
gr. sixth	*Au*	.59460	60000	$\frac{1}{13}$
b seventh	*Aw*	.56123		
※ seventh	*Ax*	.52973		
eight	*AY*	.50000	.50000	0

In this table, the 3d column shews the lengths of the vibrating string, when the scale ascends by equal degrees of sound, or when all the half notes are equal.

The 4th col. shews the length of the string to sound the pure concords.

The last col. relates only to the concords: and shews the error of the 3d col. expressing what part of a whole tone it is; and whether it is below (expressed by *b*) or above (by ※). By this col. we can judge how the scale in the 3d col. will perform. And these errors are found by comparing the 3d and 4th col. together. As suppose you would know what it is in the fifth, we shall have $66742-66666=76$, and $70711-62996=7715$. which represents a tone in that place. Then $\frac{76}{7715}$ or $\frac{1}{100}$ is the error, which is but the hundredth part of a whole tone. And as the number in the 3d col. is greater, it shews, that by this scale, the fifth is flatter than it ought to be. And so are the rest of the errors found out, and examined.

Now its evident, that the error in a fifth or a fourth is quite insensible in practice; but the thirds and sixths suffer the most, being in some but the 13th part of a note, which perhaps may be sensible to a good ear; but then it will not be so perceptible in a third, as it would be in a fifth; because a third is less perfect than a fifth. And the sweeter the cord, the more easily is an imperfection discovered. Now although these trifling errors will take away something from the sweetness of the harmony,

mony, and will hinder the ſcale from being abſolutely perfect; yet there is no remedy for it, but what is worſe than the diſeaſe. FIG. 314.

As to the tuning this inſtrument, it is plain that the notes ought not to be tuned by perfect fifths, for the upper note will always be the hundredth part of a note too high. And ſince one muſt take 12 fifths, before he can come at the ſame note again, whence he ſet off; there will at laſt be an error of $\frac{12}{100}$ or $\frac{1}{8}$ of a note, which is very diſcoverable in a fifth. The method therefore to be taken, is, to make the upper note a very little flatter than a perfect fifth; by firſt tuning it perfect, and then lowering it a ſmall matter, but not ſo much as to offend the ear. And after you have thus gone through the octave, if you find the laſt note either too high or too low, begin a new, and alter them all a little, according to your judgment, till the laſt does agree: but this judgment is to be attained principally by practice. Upon the firſt octave being rightly tuned, all the reſt depend, and therefore one ought to be very exact in it; for in all the reſt, there is nothing to do but to take the eights above and below. And you ought to begin to tune about the middle of the inſtrument.

Thoſe that tune by thirds, ought to take the upper note of the greater third, as ſharp as the ear will bear. And leſſer thirds ſhould be taken as flat as they may.

Moſt people in tuning, take ſome of the fifths perfect, and leave others imperfect; which they call bearing notes. But this is attended with great inconvenience: for the muſick ought to be ſo ſet, that no fifth ought to fall on any of theſe bearing notes, which inſtead of being a perfect concord, will be no better than a diſcord; ſince the error in theſe bearing notes is very great. For if there is but one in an octave, its error is $\frac{1}{8}$ of a note, if two of them, and both alike, $\frac{1}{16}$ of a note. Now if theſe people be ſo nice as to diſtinguiſh $\frac{1}{100}$ part of a note, much more would they be offended at $\frac{1}{8}$ or $\frac{1}{16}$. And always to avoid taking the fifths upon theſe notes, when they come naturally in the way, would be cramping, and even ſpoiling the muſic. And another diſadvantage would ariſe, that a piece of muſic could not be tranſpoſed upon any key at pleaſure, whatever need there might be for it: but muſt be tied down to a very few. And if this method could cure in any meaſure, the errors of the fifths; yet it would not at all mend thoſe of the thirds, which are far greater. And if any one third ſhould happen to be bettered, it is certain that others will be made as much worſe, and will be turned into diſcords.

Some

FIG.

Some that like not the *equiharmonic* or *isotonic scale*, above described, would compose a scale of several sorts of tones and semi-tones, as $\frac{1}{9}$, $\frac{1}{10}$, and $\frac{1}{16}$, $\frac{1}{17}$, $\frac{1}{18}$, $\frac{1}{19}$, $\frac{1}{20}$, *&c.* but what end can this answer? it is very easy to shew, that if some cords may be taken perfect, others will be miserably bad, and degenerate into discords. But this scheme seems to be built only upon the consideration of abstract numbers; for it regards only this, how to make several fractional quantities resolve themselves into others more simple, by multiplication; which is a thing of no manner of use in music.

Others, to avoid the badness of the cords which fall in some places, have invented quarter notes; which makes the music extremely hard to play; and is, besides, far from answering the end proposed. Therefore upon the whole, I cannot but think the scale above described, to be the best for practice. For so small an error as $\frac{1}{100}$ of a note in a fifth, and every where the same, cannot be sensible, or do much hurt to the music. And I will venture to say, that the very alteration of the weather, in 24 hours time, by heat or cold, drought or moisture, will have such an effect upon either strings or pipes, as to cause a greater difference than this amounts to. There are imperfections in every thing, which we cannot quite take away; all we can do, is to make them as little as possible.

More examples of the constructions of engines might here be added. But as there is such an infinite variety in the world, it would be an endless task to describe all the kinds of them. Therefore I think it needless to produce any more, especially, since their construction and use depend upon the same principles as these already described. And if the reader does but thoroughly understand the powers and forces of these before mentioned; he cannot be at a loss to find out the powers, forces, or motions, of any other machine, though never so compounded.

AN ALPHABETICAL INDEX OF THE TERMS used in MECHANICS.

A.

AIR PUMP, a machine to draw the air out of a glaſs. *Fig.* 277.

Ajutage, the ſpout for a *jet d'eau* in a fountain.

Amplitude, the diſtance a ball is ſhot to.

Anemoſcope, a machine for ſhewing the point of the wind. See *Fig.* 253.

Angle of application, is the angle which the line of direction of a power makes with the leaver it acts upon.

—— *of inclination*, is the angle an inclined plane makes with the horizon.

—— *of traction*, the angle which the direction of a power makes with an inclined plane.

Aqueduct, an artificial river, or tube to convey water.

Arbor, the axle or ſpindle of a wheel. *Fig.* 185. *ef.*

Arch, a hollow wall made of a circular form, to ſupport any building.

Areometer, an inſtrument to meaſure the weight of liquors.

Arm, any piece of timber or metal, that projects horizontally from ſome part of a machine.

Axle, or *Axis*; the line or ſpindle about which a wheel turns round. *Fig.* 185. *ef.*

Axis in peritrochio, a machine for raiſing weights, conſiſting of a wheel, fixed upon a cylinder for its axis.

B.

Balance, a machine to weigh bodies in; one of the mechanic powers. *Fig.* 188.

Balance wheel, the fly or pendulum of a watch.

Barometer, a machine to ſhew the weight of the air or atmoſphere. See *Fig.* 218.

Baroſcope, the ſame as barometer: a weather-glaſs.

Barrel of a wheel, is the axle, or cylindrical body, about which the rope goes.

—— *of a pump*, is the hollow part of the pump where the piſton works.

Barrs, ſtrait pieces of timber or metal, that run croſs from one part of a machine to another.

Baſe, the foot of a pillar.

Baſil, that angle, the edge of a tool is ground to.

Batten, a piece of timber three or four inches broad, and an inch thick.

To *Batter*, to lean backward.

Bauk, a long piece of timber.

Beak, the crooked end of a piece of iron, to hold any thing faſt.

Beam, a large piece of timber lying acroſs any place.

Beetle, a wooden inſtrument, or mallet, for driving piles.

Bevil, any angle that is not right. An oblique angle.

Block, a lump of wood.

Blocks, pieces of wood in which the ſheevers or pullies run, and through which the ropes go. *Fig.* 42. *A*, *B*.

Bolts, large iron pins.

Bond, the faſtening ſeveral pieces of timber together, either by mortiſe and tenant, dove-tailing, *&c.*

Brace, a piece of timber fixed obliquely into others, to ſtay them from moving any way.

Brackets, the cheeks of the carriage of a mortar. A cramping iron to ſtay timber work; alſo ſtays ſet under a ſhelf to ſupport it.

Brads, nails having no broad heads.

Bridge, any horizontal beam, *&c.* that is to ſupport ſomething.

Butments, thoſe ſupports on which the feet of arches ſtand.

Buttreſs, a piece of ſtrong wall that ſtands on the outſide of another wall to ſupport it.

C.

Capſtand, a machine on board a ſhip, to hoiſt the maſts, or raiſe any weight. *Fig.* 248.

Caſcade, a fall of water.

Cataract, is a precipice, or violent fall of water in a river, thro' high rocks, cauſing the water to fall with a great noiſe and force.

Catch, ſome ſmall part of a machine; which, in its motion, hooks or lays hold of ſome other part to ſtop it.

Center

Center of motion, the point about which a body moves.
—— *of gravity*, the point upon which a body being fuſpended, it will reſt in any poſition.
—— *of magnitude*, a point equidiſtant from the oppoſite extremes of a body. The middle.
—— *of percuſſion*, the point of a vibrating body that gives the greateſt ſtroke.
Center pin, a pin about which, as a center, a body moves.
Chain pump, a pump having ſeveral buckets fixed to an endleſs chain, which goes through it, and is moved round upon an axle. *Fig.* 254.
Chaps, two ſides of a machine which take hold of any thing.
Cheeks, two upright parts of a machine, anſwering to one another in poſition and uſe.
Chronoſcope, a pendulum to meaſure time.
Clack, a ſort of valve which is flat, like a board; ſerving to ſtop a fluid from running out. *Fig.* 268. *T*, a flap.
Clampt, when the edges of two pieces of boards are joined together, ſo as the grain of one may lie croſs the grain of the other.
Claſp, a ſort of buckle to faſten any thing.
Claſp nails, thoſe with little heads to ſink into the wood.
Claws, ſlender crooked pieces of metal in a machine, which ſerve to move or hold any thing. Long teeth.
To *Clench* or *clinch*, to double back the end of a nail, that it may not draw out again. To rivet.
Clench nails, nails that may be clinched.
Cock, a braſs ſpout to let a fluid run out, or ſtop it by turning.
Cogs, the wooden teeth of a great wheel. *Fig.* 185, *a*, *a*, *a*.
Cog-wheel, a large wheel made of timber, where the teeth ſtand perpendicular to the plane of the wheel. *Fig.* 185, *CD*.
Collar, a ring of metal that goes about any thing, near the top, in which it turns round.
Column, the ſhaft or trunk of a pillar.
Contrate-wheel, a wheel in a clock, where the teeth are parallel to the axis, and ſtand on the under ſide of the rim.
Corbel, a piece of timber, or ſtone, ſet under another piece, to diſcharge the weight.
Crab, a ſmall capſtand with three claws, to be placed on the ground, moveable from one plane to another. This is called a flying capſtand.
Crane, a machine for hoiſting goods out of a ſhip, or for raiſing timber or ſtone. *Fig.* 233.

Crank, that part of an iron axis which is turned ſquare with an elbow. *Fig*. 238. *I I*.

Croſs-tree, a horizontal beam fixed acroſs another.

Crow, a ſtrong ſquare bar of iron, forked at the end, to remove heavy timber, *&c*. by uſing it with the hands.

Crown-wheel, in a clock or watch, is that next the balance, its teeth ſtand in the upper ſide of the rim, and not in the edge. *Fig*. 166. *FG*.

Cupelo, a hollow arched tower, in form of a hemiſphere, or of a bowl turned upſide down.

D.

Denſity, is a greater or leſs quantity of matter contained in a given ſpace.

Detents, are thoſe ſtops, which being lifted up, the clock ſtrikes; and falling down, ſhe ſtops.

Dog nails, nails uſed for faſtening hinges.

Dome, a round vaulted roof or tower. A cupola.

Dormant, a great beam lying croſs a houſe. A ſummer.

Dormer, a window in the roof of a houſe.

Dove-tailing, letting one piece of timber into another, with a joint in form of a dove's tail, being broader at the end, that it may not draw out again.

Drum, the lantern or trundle, which is carried by a great wheel. *Fig*. 266. *EF*.

Drum head, a timber head, or lump of timber, in form of a drum.

E.

Edging, the outſide or border.

Endleſs chain, a chain with the ends joined together; by which any part of a machine is wrought.

Endleſs ſcrew, a ſcrew working in the teeth of a wheel; which may be turned about for ever. *Fig*. 193. *E*.

Engine, a mechanical inſtrument compoſed of wheels, leavers, ſcrews, *&c*.

Eolipyle, a hollow globe of metal, filled with water, and put in the fire; the heat and vapour ruſhes out at a ſmall hole, with a great noiſe. *Fig*. 264.

Equilibrium, the equality of weight, of two or more bodies, *&c*. keeping one another at reſt.

Eye, a hole in ſome part of a machine, through which any thing is put.

F.

F.

Face, the foot or foreſide of a machine, or of ſome principal part of it.

Fang, ſome ſmall piece of metal like a long tooth, that by its motion moves ſome other part.

Felhes, pieces of wood on the outſide of a wheel, which make the rim.

Ferril, a ſort of hoop.

Floats, the flat boards ſet perpendicular on the edge of a water-wheel; by which the water drives the wheel about. *Fig.* 236. *D, D.*

Fly, that part in a clock, &c. that regulates the motion, and makes it uniform. *Fig.* 168, 169, 170.

Force, any thing that acts upon a body to put it in motion.

Force pump, a pump that diſcharges water by preſſing it upwards. *Fig.* 267, 268.

Frame, the outwork of any machine, or what holds all the reſt together.

Free, clear of all impediment.

Friction, the reſiſtance that bodies have by rubbing againſt one another.

Fulcrum, that which ſupports a leaver in moving any heavy body.

G.

Gain, the levelling ſhoulder of a joiſt or other timber.

Gin, a machine to raiſe great weights. *Fig.* 257.

Gravity, the weight of bodies.

——— *ſpecific*, by this one body weighs more or leſs than another of the ſame bulk.

——— *relative*, is the weight of a body in a fluid, or on an inclined plane.

Groove, a channel cut in wood or ſtone.

Gudgeons, the eyes in the ſtern of a ſhip, on which the rudder hangs. The center pins of an axle.

Gyration, a whirling round.

H.

Hand, an index or pointer.

Handle, the part of an inſtrument to take hold on with the hand.

Hand ſpike, a wooden leaver to be uſed with the hand, in moving any thing.

Head, the top part of any thing.

Hem,

Hem, the edge of ſome cloth turned down and ſewed.
Hinge, an iron joint on which a door turns, *&c.*
To *Hitch*, to catch hold on, with a hook or turn of a rope.
To *Hoiſe*, or *hoiſt*, to heave up, or raiſe by force.
Hook pins, taper iron pins with a hook head, by which they are ſtruck out again. They ſerve to pin the frame of a roof or floor together, till wrought off.
Hoop, a circular ring to put about any thing, to keep it faſt.
Hydraulics, the art of making engines for water works.
Hydrometer, an inſtrument to meaſure the denſity of fluids. *Fig.* 269.
Hydroſtatics, a ſcience teaching the weights, preſſures, motions, and properties of fluids.
Hydroſtatical balance, an inſtrument for finding the ſpecific gravity of bodies.
Hygrometer, } an inſtrument for meaſuring the moiſture and dry-
Hygroſcope, } neſs of the air. *Fig.* 265.

I.

Jack, an engine to lift up a loaded cart, or the roof of a houſe, *&c.* See *Fig.* 249. Alſo an engine to roaſt meat. *Fig.* 258.
Jack pump, a chain pump.
Jaums, door poſts, or window poſts, *&c.*
Jet d' eau, the pipe of a fountain, which ſpouts up water into the air. *Fig.* 287.
Impetus, any blow or force wherewith one body ſtrikes or impels another.
Joint, the place where one part is fixed into another, or moves about it.
Joiſts, thoſe pieces of timber framed into the dormant, in a houſe, and on which the boards of the floor are laid.

K.

Keys, ſtones in the top of an arch, to bind the ſweeps together.
Kenk, a ſnarle, or little turn upon a rope, that it cannot run.
Knee, a crooked or angular branch of timber.

L.

Lantern, that part which is moved by the cogs of a great wheel, acting againſt the ſpindles or rounds. The drum. *Fig.* 266. *EF.*
Latch, that which faſtens a door, *&c.* A *ſneck*.

Leaver,

Leaver, or *lever*, a bar of iron or wood to raiſe a weight: one of the mechanic powers.

Leaves, the teeth of a pinion. *Fig.* 185. *c*, *c*, *c*.

Ledge, a flat border or plain, adjoining to a thing.

Level, an inſtrument to place any thing horizontal.

Linch pin, a pin that keeps a wheel from coming off its axle.

Lip, a thin edge turned hollow.

Loop, a piece of metal having a hole in the end, which goes over ſomething. A nooſe in a rope that will ſlip.

M.

Machine, a mechanical inſtrument for moving bodies.

Mechanics, a ſcience that teaches the principles of motion, and conſtruction of engines, to move great weights.

Mechanic powers, are theſe ſix; the balance, leaver, wheel, pulley, ſcrew, and wedge: and according to ſome, the inclined plane.

Mitre, an angle of 45 degrees, or half a right one. And *half mitre* is a quarter of a right angle.

Momentum, quantity of motion; or the force or power a body in motion has to move another.

Mortiſe, a ſquare hole cut in a piece of ſtuff, to receive the tennant.

Motion, is the ſucceſſive change of place of a body; or its paſſing from one place to another.

Moving force, any active force or power that moves a body.

Mouth, the part or parts of a machine, that take hold of any-thing. The entrance into any cavity.

N.

Nave, or *Naff*, the block in the middle of a wheel.

Neck, a part near the end, cut ſmall.

Notch, a dent, nick, or ſlit made in any thing.

Nut, the pinion of a wheel. *Fig.* 185. *AB*. A ſmall piece of metal going upon the end of a ſcrew nail.

O.

Oſcillation, the vibration or ſwinging of a pendulum.

P.

Paddles, a ſort of oars. The laddle boards on the edge of a water-wheel.

Pedeſtal,

Pedestal, the base or bottom of a pillar.
Peers, or *Piers*, a sort of buttresses, for support and strength.
Peg, a pin to go into a hole.
Pendulum, a weight hung by a string or wire, swinging back and forward, to measure time.
Penstock, the sluice or door, that opens or shuts the passage of the water to a water-wheel.
Percussion, the striking of one body against another.
Pestil, a long piece of wood or metal, which rises up and falls down again to beat or bruise something.
Pevets, or *Pivots*, the ends of the spindle of the wheel in a clock or any machine, which play in the
Pevet holes, the holes in which the ends of a spindle or axle of a wheel turn. *Fig.* 185, *e*, *f*.
Picket, a stake pointed with iron, to drive into the ground.
Pillar, a perpendicular column supporting one end of an arch, *&c.*
Pinion, a little wheel at one end of the spindle, consisting but of a few leaves or teeth. *Fig.* 185, *AB*.
Piston, a round piece of wood moving up and down within the body of a pump, to draw up the water.
Plate, a piece of timber, on which some heavy work is framed, as wall plate, *&c* A flat piece of metal.
Pneumatics, a science teaching the properties of the air.
Pole, a long staff, or slender piece of wood.
Post, a perpendicular or upright beam of wood.
Power, the force applied to an engine to raise any weight. Or any force acting upon a body to move it.
To *Project*, to jet out or hang over.
Projectiles, balls or any heavy body thrown into the air.
Prop, a stay or support for any thing, to bear it up.
Pulley, a small wheel with a channel in the edge of it, moving about an axis fixed in a block; the channel is to receive a rope that goes over it. One of the mechanic powers.
Pump, an engine to raise water. *Fig.* 204.
Punchins, short pieces of timber, placed upright to support some weight.

R.

Rag-wheel, the barrel or wheel in a chain pump.
Rails, small pieces of wood joined into others; those pieces into which the pannels of doors, *&c.* are fitted.
Rammer, an instrument for driving stones or piles into the ground, or beating the earth.
Random, the distance to which a ball is shot.

Range,

Range, the direction a ball is ſhot in, from a piece of ordnance.
To *Range*, to run ſtrait, or directly in a line.
To *Reeve*, to paſs a rope through any hole.
Return, the ſide that turns off from any piece of ſtreight work.
Ribs, ſlender pieces of timber, ſerving for ſtrength and ſupport.
Riglets, little flat, thin, ſquare pieces of wood.
Rim, the circular part or outſide of a wheel.
To *Rivet*, to batter down the end of a nail, that it draw not out again.
Rod, a long ſlender piece of wood or metal.
Roll, or *roller*, an engine turned by a handle, to raiſe weights, *Fig.* 243.
Rounds, the ſtaves or ſpindles in a lantern, againſt which the teeth of a great wheel work. *Fig.* 266, *c*, *c*. The ſteps in a ladder, *&c.*
Ruler, a thin ſtrait piece of wood.
Rungs, ſpindles or rounds. *Fig*, 266, *c*, *c*.
Runner, a flat circular ring, between the nave and linpin of a wheel. Alſo a ſort of rope on board a ſhip, to hoiſt with.

S.

Sails, large pieces of canvaſs, by which ſhips, windmills, *&c.* are carried, by help of the wind.
Scantlin, ſtuff cut to a proper ſize.
Screw, one of the mechanic powers. The tap with the thread is the *male ſcrew*, the hollow that receives it, is the *female ſcrew*.
Scribing, drawing an irregular line upon one piece of ſtuff, parallel to the irregular ſide of another, with a pair of compaſſes opened to a due diſtance; and carried along the ſide of it. Then the wood in the firſt piece being cut away, theſe two pieces will fit each other.
To *Seaze*, to bind or faſten a rope, *&c.*
Shaft or *ſhank*, any long part of an inſtrument, eſpecially that is held with the hands.
Sheers, two poles ſet up an end ſloping, and tied together at top; and ſecured by a rope from falling. Their uſe is to raiſe any weight by help of a block and tackle at top.
Sheevers, pullies, the little wheels that run in blocks, by a rope going over them.
Shelf, a board, *&c.* fixt horizontally.
Shoulder, a part of timber or metal, cut thicker than the reſt, in order to ſupport ſomething.
Shrouds, the ledges on the edge of a guttered wheel.

Sills, *sells*, or *groundsils*, pieces of timber that lie on the ground, into which others are fixed. Sole trees.

Siphon, a crooked glass tube, for drawing off liquors. *Fig.* 216.

Sleepers, pieces of timber laid as a foundation and support for others that are to lie upon them.

Slings, these are made of a rope spliced with an eye at either end, to go over a cask or some heavy thing, which is to be hoisted.

Snatch block, a great block with a sheever in it, and a notch cut through one of the cheeks of it; to hitch the rope into the pulley, for readiness.

Socket, a hollow piece of metal, in which any thing moves.

Sole, the bottom of the gutter or channel, in a guttered wheel.

Sole tree, the lowest piece of timber which lies flat on the ground, into which the upper works are framed. The groundsel.

Spanish burton, a sort of tackle to hoist goods, like *Fig.* 197.

Spear, a long pointed iron, or piece of timber.

Specific gravity, is that whereby one body weighs more or less than another of the same magnitude.

Spike, a pointed iron, or piece of wood.

Spindle, the axle of a wheel. *Fig.* 185. *ef*.

Spires, the turns of a rope about a cylinder or roller.

To *Splice*, to join two ropes together by working the strands into one another.

Spokes, pieces of wood running from the center of a wheel to the circumference, like rays.

Spring, an instrument made of steel, that being bent, it continually exerts a great force, that it may unbend itself again. Springing plates are sometimes made of brass.

Spur, a sort of prop, set aslope to thrust.

Spurs, long wooden teeth standing in the edge of a large timber wheel. *Fig.* 192, *a*, *a*, *a*.

Spur wheel, a wooden wheel where the teeth stand in the edge of the rim. *Fig.* 192. *CD*.

Staff, a stick or small piece of wood.

Staticks, a part of mechanics, teaching the motions and properties of heavy bodies.

Stay, a piece of timber, or other thing fixed as a prop or support to some heavy body.

Steelyard, an instrument to weigh bodies, consisting of a long beam and a moveable weight, *Fig.* 190.

Stock, the wooden part of a thing, and into which it is fixt.

Stopple or *stopper*, a plug, that fits into a hole.

Stops, any small pieces in a moving machine, that serve to stop the motion.

Strain, the ſtreſs or violence any thing ſuffers by a weight or force acting againſt it.

Stroaks or *ſtraiks*, the iron going round the circumference of carriage wheels.

Stud, a knob, or little button. A ſolid piece of metal fixt to a plate.

Stuff, any wood that joiners work upon.

Swivel, a metal ring that turns about any way.

Syphon, the ſame as ſiphon. *A crane*. *Fig*. 216.

Syringe, an inſtrument for injecting liquors into any place.

T.

Tackles, blocks with pullies and ropes in them, to heave up goods. *Fig*. 196, 197.

Tenon, the ſquare end of a piece of wood, made to fit into a mortiſe hole.

Thermometer, } an inſtrument to ſhew the degrees of heat and
Thermoſcope, } cold. *Fig*. 270.

Thread, the ſpiral ridge that goes winding round a ſcrew.

Thruſt, the action againſt a body to puſh it forward.

Tight, ſtiff, cloſe.

Tongue, a thin ſlender piece of metal in a machine.

Tool, an inſtrument to work with.

Tooth, the indented part on the edge of a wheel that moves ſome other wheel. Or what ſerves to cut, or take hold on.

Tranſom, an overthwart beam in a building.

Triangle, an engine ſtanding on three legs, to raiſe weights with. *Fig*. 195.

Trundle, the part which is carried about by the teeth of a wooden wheel. The lantern or drum. *Fig*. 266. *EF*.

Trunk, a hollow tube or box.

Tumbler, a part in a machine that rolls about upon an axis, and plays back and forward.

Tumbrel, a roller, or cylindrical beam of wood.

Tympanum, a kind of wheel placed on an axle, and has ſtaves or rounds inſtead of teeth, and is carried about by a great wooden wheel. A trundle or drum. *Fig*. 266. *EF*.

V.

Valve, a piece of wood, &c. ſo fitted into a hole, that it opens and lets a fluid paſs through one way; and ſhuts and ſtops it the other. *Fig*. 268. *V*. *W*. *A ſucking valve*, is that where the water follows the piſton. *A forcing valve* when it is driven through before it.

Vane, a ſail, or fan; generally to ſhow the point of the wind.

Velocity, an affection of motion, and is that by which a body paſſes over a certain ſpace in a given time. Swiftneſs, or celerity.

Vibration, the moving or ſwinging of a pendulum back and forward.

Vis inertiæ, a property of body, by which it reſiſts any impreſſed force, and endeavours to continue in the ſame ſtate.

W.

Wallower, a trundle upon a horizontal axis. *Fig.* 257, *F.*

Waterpoiſe, an inſtrument to try the ſtrength of liquors. A hydrometer.

Web, the thin broad part of an inſtrument, as the web of a key, *&c.*

Wedge, an inſtrument to cleave wood. One of the mechanic powers.

Weight, the tendency of bodies downward. The matter raiſed by an engine.

Wheel, a machine conſiſting of an axis and a circular rim, with teeth in it, and then it is called a toothed wheel.

—— *Smooth*, a wheel without teeth, turned by a rope.

Wheel and axle, a machine to raiſe weights. One of the mechanic powers, *Fig.* 30.

Winch, an inſtrument with a crooked handle, to turn any thing about with.

Winder, a winch or handle to wind about.

Windlaſs, a machine to raiſe great weights. On board a ſhip, it ſerves to hoiſt the anchor. It is an horizontal roller, turned round by handſpikes.

Windmill, a mill to grind corn, moved by the wind. *Fig.* 266.

Wing, a thin broad part that covers ſomething, or hangs over it. Alſo what helps to give due motion to any thing, as the hands in a water wheel; a part of a ſail, *&c.*

Worm, a ſpiral thread running round a cylinder, forming a ſort of ſcrew.

A LIST

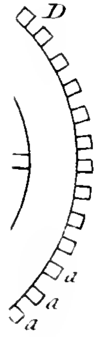
A
L I S T
OF THE
PRINCIPAL MACHINES deſcribed in this BOOK.

Lifting

FINIS.

E R R A T A.

Page 2. line 3, *for* whence *read* where. Page 12. line 22. *dele* and in theſe directions. Page 23. line 14. *for* are the ſquares, *read* are as the ſquares. Page 87. line 2. *for* their weights, *read* any weights. Page 107, lines 19, 20. *dele* the power of all. Ibid, line 25. *for* $-\frac{1}{2}d$ *read* $=\frac{1}{2}r$.



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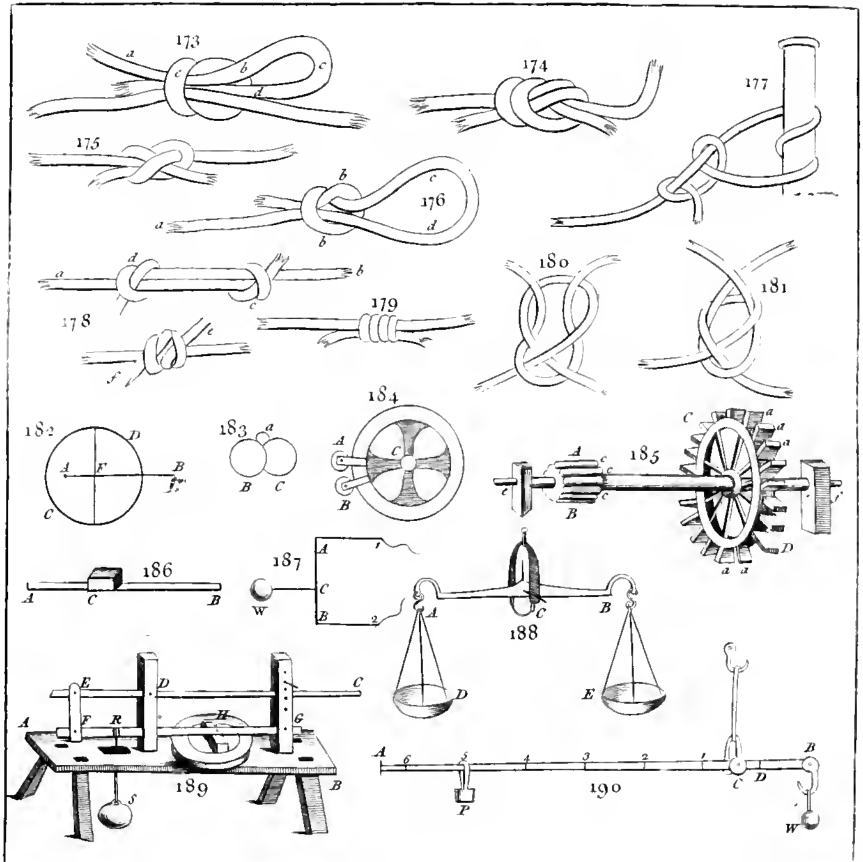
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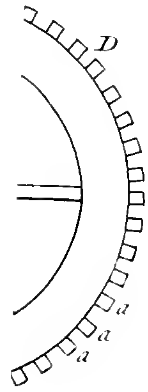

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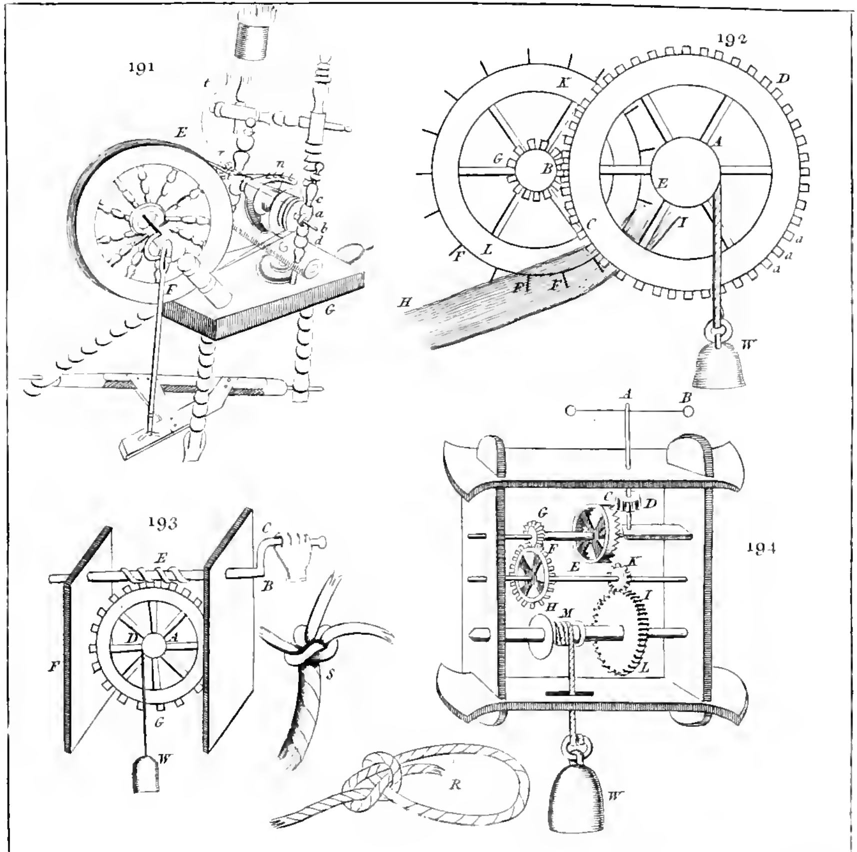

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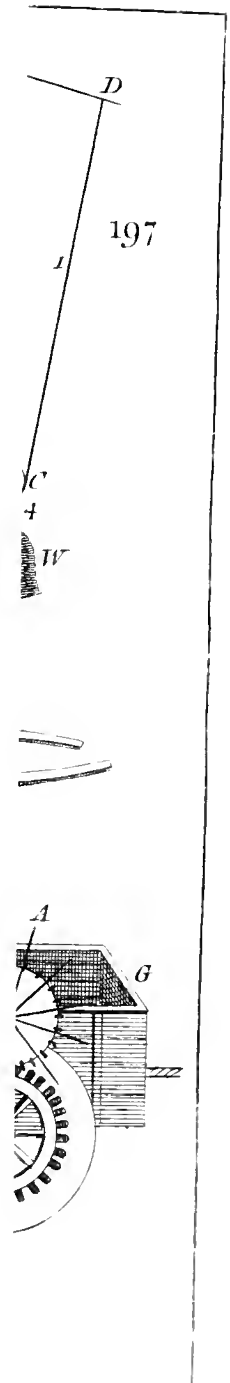

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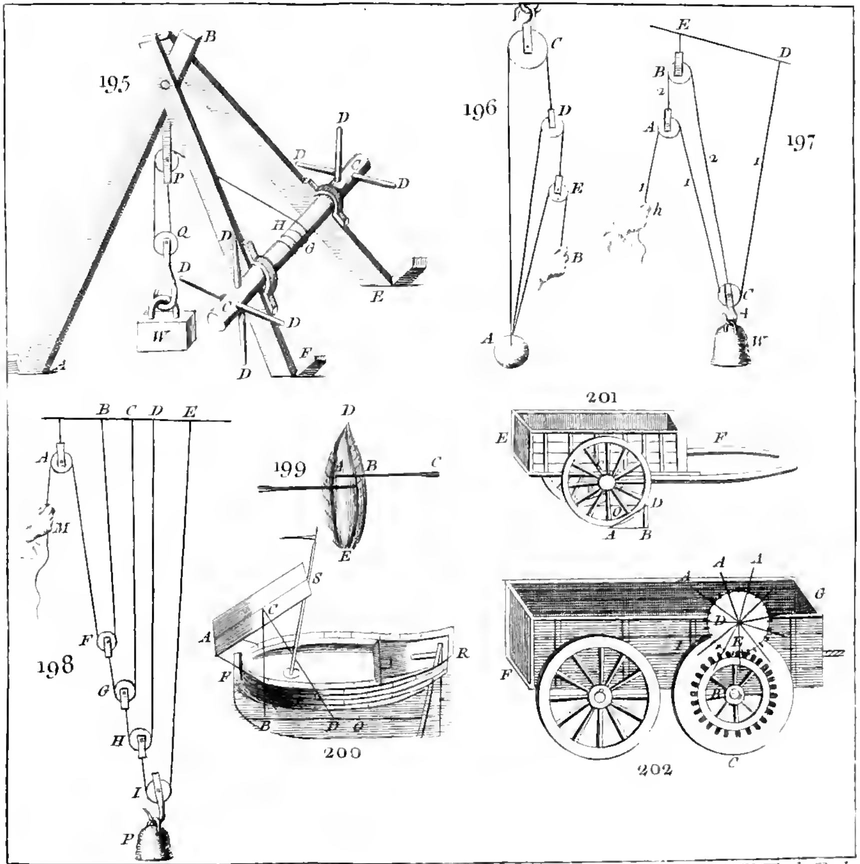

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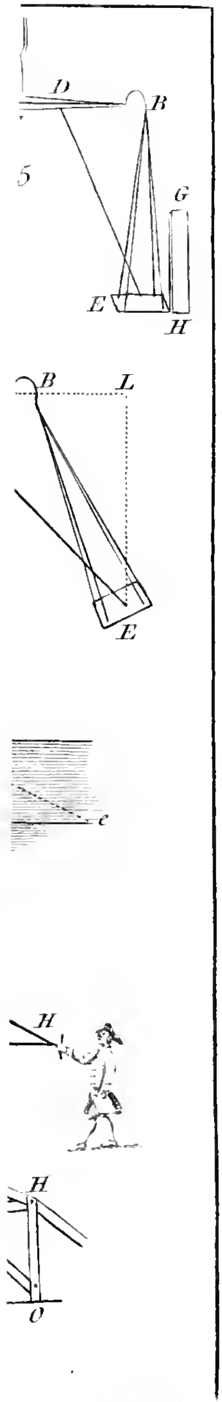

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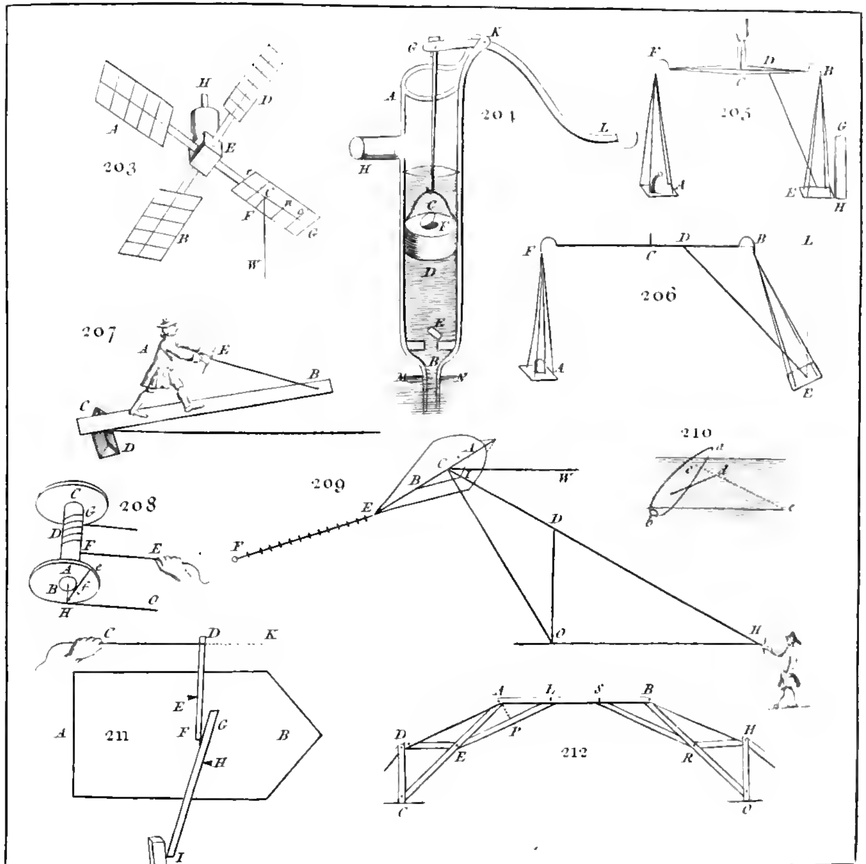

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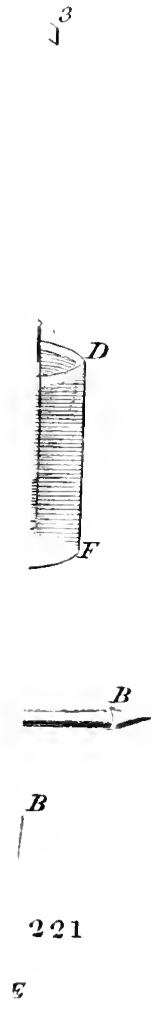

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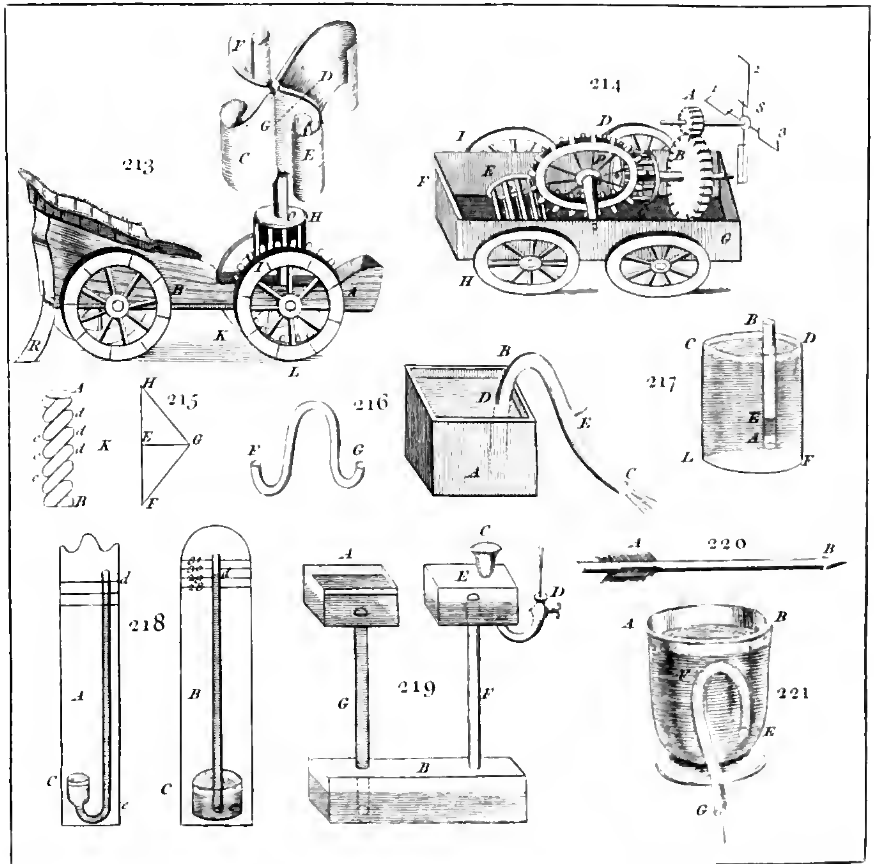

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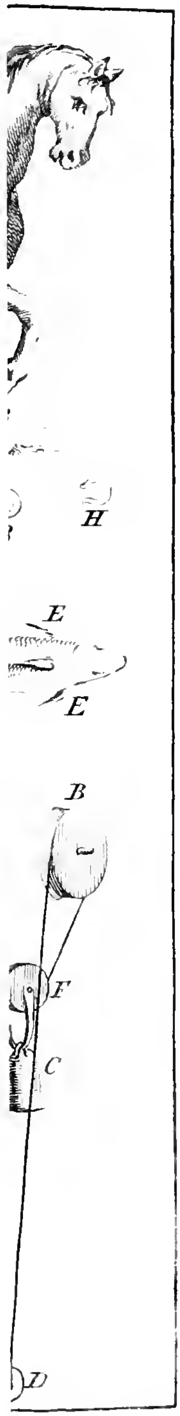

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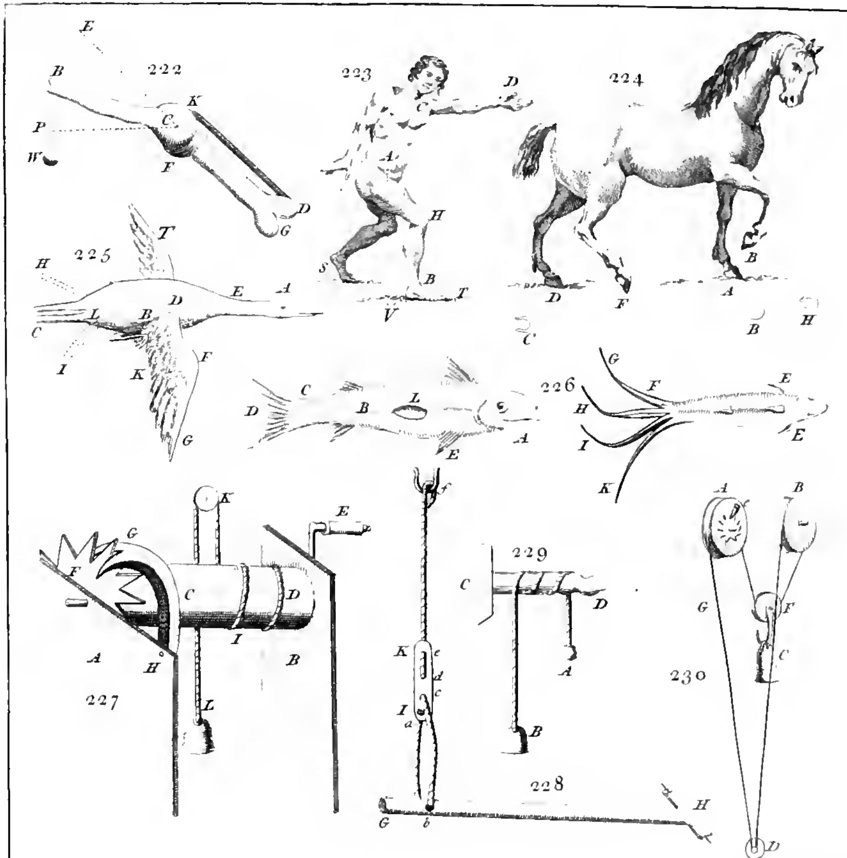

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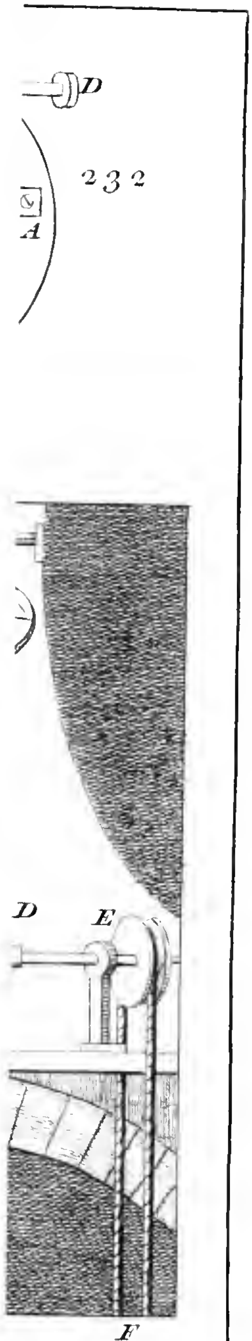

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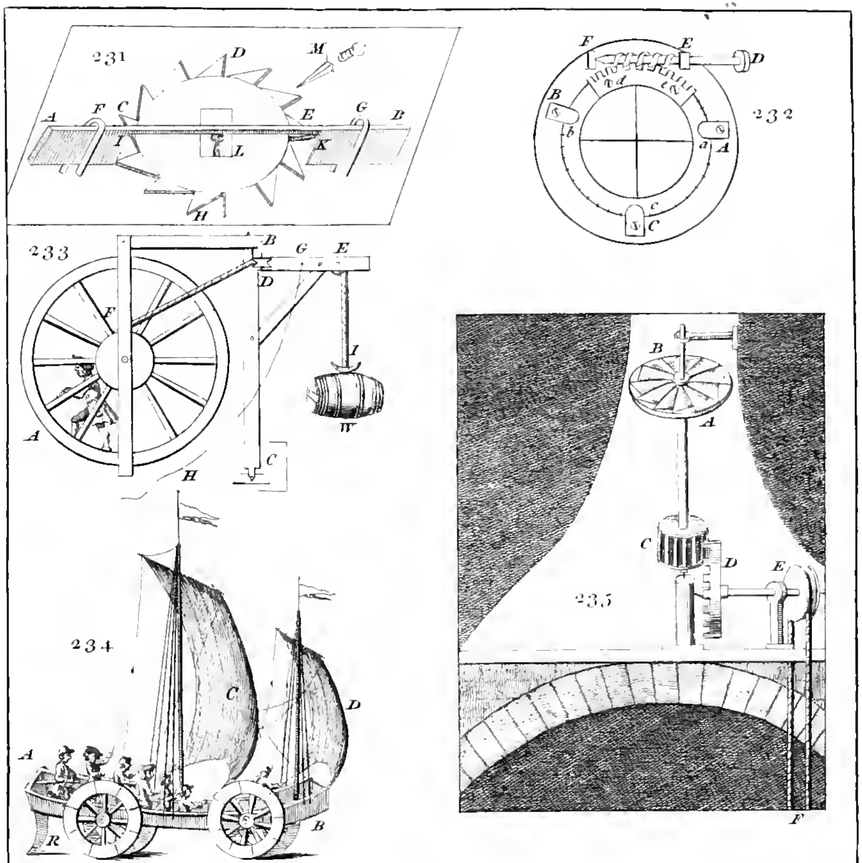

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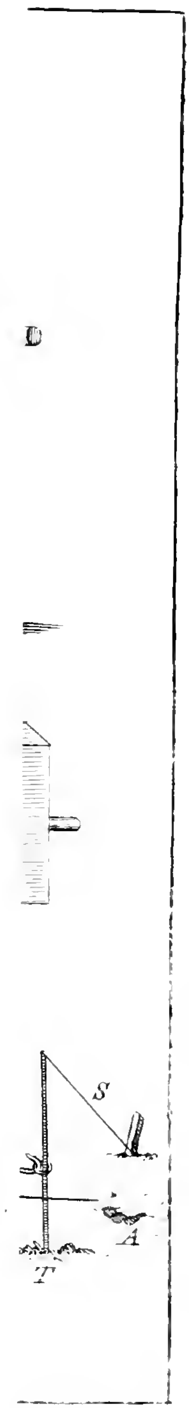

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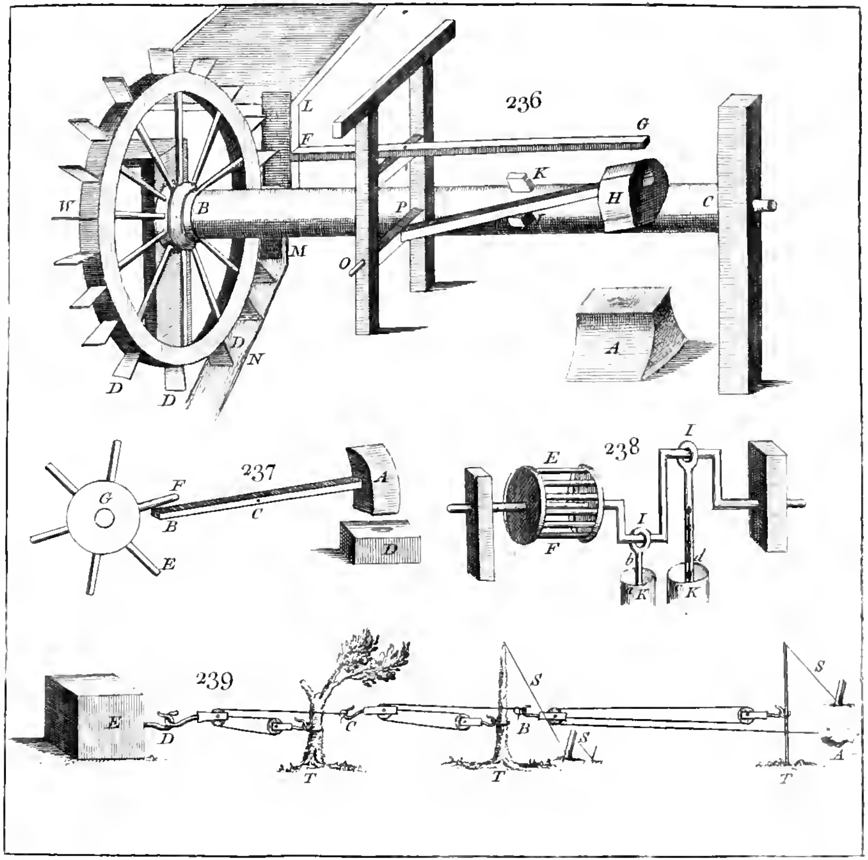

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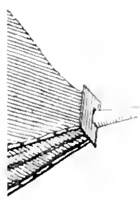

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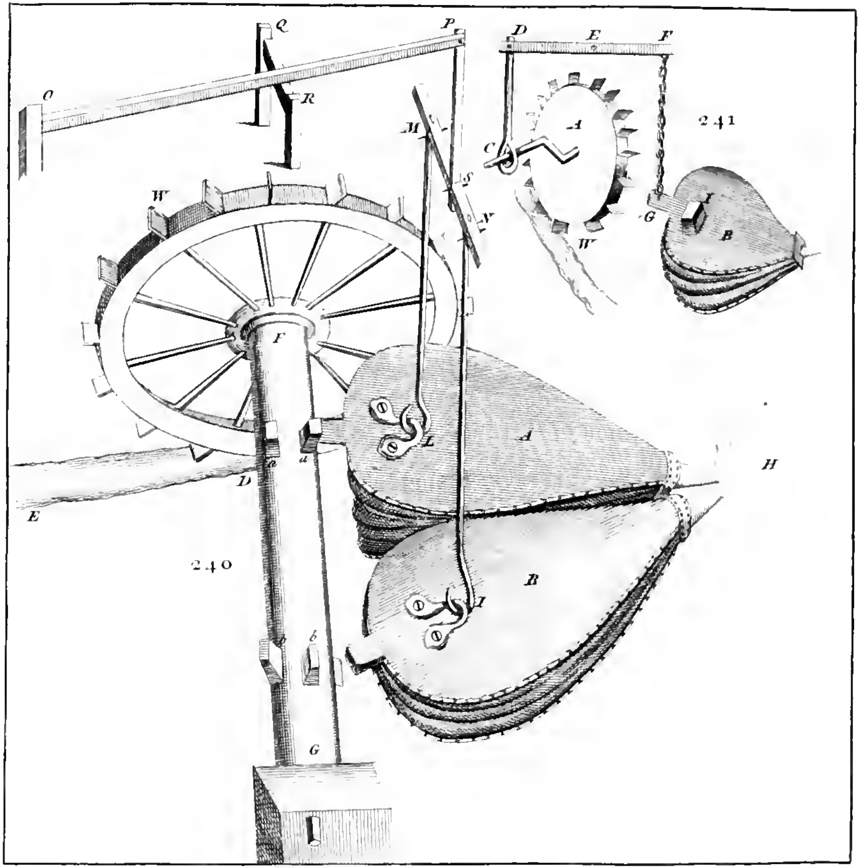

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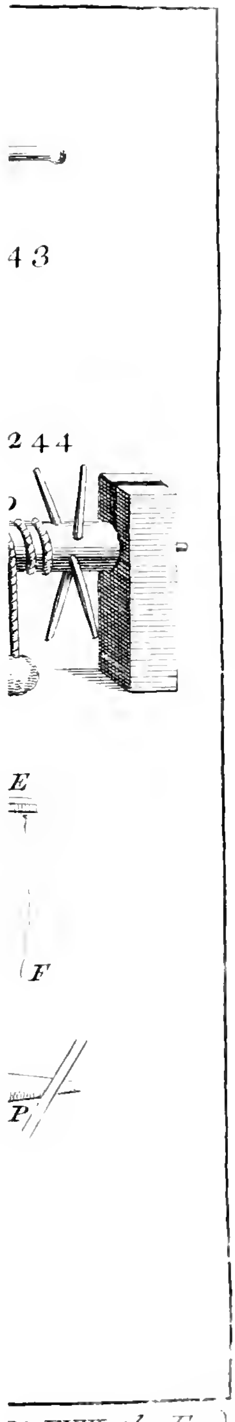

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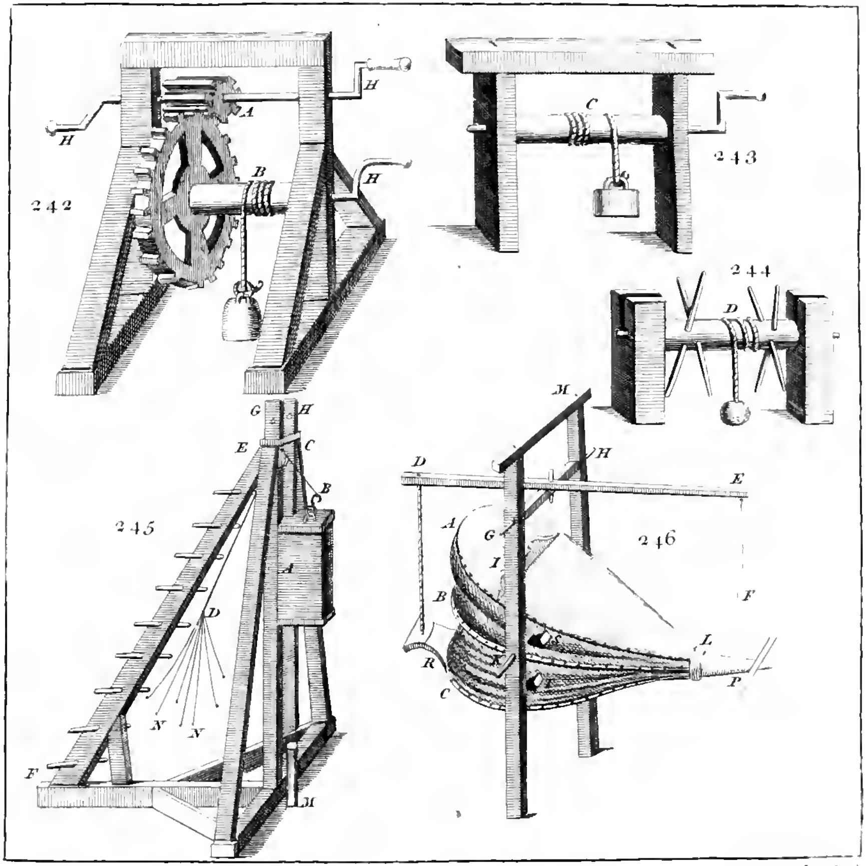

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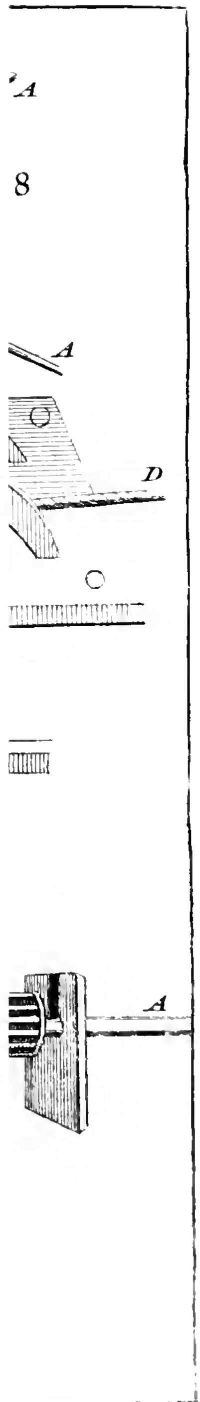

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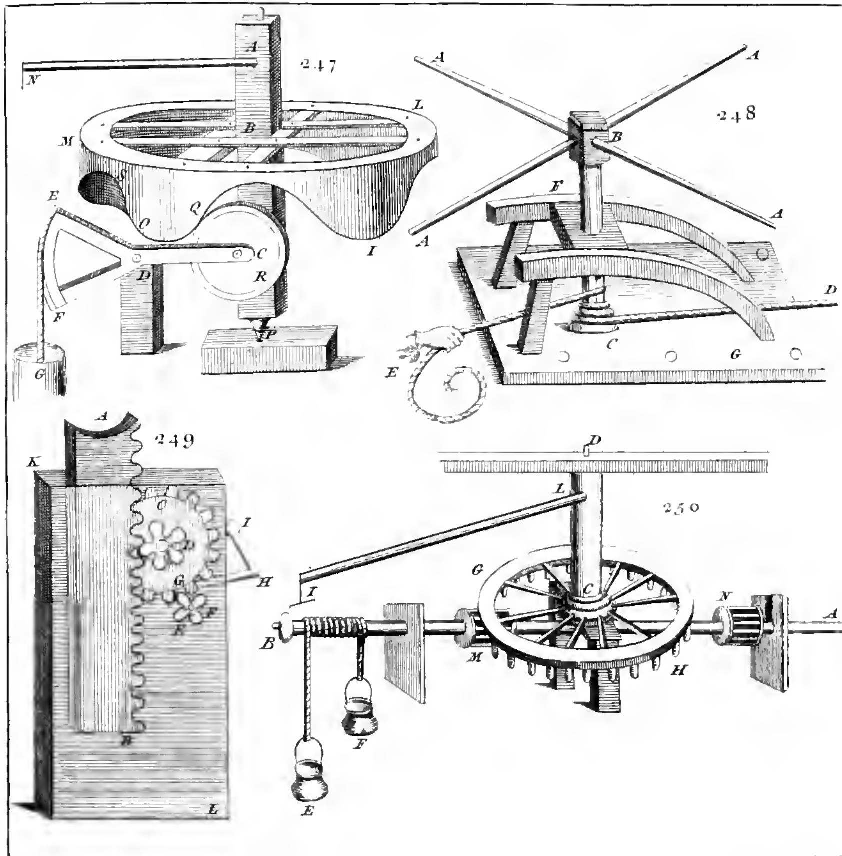

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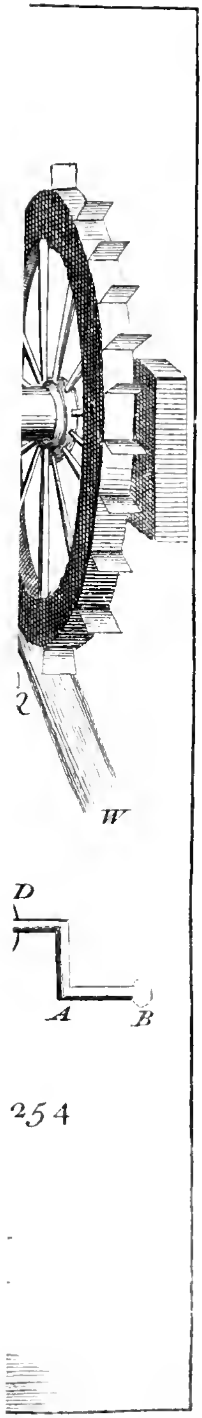

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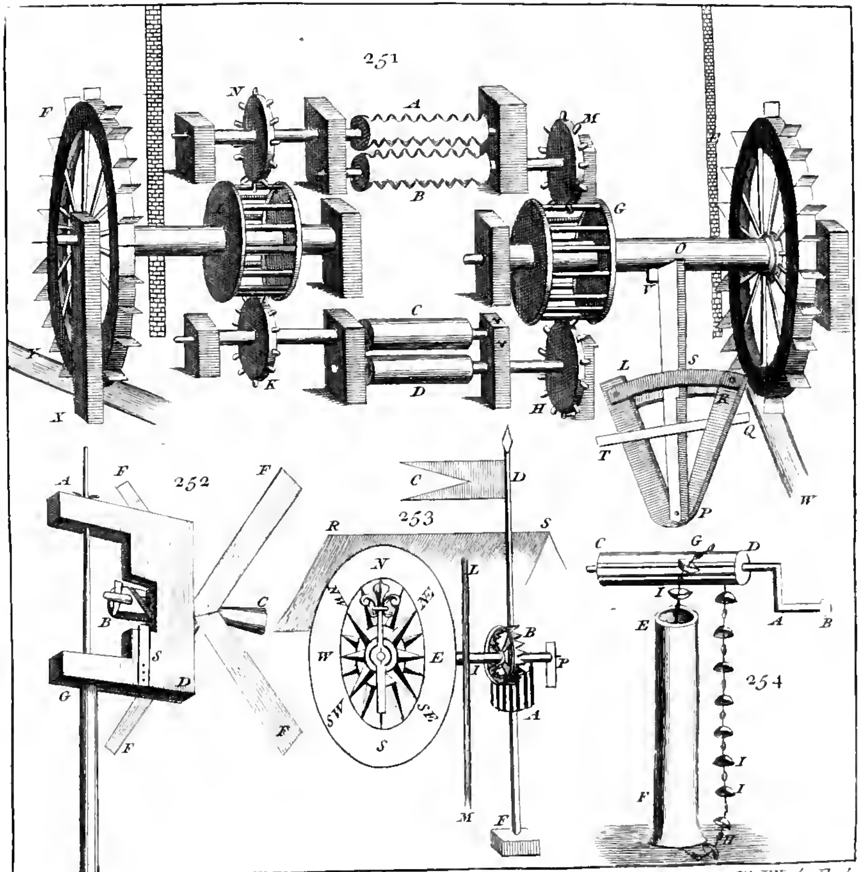

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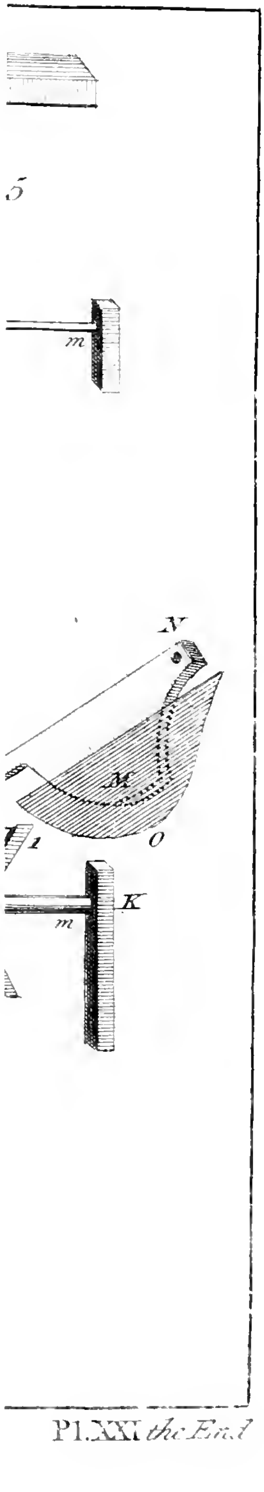

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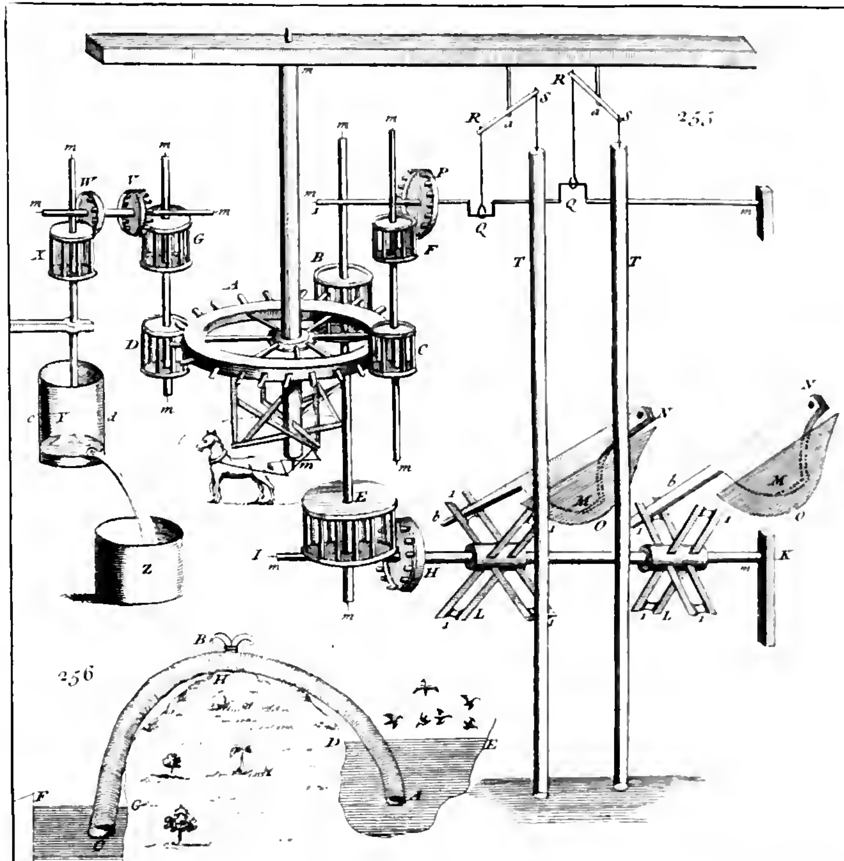

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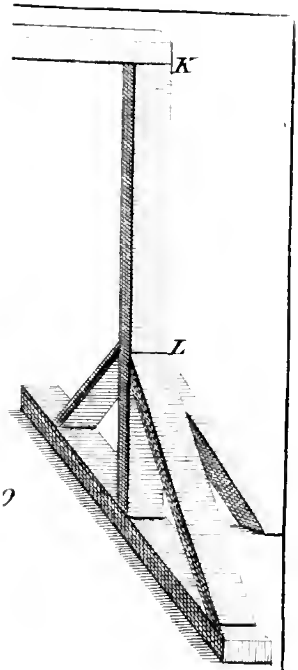

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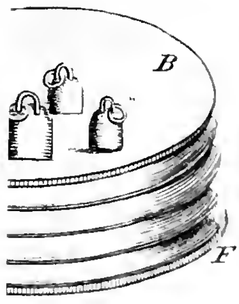

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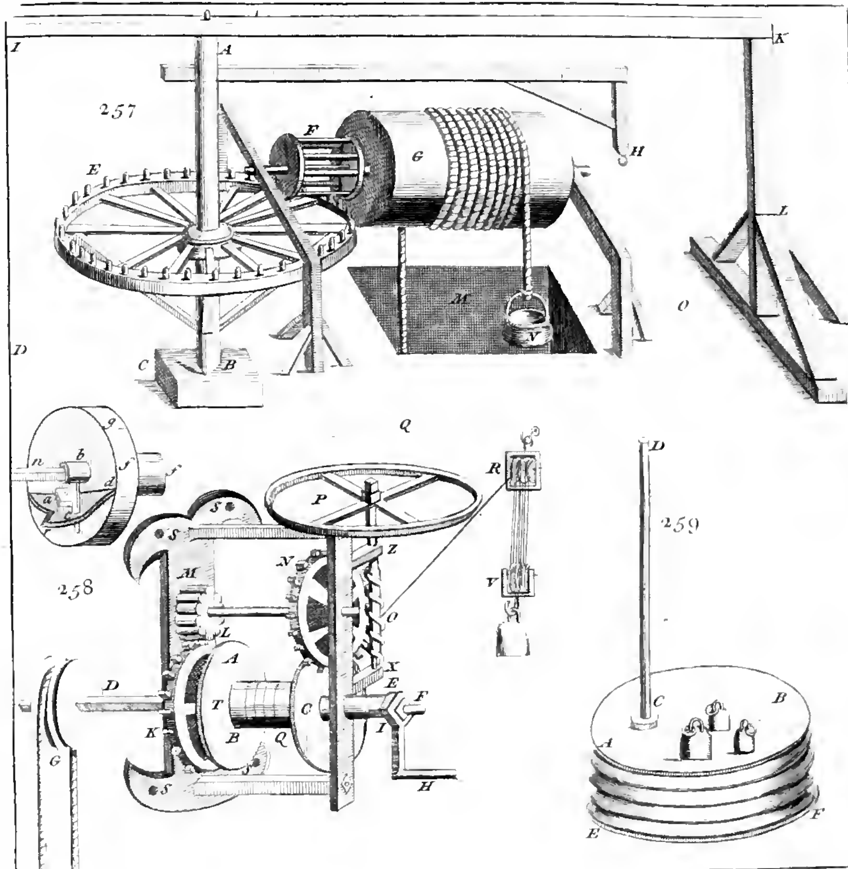

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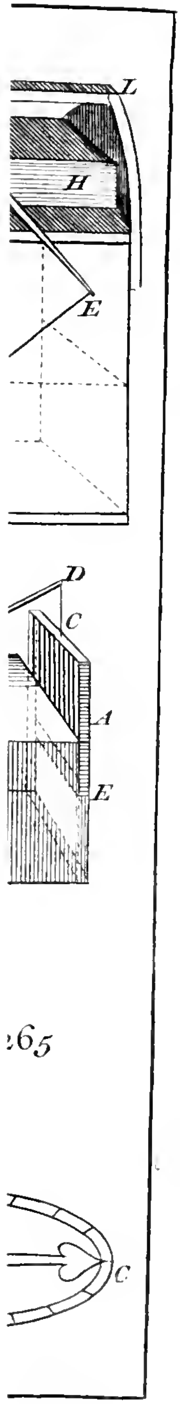

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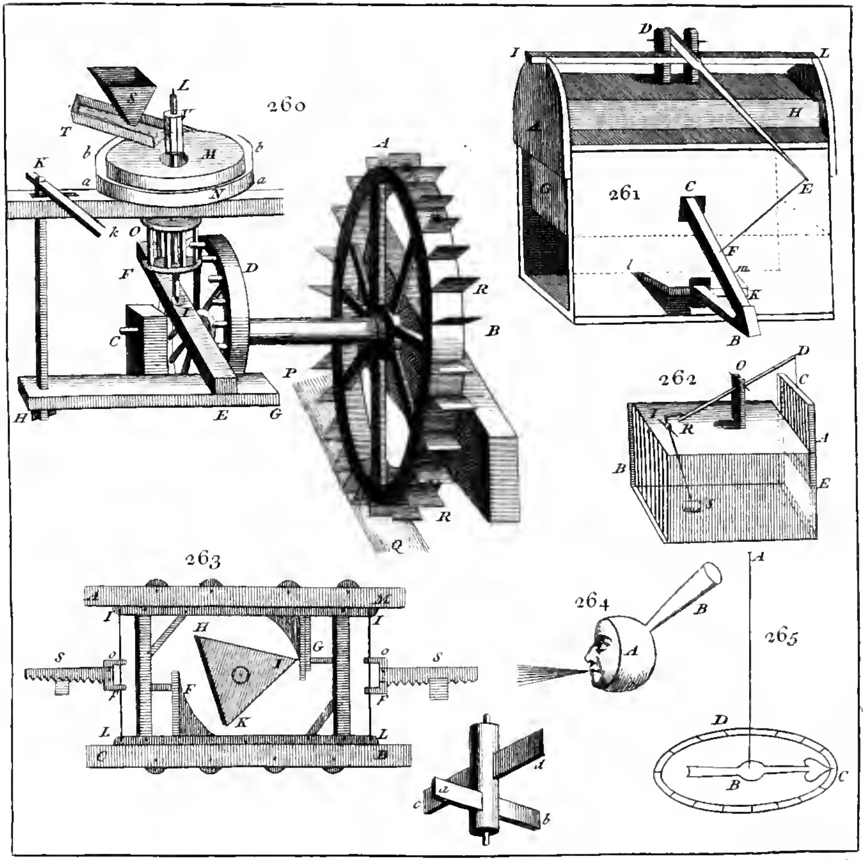

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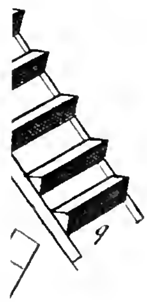

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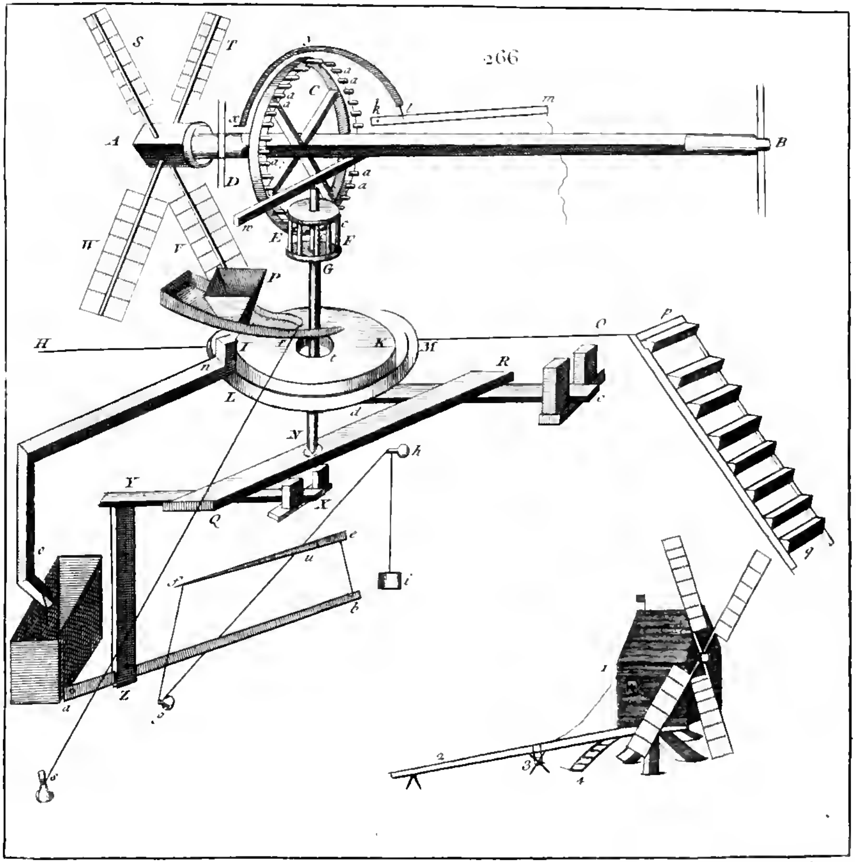

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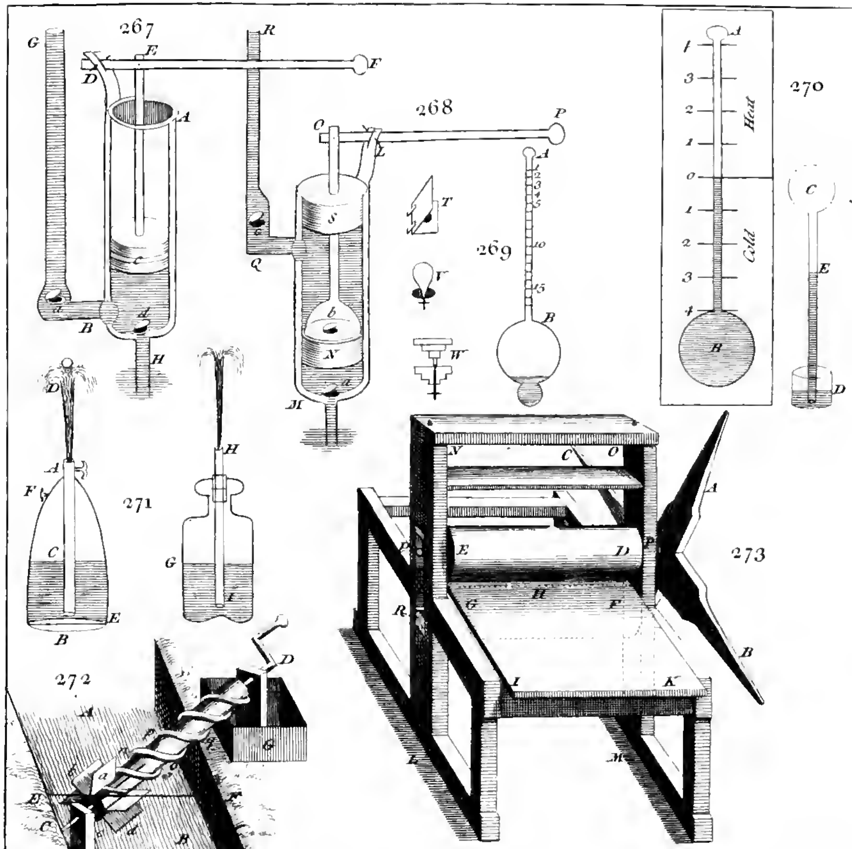

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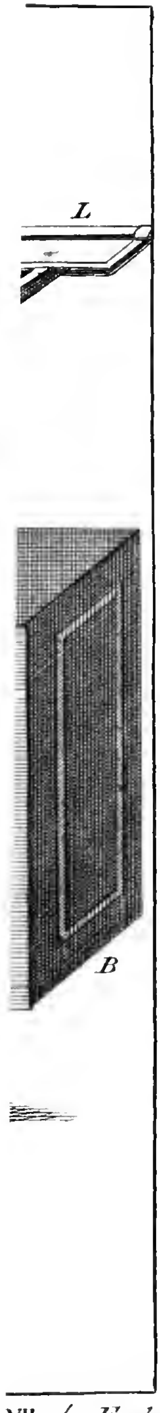

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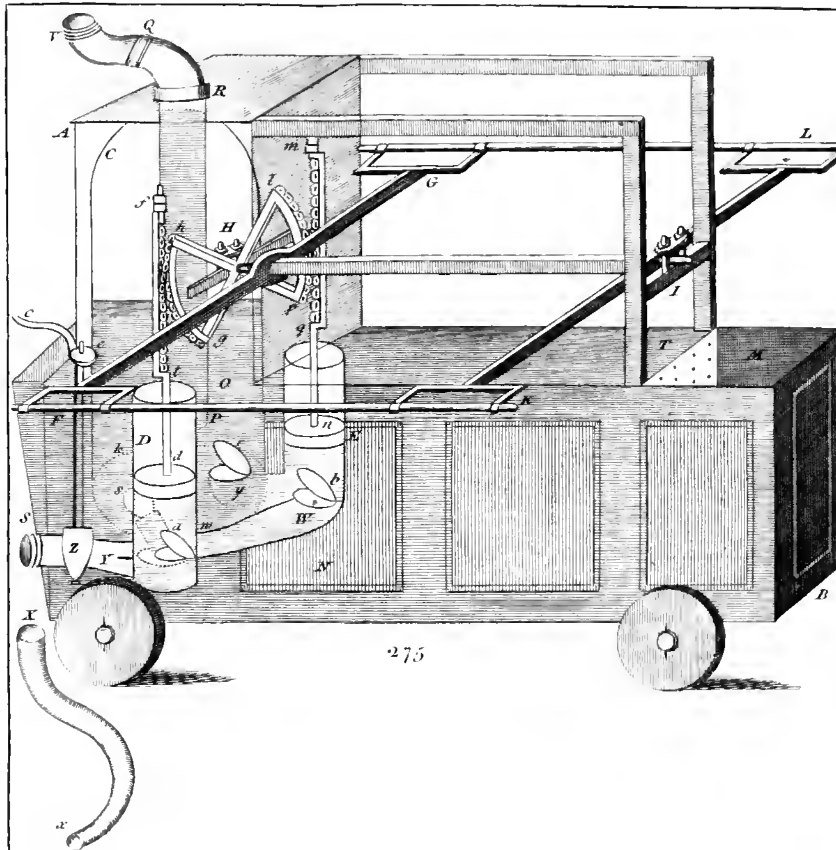

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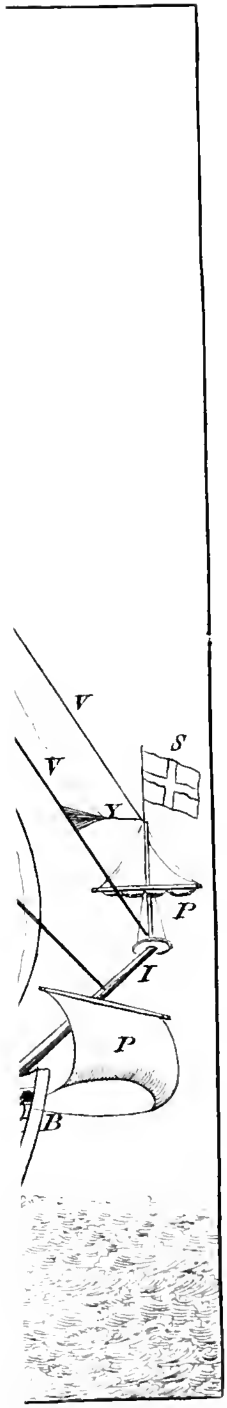

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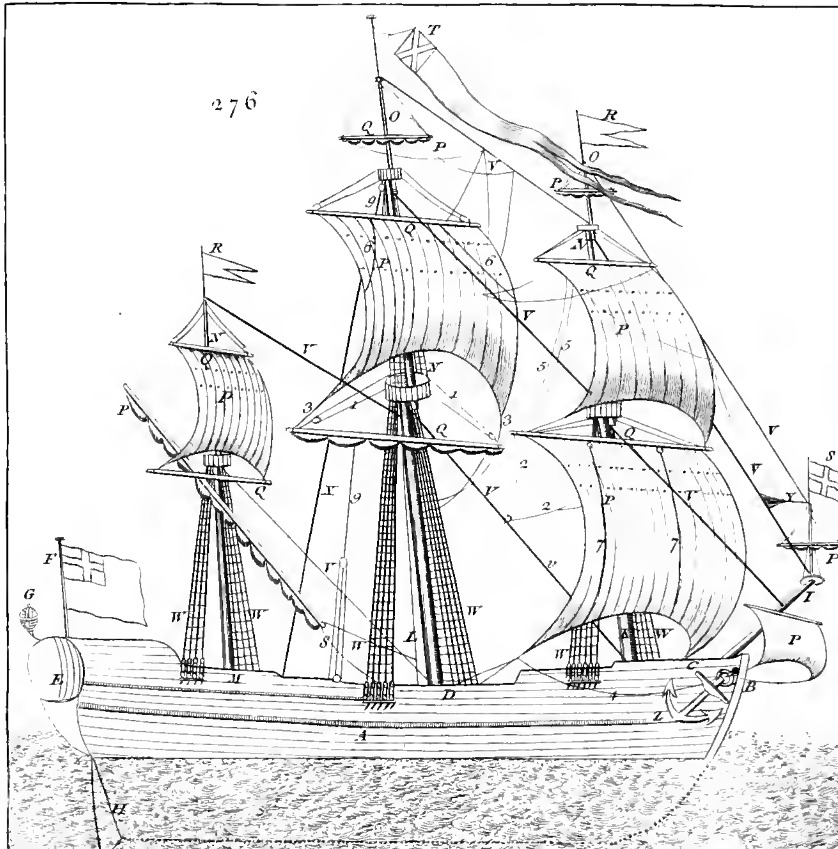

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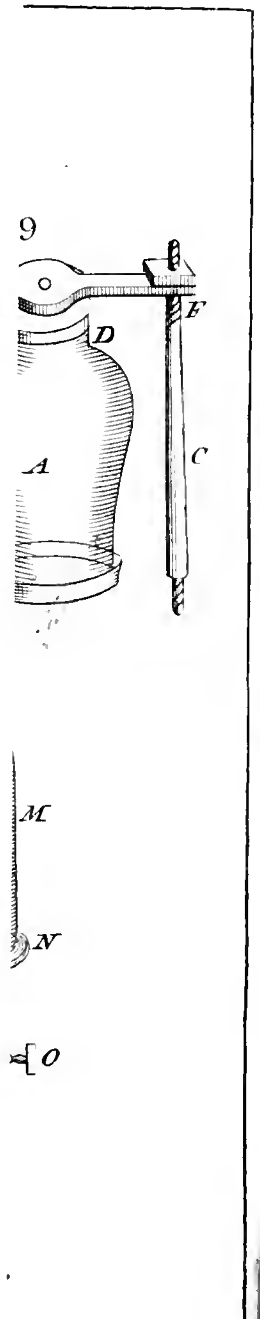

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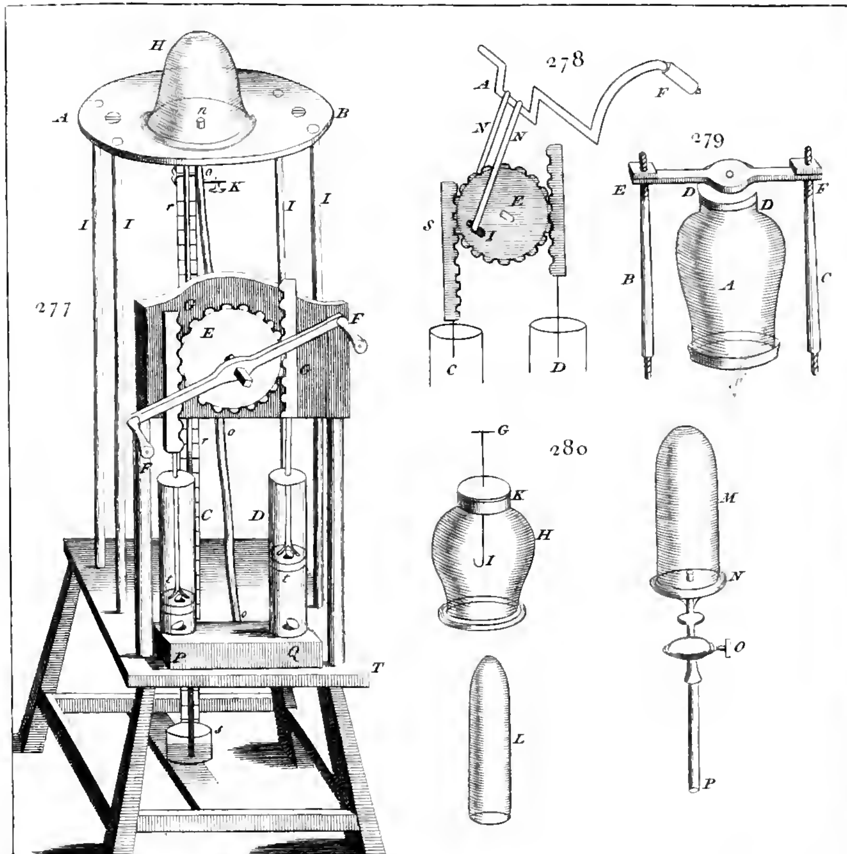

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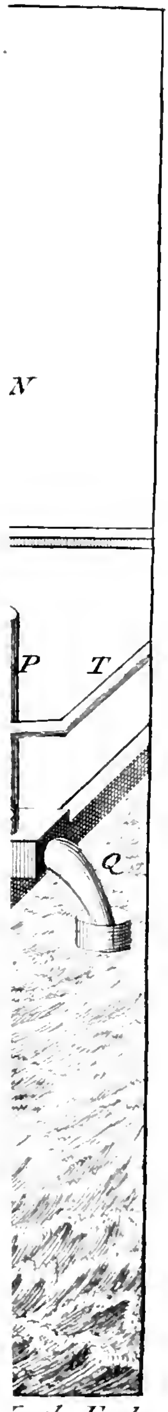

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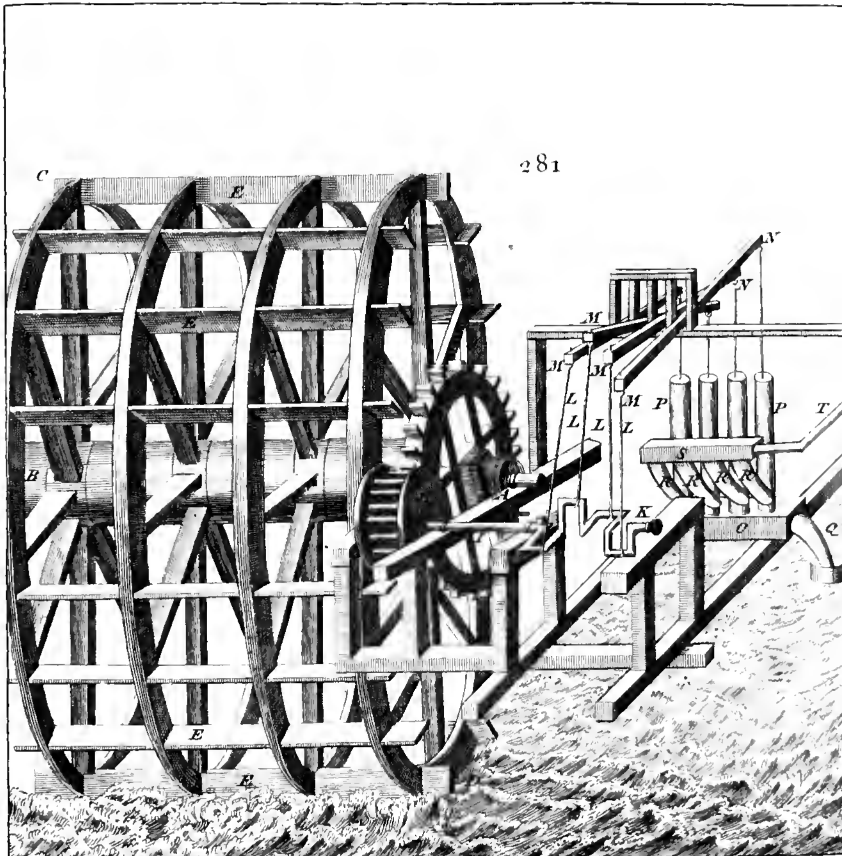

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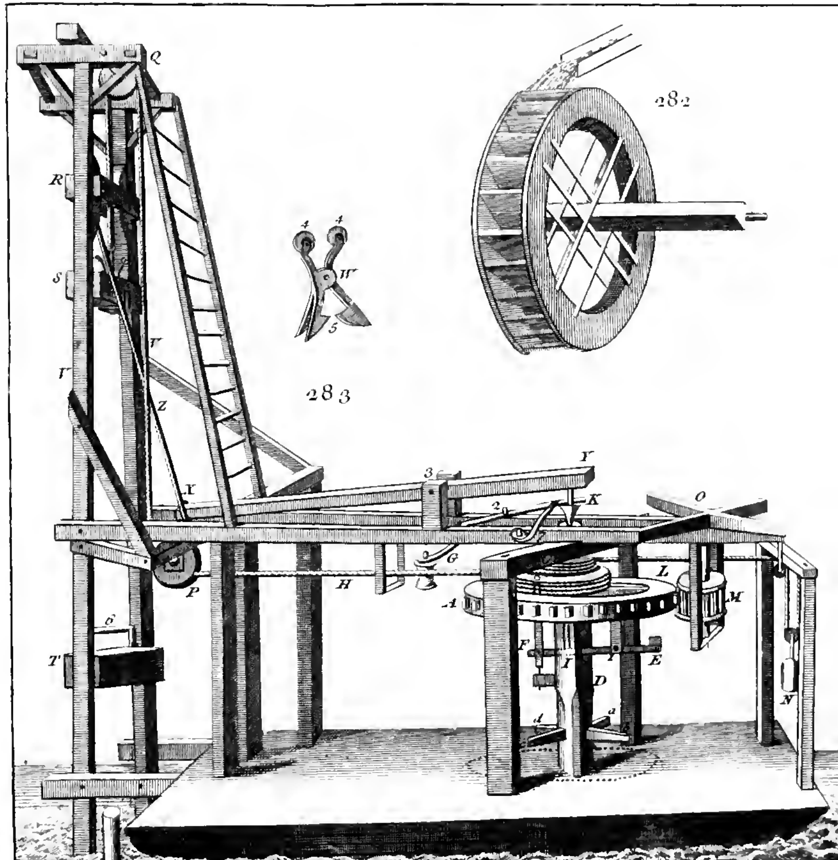

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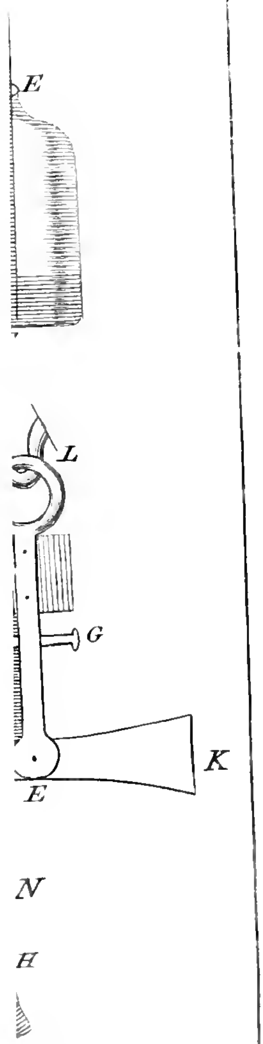

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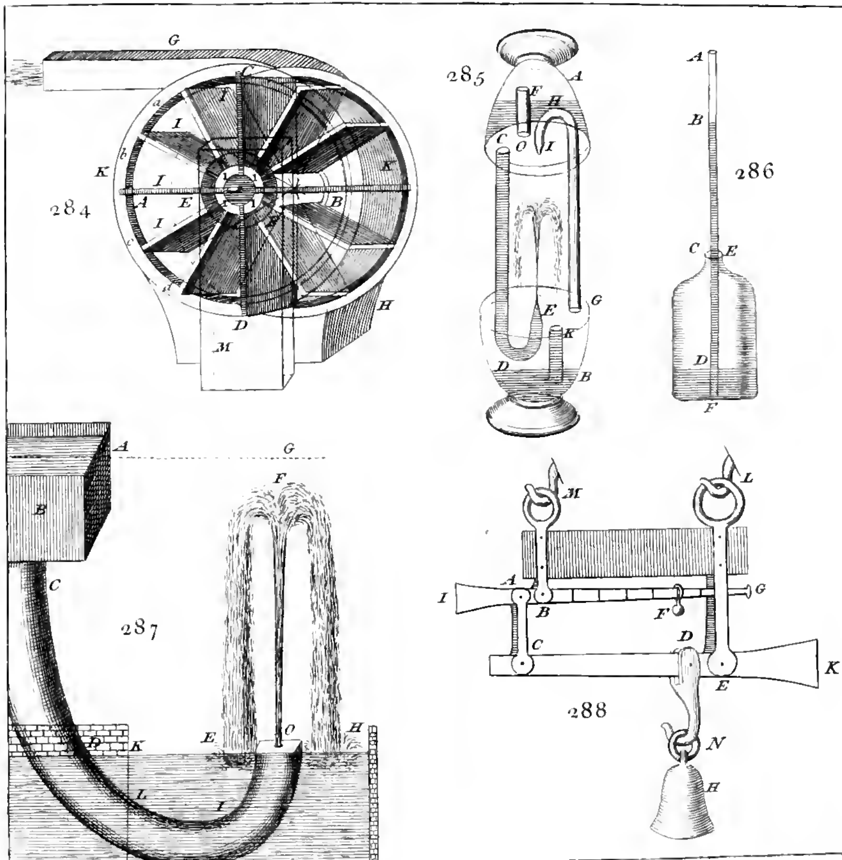

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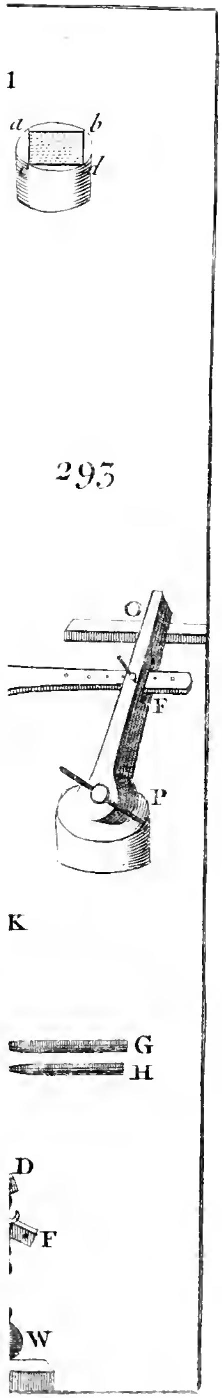

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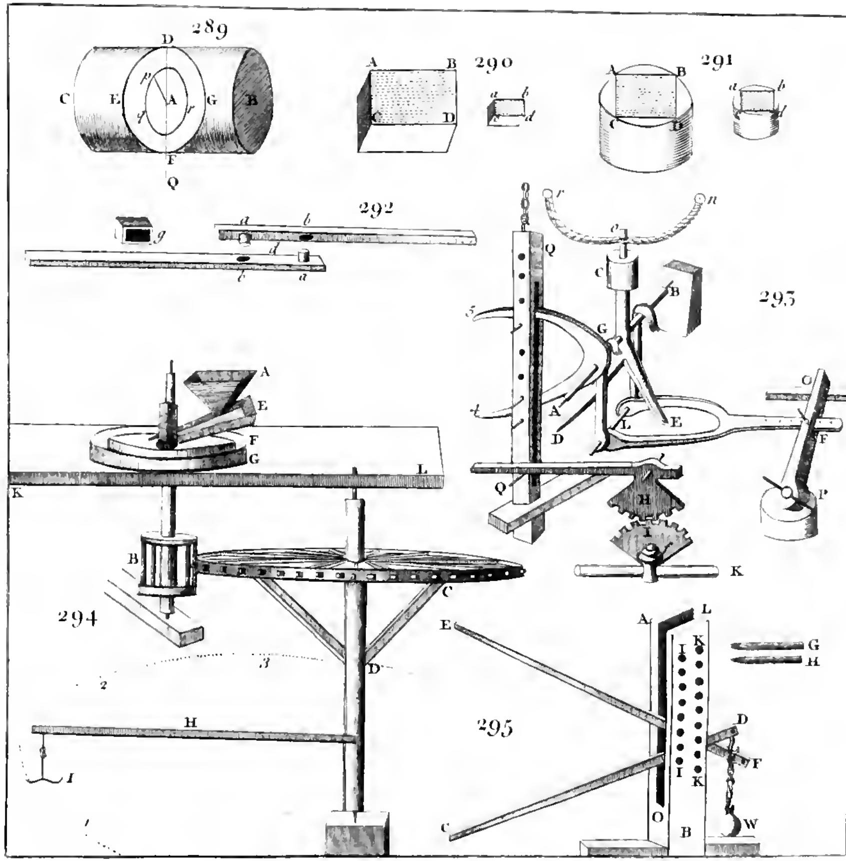

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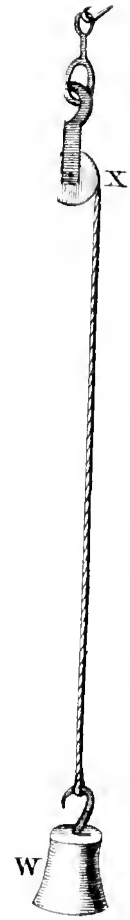

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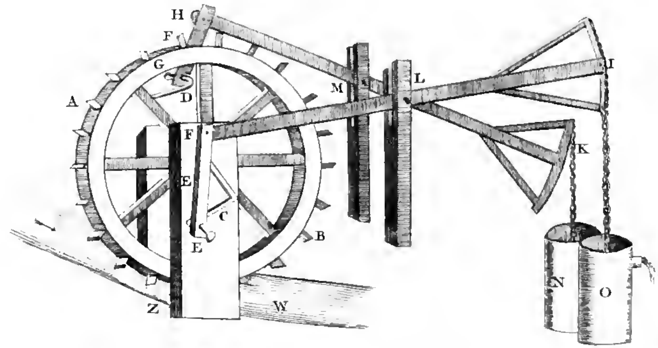

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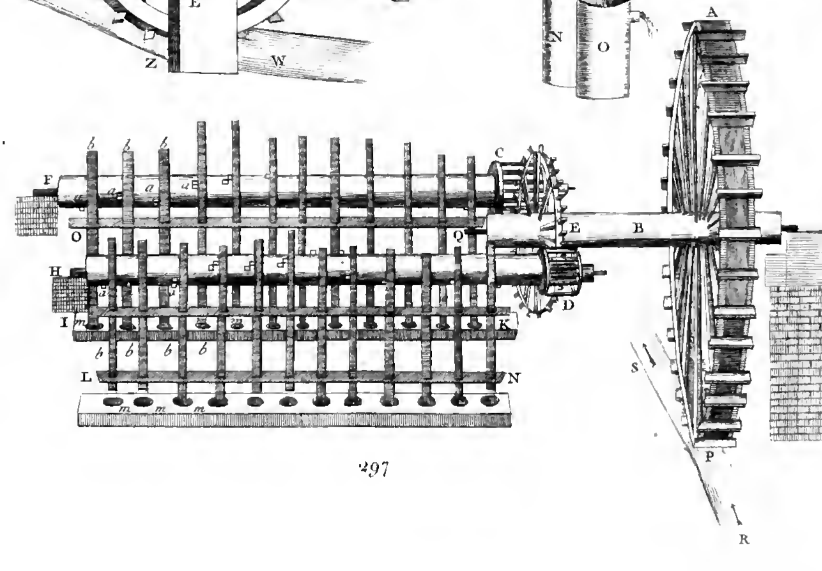

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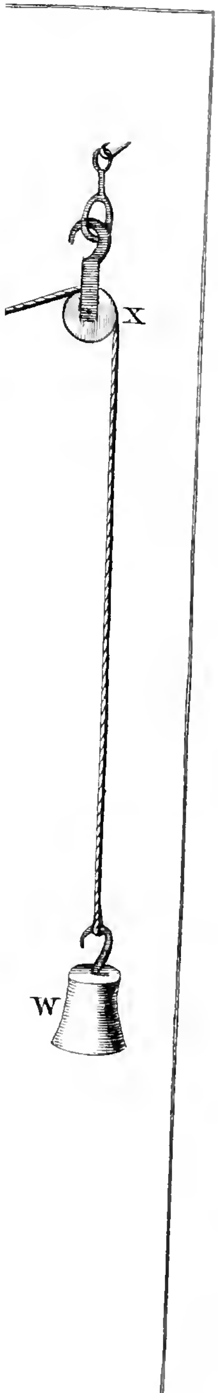
Pl. XXXIV *the En.*



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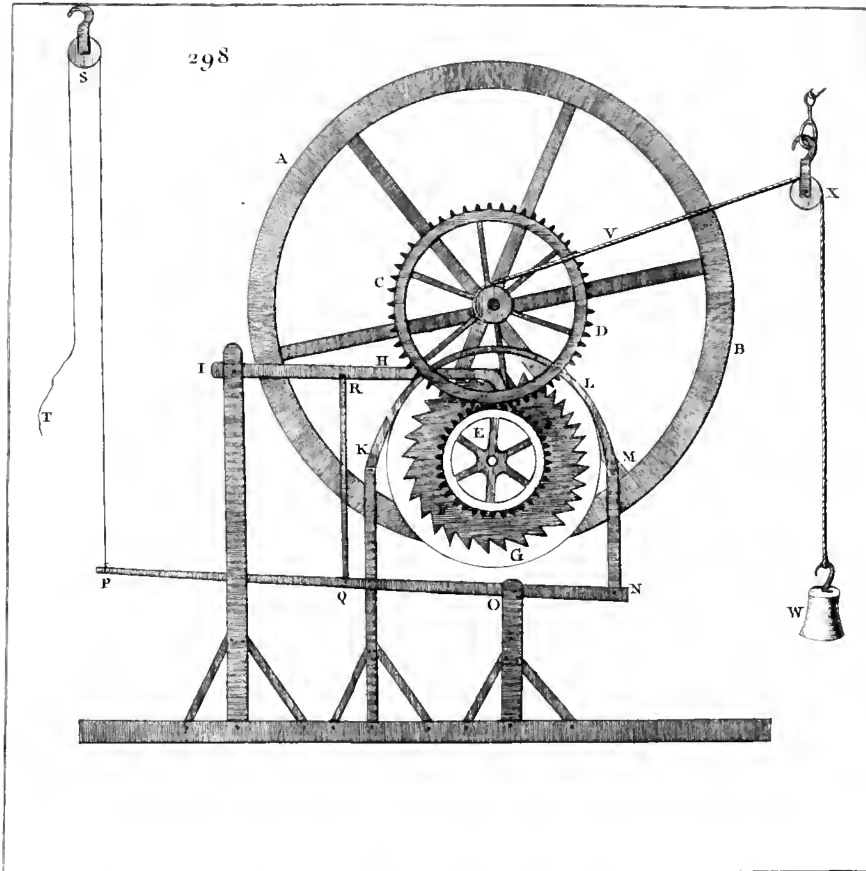

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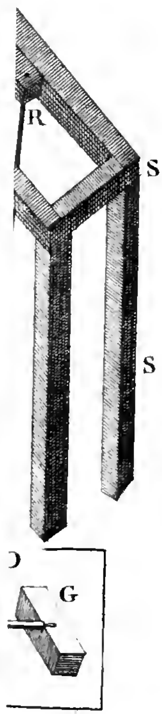

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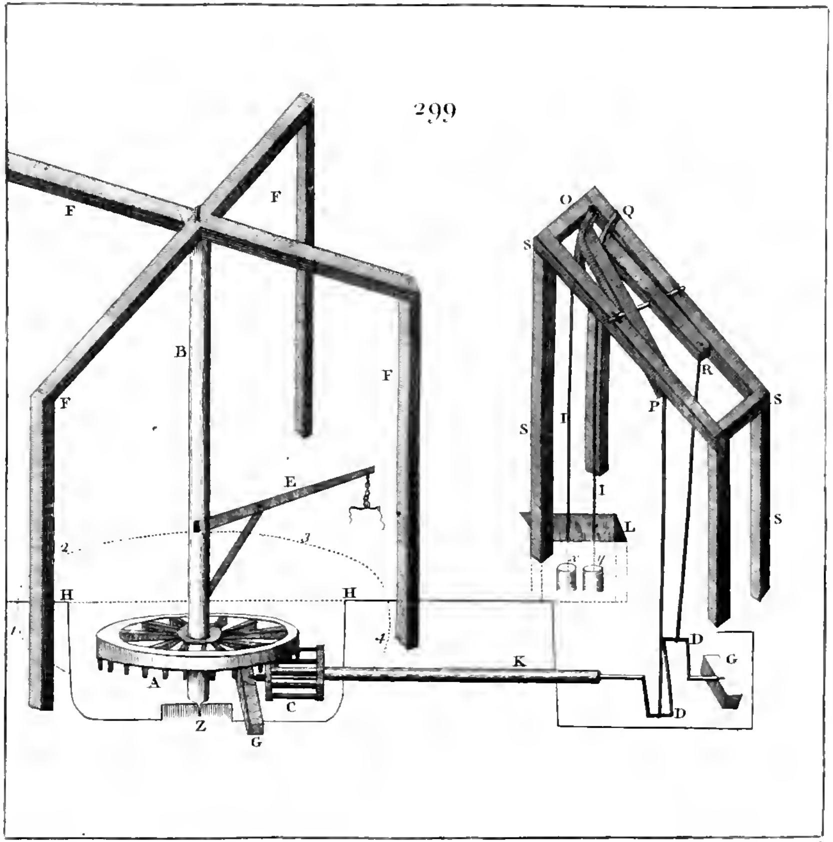

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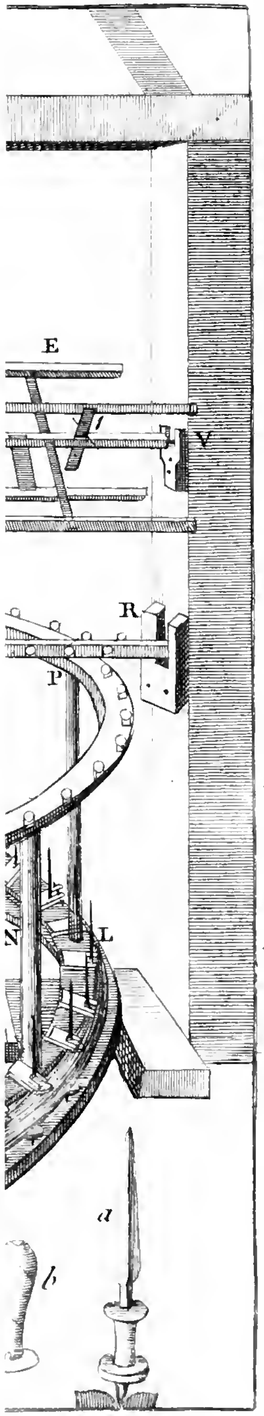

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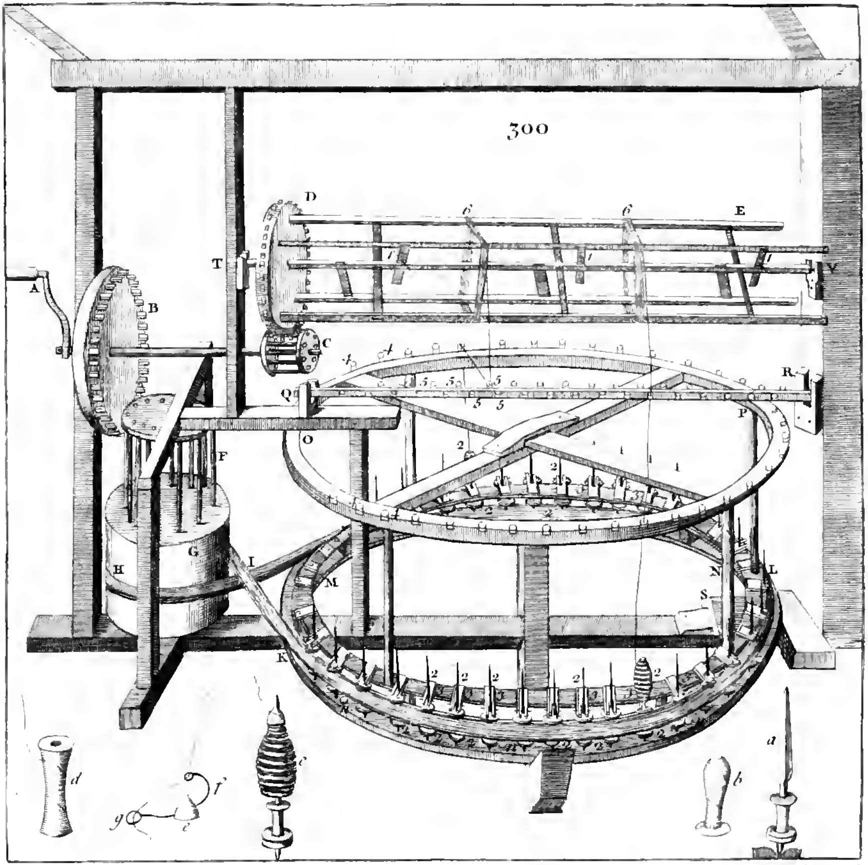

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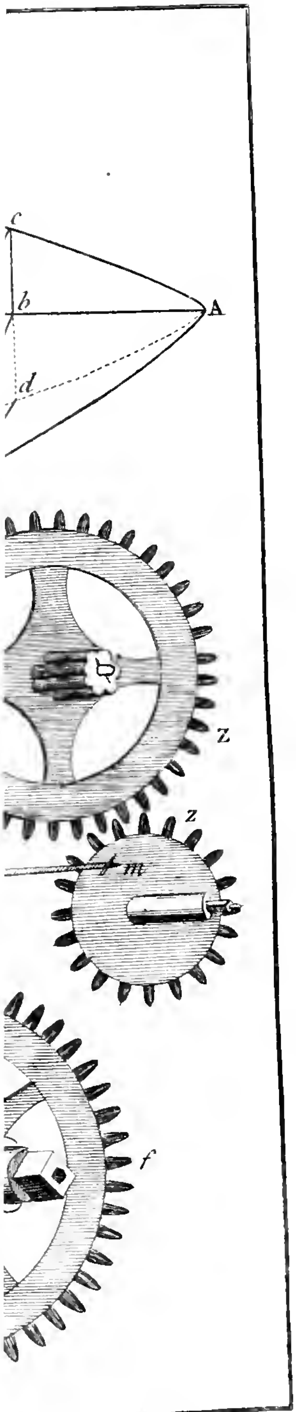

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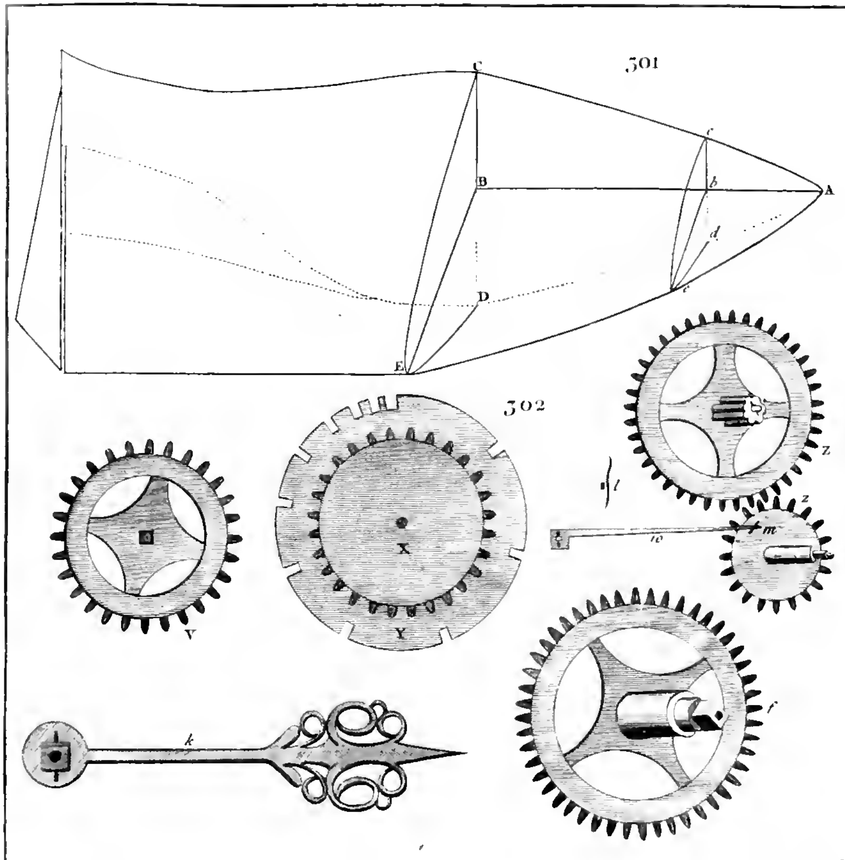

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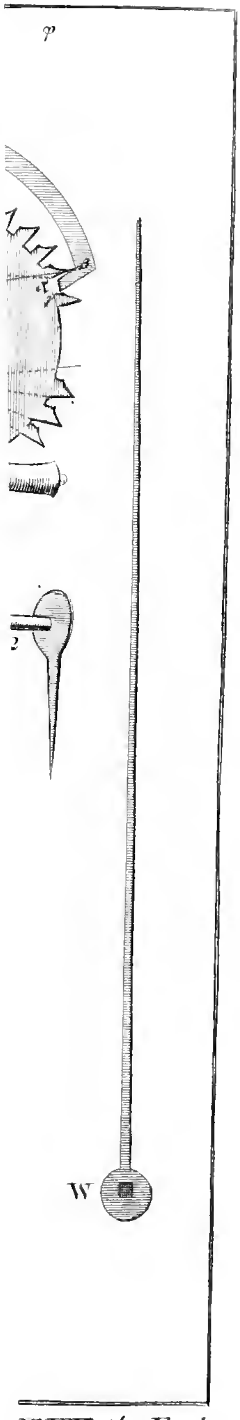

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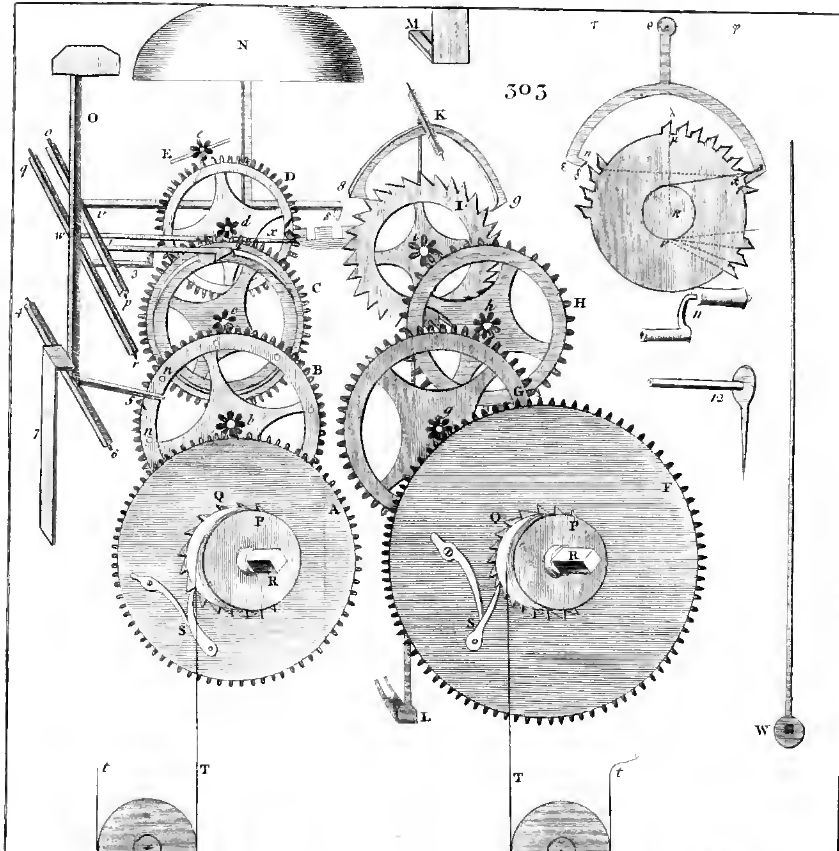

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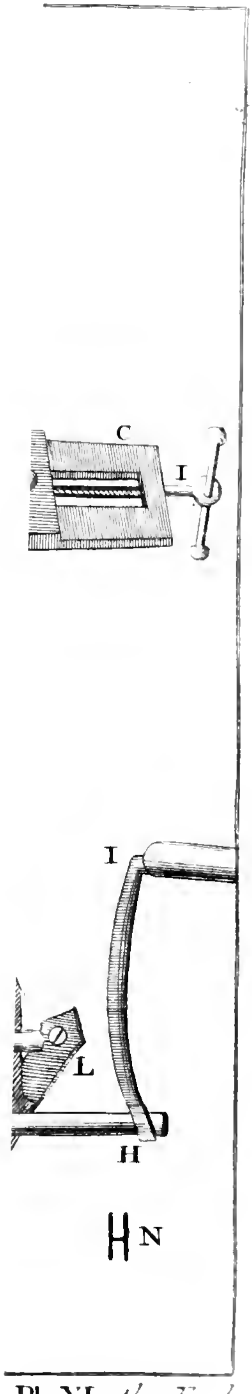

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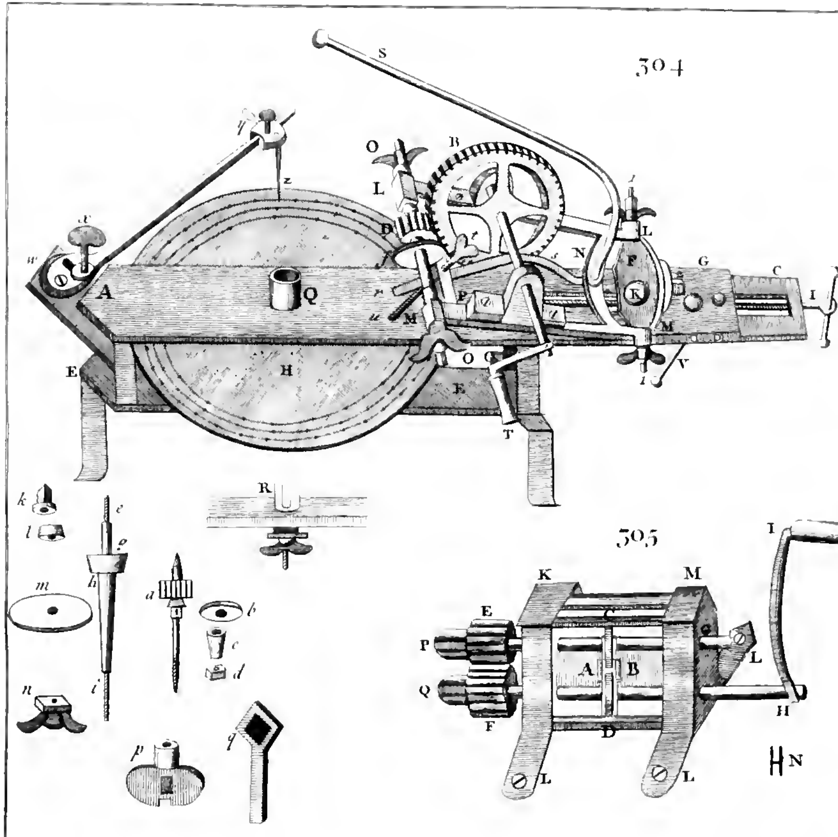

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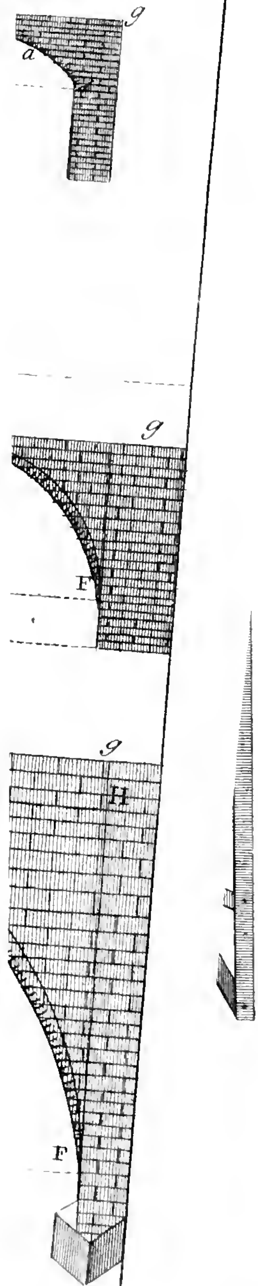

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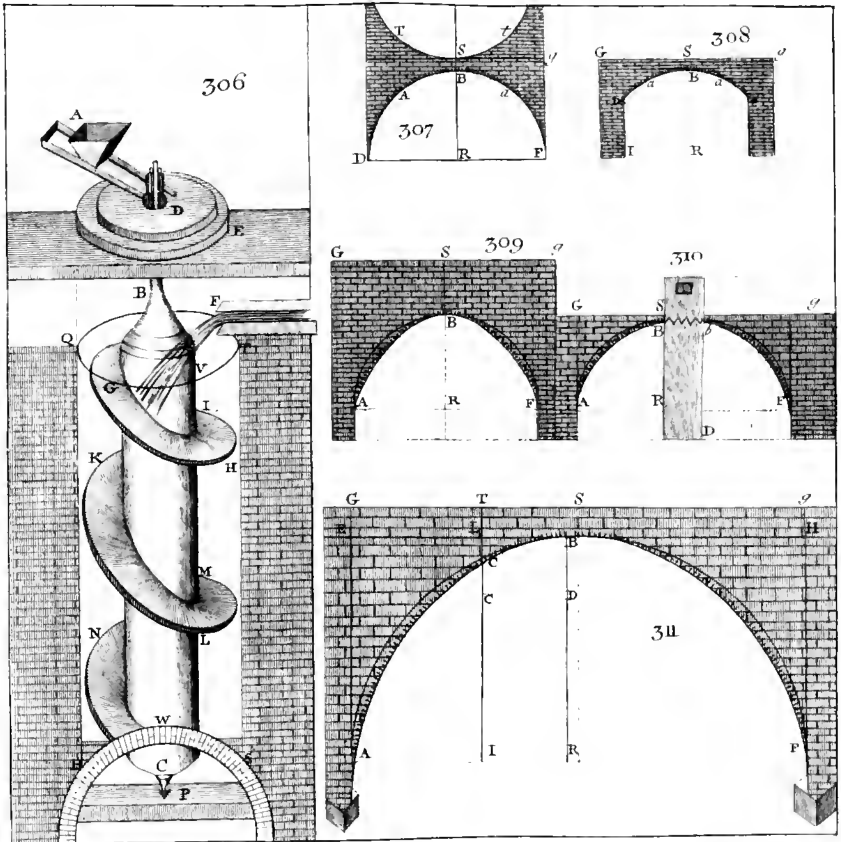

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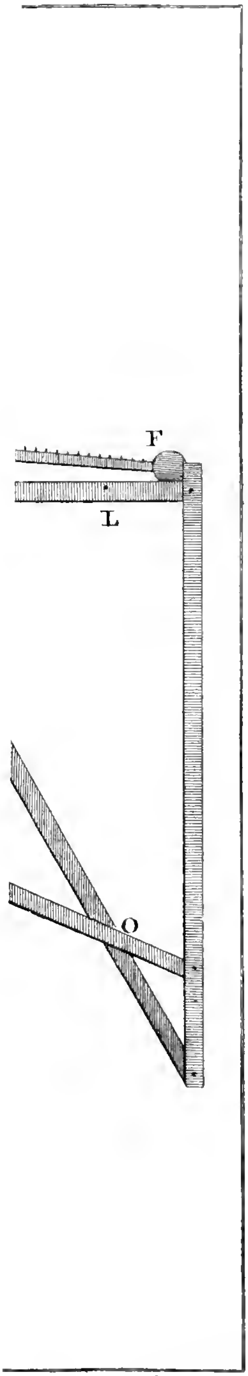

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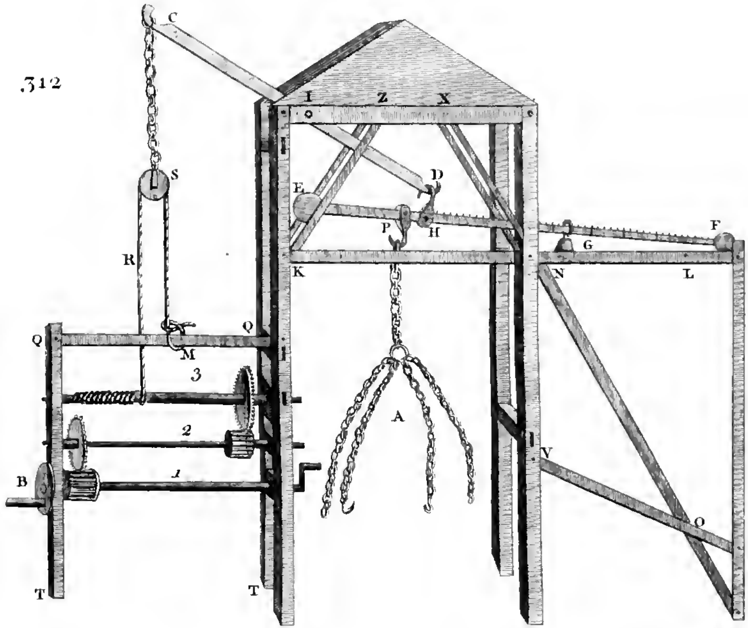

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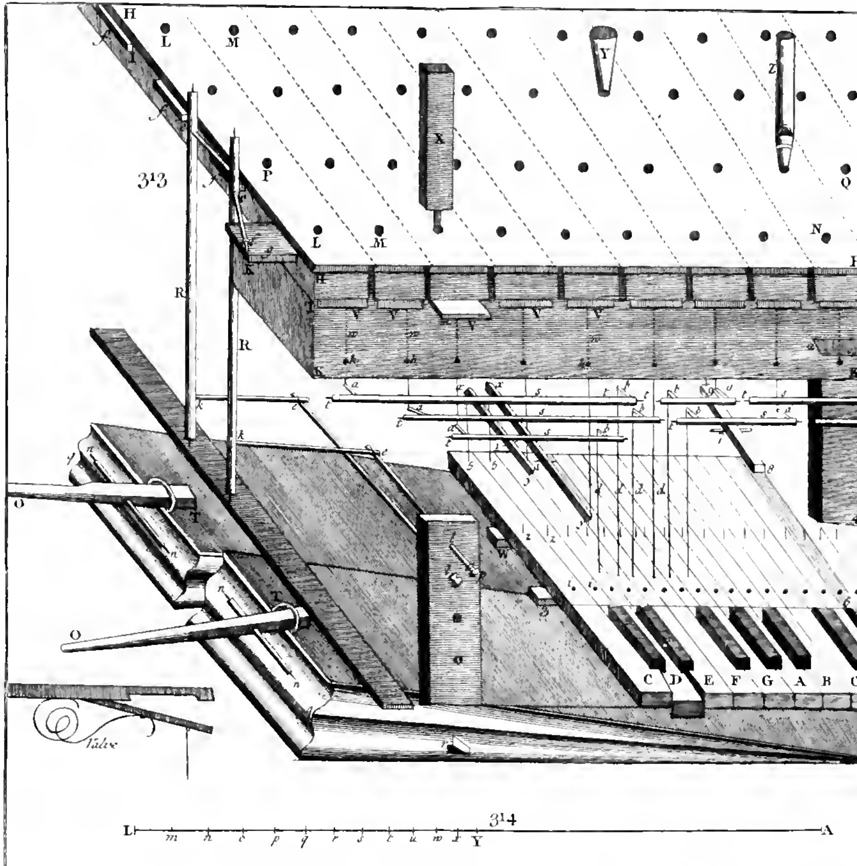

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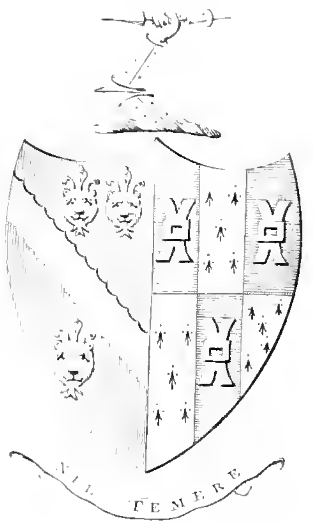

George Tennyson